UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE MA1 - 104

TITLE	Matrix Multiplication Floating Decimal Auxiliary
TYPE	Floating Decimal Auxiliary; uses interpretive orders and control registers 0, 1, and 2.
NUMBER OF WORDS	21 without punchout -- 26 with punchout
DURATION	Without punchout, 9 + 3n (2p + 2m + 11) + 10mp millisec. plus input of elements. With punchout, 9 + 3n (2p + 2m + 11) + 19mp milliseconds plus input and output time.
ACCURACY	Usually 9 decimals, but depends on condition of matrices i.e. inversely proportional to the range of the elements in any one matrix.
TEMPORARY STORAGE	0, 1, 2, 3 mp words specified by SN, n words specified by SS, np words specified by SK
PARAMETERS	S3 - SJ

3 = (Address of Floating Accumulator) x 2^{-39}

4 = (address of Code A1) x 2^{-39}

5 = np x 2^{-39}

6 = n x 2^{-39}

7 = p x 2^{-39}

8 = mp x 2^{-39}

9 = e x 2^{-39}

K = a_{00} x 2^{-39}

S = b_{io} x 2^{-39}

N = c_{00} x 2^{-39}

Where: B is an (m by n) matrix, A is an (n by p) matrix, and C = BA is an (m by p) matrix. a_{00} is the location of the first element of A. b_{io} is the location of the first element of any row i of B. c_{00} is the location of the first element of C = BA. e is the location of the left hand order to which control is transferred upon completion of this routine. If punch out is desired, set e = 21L in this routine.

MAXIMUM MATRICES	Square matrices maximum m = n = p = 20. Always, mp + n + np $\leq$ 825
DESCRIPTION	This auxiliary routine performs matrix multiplication of two matrices B = m by n matrix and A = n by p matrix to obtain BA = C = m by p matrix in floating decimal form. The elements of C are stored in row form into successive locations beginning with SN. The routine is entered by an interpretive 8J order or by an immediate 26 or 24 xN directive.

METHOD

The method used is the same as that of the regular matrix multiplication routine, Code M1 except that no terminating symbol proceeds each row.

PUNCHING AND SCALING

To insure that no element c_{ik} of C exceeds 10^{64}, $|a_{jk}|$ and $|b_{ij}|$ should be $\leq \sqrt{10/n} \times 10^{32}$. The elements of A and B are punched in row form for regular floating decimal input. The elements of A are punched first and then the elements of B. When started, the routine inputs all of A and then inputs B one row at a time.

NOTE

(1) If the elements in any one matrix vary over an excessive exponential range (> 10) the subproducts $(b_{io}a_{jk})$ may induce an error. e.g. If subproducts of an element c_{ik} are 10^{-20}, $+ 10^{10}$, $+ 10^{15}$, $- 10^{10}$, $- 10^{15}$, $- 10^{20}$, in that order, c_{ik} will be represented by -10^{-20} since adding first 10^{-20} $+ 10^{10}$ gives 10^{10} in the machine, and by adding $10^{15} - 10^{10}$ -10^{15} to this we get 0. Then adding $0 - 10^{-20}$ gives 10^{-20} due to the range of the exponents exceeding 10.

(2) If an element of A or B is represented by $1/10 \times 10^{p}$, it must be remembered that 1/10 has only 28 significant binary digits. Hence, for example, if subproducts of an element c_{ik} of C are (1/10) (4/10) x 10^{p}, we may not get zero since the two terms in this sum will not be represented in the same way in the machine. The size of the error induced in such a case is proportional to the size of p.

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CODED BY	Bessie Cobb
APPROVED BY	J.P.Nash

LOCATION	ORDER		NOTES
0	41 3F		Clear 3 for storage
	50 L		
1	26 S4		Enter interpretive code
	0K S5		Set loop to input np elements of A.
2	88 F	From 3	Input a_{jk} and store
	0S (SK)	by 3	
3	03 2L	Yes	Is A input?
	1K S8		Set loop for counting elements of C.
4	0K S6	From 13'	Set loop to input elements of B
	88 F	From 5'	Input row i of B and store.
5	0S (SS)		
	02 4L	Yes	Is row i of B input?
6	2K S7		Set loop for p Σ s using row i of B.
	0K S6	From 12'	Set loop for n products to sum.
7	05 (SS)	From 9'	b_{ij}
	87 (SK)	By 15	$(b_{ij}a_{jk})$
8	84 3F		
	8S 3F		$\Sigma b_{ij}a_{jk} = N(3)$
9	8J 14L		Escape to 14L
	03 7L	Yes	Is $\Sigma b_{ij}a_{jk}$ complete for column k of A?
10	85 3F		
	1S (SN)		Store c_{ik}
11	8J 16L		Escape to 16L
	12 12L	Yes	Has C been computed?
12	8J S9	Yes	Escape when finished
	22 6L	From 11'	Have p sums been formed using row i of B?
13	8J 19L	Yes	Then escape to 19L
	83 4L		
14	L5 7L	From 9	Increase address of a_{jk} by p
	L4 18L		
15	42 7L	From 18,20	
	26 29S4		Reenter interpretive code
16	41 3F	From 11	Clear 3 for storage
	L5 7L		

LOCATION	ORDER		NOTES PAGE 2 MA1
17	L0 1L		Reset address of a_{jk} for next summation
	L4 43S4		
18	26 15L		
	00 S7		Address used
19	41 S3	From 13	Clear S3 for 83 order.
	L5 20L		
20	26 15L		
	00 SK		Address used.
21	32 21L	From 12	For Print out
	50 21L		
22	26 S4		Enter Code A1
	0K S8		Set cycle for mp elements of C.
23	05 (SN)	From 24	c_{ik}
	89 9F	By 24	Print c_{ik} to 9 decimals
24	03 23L	Yes	Have mp elements of C been printed?
	8J 25L		Escape to stop.
25	0F F	From 24'	
	0F F		