

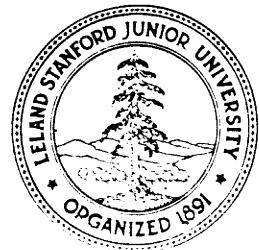
SORTING AND SEARCHING - ERRATA AND ADDENDA

BY

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Abstract

This report lists all the typographical errors, in The Art of Computer Programming / Volume 3, that are presently known to its author. Several recent developments and references to the literature, which will be incorporated in the second printing, are also included in an attempt to keep the book up-to-date. Several dozen corrections to the second (1971) printing of volume two are also included.

Since a reward of \$1.00 is paid to the first finder of each error, I hope this list will curtail the number of people finding the same mistakes.

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SORTING AND SEARCHING - ERRATA AND ADDENDA

Volume 3 (all known errors and corrections as of October 1973):

Page vi, line 21, that long word ends with "rad" not "rad" (the Norwegian-Danish letter ).

Page ix, change layout of the box as in the second edition of volume one.

Page xii, raise this illustration about half an inch, so it just covers the running headline (which is just barely visible from contents page xi).

Page 5, line 25, grows with  $\underline{N}$   $\rightarrow$  must grow at least as fast as  $\log \underline{N}$

Page 5, line -2,  $\mathcal{R}_{p(i_{j-1})+1} \rightarrow \mathcal{R}_{p(i_{j-1}+1)}$

Page 8L, line 23, siècle

Page 10, line 5, insert PRESA, after PRASE,

Page 23, line 10, put a dot under the z in Yezirah.

Page 24, Eq. (6) should be  $(\alpha_{\mathbf{T}\beta})_{\mathbf{T}}^{\gamma} = \alpha_{\mathbf{T}}(\beta_{\mathbf{T}}^{\gamma})$ , (6)

Page 33, line 5, change  $(1-z)(1-z^2)\dots(1-z^m)$  to  $\prod_{1 \leq k \leq n} (1 + z + \dots + z^{k-1})$

Page 61, last line, "Wissenschaften"  $\rightarrow$  "Wissenschaften"

Page 76, line 16,  $1 \leq j \leq i \rightarrow 1 \leq j < i$

Page 79, line -5, D4  $\rightarrow$  C4

Page 91, last line of Lemma L, "(7)" should not be in italics.

Page 96, Eq. (9), change  $1 \leq i \leq N$  to  $1 \leq i < N$

Page 102, line -3, in step D3  $\rightarrow$  when we begin step D3

Page 109, line 6, 1030u  $\rightarrow$  960u

Page 122, line -8, !  $\rightarrow$  .

Page 129, lines 5-6, change "it requires . . . comparisons\*" to: it is slightly slower than quicksort. Its running time appears to be difficult to analyze; in fact,

Page 137, line -1, change the rating to M46.

Page 146, line 10, N should be in italics.

Page 149, line 8, change 'had' to 'hard'.

Page 152, at bottom, add new paragraph:

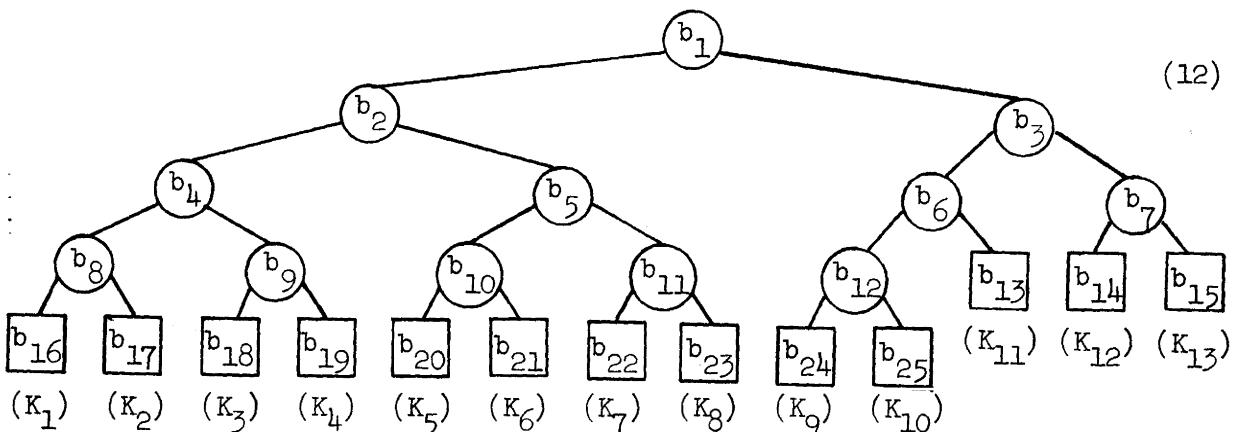
See Section 5.4.1 for another way to handle priority queues, when the number of elements in the queue stays more or less constant.

Page 153. Move Fig. 28 to page 154 and put the following copy on page 153:

A pleasant special case. If all the elements of a priority queue are known to be contained in some fixed set  $\{K_1, K_2, \dots, K_N\}$ , where  $K_1 < K_2 < \dots < K_N$ , there is a much simpler way to achieve an efficient representation. In fact, only  $2N-1$  bits of memory are required for this scheme, and the operations are extremely easy to program.

The idea is to use the complete binary tree with N external nodes, with one bit  $b_j$  of memory in each node for  $1 \leq j \leq 2N-1$ .

For example, when  $N = 13$  we have



The external nodes are implicitly associated with the keys in increasing order, from left to right; e.g.,  $b_{18}$  is associated with  $K_3$ , and

$b_{13}$  with  $K_{11}$ . By assumption, the contents of the priority queue is some subset  $S$  of  $\{K_1, \dots, K_N\}$ . We represent this by setting  $b_j = 1$  if and only if one of the associated keys below it is present. Thus for example if  $S = \{K_3, K_7, K_8\}$ , we would have  $b_1 = b_2 = b_4 = b_5 = b_9 = b_{11} = b_{18} = b_{22} = b_{23} = 1$ , and the other  $b_j$  would be 0. In effect we have a tournament like Fig. 23, with 0 in place of  $-\infty$ .

It is easy to see how to find the largest element of  $S$ , in order  $\log N$  steps: First set  $j \leftarrow 1$ , then repeatedly set  $j \leftarrow 2j + b_{2j+1}$  until  $j > N$ ; then  $b_j$  will be the external node associated with the desired key. (If  $S$  might be empty, we should also test that  $b_j = 1$  at the conclusion of the algorithm. Note that  $b_1$  is never needed, and we could omit it.)

It is just as easy to find the smallest element of  $S$ , or to insert or delete elements. All these operations will take order  $\log N$  steps. (Note that this is the logarithm of the maximum size of the priority queue, not of the actual size; however, the difference may not be crucial.)

The above method of priority queue representation is due to Luther C. Abel [Ph.D. thesis (Univ. of Illinois, 1972), 106-109].

[Now move seven lines from page 154 to page 153.]

Page 154, delete '(12)', leaving that equation unnumbered.

Move Fig. 28 to the top of page 154, tipped rightside up and reduced to about 11 lines. Also move two lines from page 154 to page 155.

Page 159, new exercise:

37. [20] For priority queues represented by Abel's bit method as in (12), design algorithms to (a) find the smallest; (b) insert or (c) delete the element associated with  $b_j$ , given the value of  $j \geq N$ .

Page 164, step S12, change "Return to step S3." to "If  $j-i < p$ , return to step S10, otherwise return to S3."

Page 180, move the quotation to the bottom of the page.

Page 205, first line of Table 2, change 051 to 061.

Page 215, add a new sentence to the paragraph preceding Table 1:

On the other hand, Frances Yao has shown [Ph.D. thesis, M.I.T. (1973)] that (11) is never worse than 2 comparisons from the optimum, when  $t = 3$ ; in fact, equality holds for infinitely many  $n$ .

Page 216, line 9, Tarjan:  $\rightarrow$  Tarjan [J. Computer and System Sci. 7 (1973), 448-461]:

Page 217, replace lines 14-16 by the following new paragraph:

Theorem L shows that selection can always be done in "linear time", i.e., that  $V_t(n)/n$  is bounded. Of course, the method used in this proof is rather crude since it throws away good information in Step 4. Deeper study of the problem has led to much sharper bounds; for example, A. Schönhage [to appear] has shown that the asymptotic maximum number of comparisons required to find the median is at most  $3n$ , and V. Pratt [to appear] has shown that it is at least  $1.75n$ .

Page 217, line 23, report 113  $\rightarrow$  reports 113 and 114

Page 217, delete (13) and the line of text between (12) and (13).

Page 218, change (14) to number (13), and replace the following sentence by:

Using another approach, based on a generalization of one of Sobel's constructions for  $t = 2$ , David W. Matula [to appear] has shown that

$$\bar{V}_t(n) \leq n + t \lceil \lg t \rceil (11 + \ln \ln n) . \quad (14)$$

Thus, for fixed  $t$  the average amount of work can be reduced to only  $n + O(\log \log n)$  comparisons. No satisfactory lower bounds for  $\bar{v}_t(n)$  have yet been obtained.

Page 219, line 11, (3,2,4,0,1)  $\rightarrow$  (3,2,5,0,1)

Page 219, change exercise numbers

from 10 to 12 (and move this exercise down to the proper place)  
from 11 to 10  
from 12 to 11.

(This is necessary to agree with the answers on pages 635-636.)

Page 229, lines 18-20, replace this paragraph by:

The best sorting networks currently known as  $n \rightarrow \infty$  were constructed by R. L. Drysdale III in his undergraduate honors project at Knox College in 1973. His remarkable networks require  $\sim \frac{1}{4} n(\lg n)^2 - \frac{371}{960} n \lg n$  comparators; in particular his construction yields  $\hat{S}(256) \leq 3657$ , compared to Batcher's 3839.

Page 235, bottom, add new sentence to the last paragraph:

Andrew Yao has determined the asymptotic behavior of  $\hat{U}_t(n)$  for fixed  $t$ , by showing that  $\hat{U}_t(n) = 2n + \lg n + O(1)$  and  $\hat{U}_t(n) = n \lceil \lg(t+1) \rceil + O((\log n)^{\lfloor \lg t \rfloor})$  as  $n \rightarrow \infty$  [to appear].

Page 246, Fig. 61, dots on the i's.

Page 259, line 6, insert "about" before 2.3P and "and about" before 2.5~.

Page 259, line 11, change "Wong." to "Wong [CACM 15 (1972), 910-913]."

Page 259, line -7, the plow) gets up to  $P' = P$  units.

Page 279, line -12, change ", to appear]." to: , 1, 454-459].

Page 299, line 15, change "to appear]." to: 1, 454-459].

Page 373, line 15, add a new sentence: Furthermore the asymptotic value of  $A_1(n)$  can be worked out as shown in exercise 9.

Page 377, change exercises 9, 10 to:

9. [HM39] (L. Hyafil, F. Prusker, J. Vuillemin.) Prove that, for fixed  $\alpha$  and  $\beta$ ,

$$A_1(n) = \left( \min_{m \geq 2} \frac{\alpha m + \beta}{\log m} \right) n \log n + O(n)$$

as  $n \rightarrow \infty$ , where the  $O(n)$  term is  $> 0$ .

10. [HM44] (L. Hyafil, F. Prusker, J. Vuillemin.) Prove that when  $\alpha$  and  $\beta$  are fixed,  $A_1(n) = \alpha mn + \beta n + A_m(n)$  for all sufficiently large  $n$ , if  $m$  minimizes the coefficient in exercise 9.

Page 381, under "Median-of-3" method, average running time, change  $1.89N$  to  $1.82N$ .

For  $N = 1000$ , change 81485 to 81431.

Page 383, 385, 387, delete "IN" from the running headline at top.

Page 395, both **programs** line 01, BEGIN  $\rightarrow$        

Page 398, line -9, change "know" to "know what"

Page 402, move three rows from the righthand side of Table 1 to the lefthand side, and add the following six entries to the righthand side:

220	47326693
222	122164747
234	189695659
248	191912783
250	387096133
282	436273009

, Page 402, caption to Table 1, lines 2-4, change to: For further information, see L. J. Lander and T. R. Parkin, Math. Comp. 21 (1967), 483-488.

Page 404, line 3 of ex. 17, schedule for  $\rightarrow$  schedule  $a_1 a_2 \dots a_n$  for

Page 411, Eq. (5),  $-15 \rightarrow -16$  and  $+13 \rightarrow +12$

Page 419, line 6, should be

1 (1946), 9.7-9.8; 3 (1946), 22.8-22.9]. The method became "well-known," but

Page 429. The remark preceding Theorem H is false and I will have to do something to fix up this discussion, after Gary Knott's thesis is written.

Page 434, Fig. 14, replace I II IV V by V IV II I respectively.

Page 436, line 20, (the displayed inequalities relating to  $r[i,j]$ ),  
 $i \leq j \rightarrow i < j$ .

Page 438, change ; to : in the caption to Fig. 16.

Page 439, line 8, (1972), 303-323] :

Pages 440 and 444, move subscripts up to boxes.

Page 447, exercise 2, the rating is 20.

Page 452, line 24,  $20 \log_2 N \rightarrow 64 \lg N$   
line 25, greater than 1024.  $\rightarrow$  greater than 256.

Page 455, Fig. 21, the node labeled R should be filled with • not +.

Page 457, line 3, " -a R ), " should be " -a,R ) "

Page 457, delete the bottom line "2029 . . . ". In page 458, add 10 lines at the top. They are to be like lines numbered 14-23 on page 425, but they should be renumbered 20-29, and on the first line change "T5. Insert into tree." to "A5. Insert."

Page 466, line 19, add sentence to the paragraph: Rut empirical tests show that only about 0.21 rotations per deletion are actually needed, on the average.

Page 470, exercise 5, change "N keys . . . increasing order, " to:

"the keys  $K_2, \dots, K_N$  successively in increasing order into a tree which initially contains only the single key  $K_1$ , where  $K_1 < K_2 < \dots < K_N$ , then"

Page 490, line -16, add a new sentence to this paragraph: A closely-related algorithm was published at almost exactly the same time in Germany by G. Gwehenberger, Elektronische Rechenanlagen 10 (1968), 223-226.

Page 494, line -3, in a binary  $\rightarrow$  in an M-ary

Page 497, line -9, Discrete Mathematics 4 (1973), 57-63.]

Page 498, (16) ,  $2^{n-1} \rightarrow 2^{1-n}$

Page 503, align the columns of fractions ( $\frac{1}{2}$  is too far left).

Page 506, fourth instruction in Table 1, change \* to \*

Page 507, change "[The birthday paradox originated in work of" to:  
[The birthday paradox apparently originated in unpublished work of  
H. Davenport; cf. W. W. R. Ball, Math. Recreations and Essays (1939),  
45. See also

Page 509, line -6, would  $\rightarrow$  could

Page 525, line 6, change "15 (1972), to appear.]" to 16 (1973), 105-109.]"

Page 534, line 1, \_  $<$   $\rightarrow$  c

Page 540, line 13, change 'output' to 'traverse'.

Page 546, line -3, change "hasing" to "hashing".

Page 549, new exercise:

66. [25] (Ole Amble.) Is it possible to insert keys into an open hash table making use also of their numerical or alphabetic order, so that a search with Algorithm L or Algorithm D is known to be unsuccessful whenever a key smaller than the search argument is encountered?

Page 549, flush with bottom, add quote:

RASH, x. There is no definition  
for this word --  
nobody knows what hash is.

AMBROSE BIERCE, The Devil's Dictionary (1906)

Page 555, Fig. 45, in the node for Davenport IA change 508 to 808 .

Page 562, line 11, change  $b_{m-1}$  to  $b_{M-1}$  .

Page 562, line -12,  $(\frac{16}{9}) \rightarrow (\frac{6}{16})$

Page 562, line -5,  $b_0 b_1 \dots b_1 \rightarrow b_0 b_1 \dots b_{15}$

Page 563, line 8, after "1971." add a new sentence and then a new paragraph:

[See his Ph.D. thesis (Stanford University, 1973).]

Suppose first that we wish to construct a 'crossword puzzle dictionary' for all six-letter words of English; a typical query asks for all words of the form N\*\*D\*E, say, and gets the reply {NEEDLE, NODDLE, NOODLE}. We can solve this problem nicely by keeping  $2^{12}$  lists, putting the word NEEDLE into list number

$$h(\underline{N})h(\underline{E})h(\underline{E})h(\underline{D})h(\underline{L})h(\underline{E}) .$$

Here  $h$  is a hash function taking each letter into a 2-bit value, and we get a 12-bit list address by putting the six bit-pairs together. Then the query N\*\*D\*E can be answered by looking through just 64 of the 4096 lists.

Similarly, let's suppose that we have 1,000,000 records [now continue as the present line 9].

Page 564, in front of the subsection on 'Balanced filing schemes', more inserts:

Rivest has also suggested another simple technique for handling basic queries. Suppose we have, say,  $N \approx 2^{10}$  records of 30 bits each, where we wish to answer arbitrary 30-bit basic queries like (4). Then we can simply divide the 30 bits into three 10-bit fields, and keep three separate hash tables of size  $M = 2^{10}$ . Each record is stored three places, in lists corresponding to its bit configurations in the three fields. Under suitable conditions, each list will contain about one element. Given a basic query with  $k$  unspecified bits, at least one of the fields will have  $\lfloor k/3 \rfloor$  or less bits unspecified; hence we need to look in at most  $2^{\lfloor k/3 \rfloor} \approx N^{k/30}$  of the lists, to find all answers to the query. Or, we could use any other technique for handling basic queries in the selected field.

Generalized tries. Rivest has also suggested yet another approach, based on a data structure like the tries in Section 6.3. We can let each internal node of a generalized binary **trie** specify which bit of

the record it represents. For example, in the data of Table 1 we could let the root of the trie represent Vanilla extract; then the left subtrie would correspond to those 16 cookie recipes which omit Vanilla extract, while the right subtrie would be for the 15 which use it. This 16-15 split nicely bisects the file; and we handle each **subfile** in a similar way. When a **subfile** becomes suitably small, we represent it by a terminal node.

To process a basic query, we start at the root of the trie. When searching a generalized trie whose root specifies an attribute where the query has 0 or 1, we search the left or right subtrie, respectively; and if the query has \* in that bit position, we search both subtries. Rivest has shown that the average amount of time will again grow as  $N^{k/m}$  if  $k/m$  of the attributes are specified.

Page 566, delete the paragraph including (12). Then in the next line:

The theory of block designs and related patterns is developed . . .

Page 568, new exercise 8 replacing the old:

8. [M35] Consider the set  $Q_{t,m}$  of all  $2^t \binom{m}{t}$  basic m-bit queries like (4) in which there are exactly  $t$  specified bits. Given a set  $S$  of m-bit records, let  $f_t(S)$  denote the number of queries in  $Q_{t,m}$  whose answer belongs to  $S$ ; and let  $f_t(s,m)$  be the minimum  $f_t(S)$  over all such sets  $S$  having  $s$  elements, for  $0 \leq s < 2^m$ . By convention,  $f_t(0,0) = 0$  and  $f_t(1,0) = s_{t0}$ .

(a) Prove that for all  $t > 1$ ,

$$f_t(s,m) = \begin{cases} f_t(s, m-1) + f_{t-1}(s, m-1) , & \text{for } 0 \leq s \leq 2^{m-1} ; \\ f_t(2^{m-1}, m-1) + f_{t-1}(2^{m-1}, m-1) + f_{t-1}(s-2^{m-1}, m-1) , & \text{for } 2^{m-1} < s < 2^m . \end{cases}$$

(b) Consider any combinatorial hash function  $h$  from the  $2^m$  possible records into  $2^t$  lists, with each list corresponding to

$2^{m-r}$  records. If each of the queries of  $Q_{t,m}$  is equally likely, the average number of lists which need to be examined per query is  $1/2^t \left(\frac{m}{t}\right)$  times

$$\sum_{Q \in Q_{t,m}} (\text{lists examined for } Q) = \sum_{\text{lists } S} (\text{queries of } Q_{t,m} \text{ relevant to } S) \geq 2^t f_t(2^r, m) .$$

Show that  $h$  is optimal, in the sense that this lower bound is achieved, when each of the lists is a 'subcube', i.e., when each list corresponds to all records satisfying some basic query with exactly  $r$  specified bits.

Page 570 receives overflow from page 569 and also the following new exercise:

15. [HM30] (P. Elias.) Given a large collection of  $m$ -bit records, suppose we want to find a record closest to a given search argument, in the sense that it agrees in the most bits. Devise an algorithm for solving this problem efficiently, assuming that an  $m$ -bit  $t$ -error-correcting code of  $2^n$  elements is given, and that each record has been 'hashed' onto one of  $2^n$  lists corresponding to the nearest codeword.

Page 574, exercise 12, line 5,  $f_1 \dots f_j$  should be  $f_2 \dots f_j$

Page 583, first line of exercise 17, change  $n!(1-z)^n$ ; the to:  
 $n!$ ; then the

Page 612, line -4, close up  $\mathbf{R}(w)$

Page 614, exercise 55, the righthand column should become

JGE	Line 13	
LDA	INPUT,4	(all typewriter style type)
LDX	INPUT,3	
CMPA	INPUT,5	
JLE	Line 21	
IDA	INPUT,5	
STX	INPUT,5	
JMP	Line 13	

Page 615, line 2, change  $52\frac{1}{6}$  to  $51\frac{2}{3}$

Page 619, line -8, change  $>$  to  $<$

Page 619, line -2, transpose " $\underline{R} \leftarrow \underline{Q}$ " and " $\underline{P} \leftarrow \underline{R}$ ".

Page 620, exercise 34,  $L_{N+1} \rightarrow L_N$  and  $< \rightarrow \geq$

Page 620, new answer:

37. (a) Set  $j \leftarrow 2j+1-b_{2j}$  until  $j > N$ . (b) Set  $b_j \leftarrow 1$  and  $j \leftarrow \lfloor j/2 \rfloor$  until  $b_j = 1$ . (c) Set  $b_j \leftarrow 0$  and  $j \leftarrow \lfloor j/2 \rfloor$  until  $b_{2j}+b_{2j+1} = 0$ .

Page 621, line 2 of ex. 8, replace "p, and the situation is" by:

p. Step S12 handles the cases where the short run is to be "merged" with an empty run, or where  $t=0$ ; otherwise we have

Page 621, line 5 of ex. 8, delete "t = 0 or"

Page 632, change "8." to "8, 9." and delete "9."

Page 634, at end of line 2, add: [JACM 19 (1972), 649-659.]

Page 639, lines -11, -10, sta td 3 stated  
inteh in the

Page 640, lines -12 to -10, replace [An amusing ... order.)] by:

[If  $\alpha$  is primitive sorting network, so is  $\alpha^R$  (the comparators in reverse order). For generalizations and another proof of (c), see N. G. de Bruijn, Discrete Mathematics 6 (1974), to appear.

Page 652, add to ex. 11: (This explicit form for the denominator was first discovered by David E. Ferguson.)

Page 664, add a new answer (somehow):

2. Let  $d$  minimize  $(\alpha m + \beta)/\ln m$ . A simple induction using convexity shows that  $A_1(n) \geq (\alpha d + \beta)n \log_d n$ , with equality when  $n = d^t$ . A suitable upper bound comes from complete  $d$ -ary trees, since these have  $D(7) = dE(\mathcal{T})$ ,  $E(\mathcal{T}) = tn + dr$  for  $n = d^t + (d-1)r$ ,  $0 \leq r \leq d^t$ .

Page 664, new answer to exercise 16:

16. Eleven stops: 123456 to floor 2, 334466 to 3, 444666 to 4,  
256666 to 5, 466666 to 6, 123445 to 4, 112335 to 5,  
222333 to 3, 122225 to 2, 111555 to 5, 111111 to 1.

[This is minimal, for a 10-stop solution with any elevator capacity can by symmetry be arranged to stop on floors 2, 3, 4, 5, 6,  $p_2$ ,  $p_3$ ,  $p_4$ ,  $p_5$ , 1 in that order, where  $p_2 p_3 p_4 p_5$  is a permutation of  $\{2, 3, 4, 5\}$ ; such schedules are possible only when  $b > 8$ . Cf. Martin Gardner, Scientific American 228 (May, 1973), 107.]

Page 666, bottom line, 256.]  $\rightarrow$  256; some interesting extensions have been obtained by James R. Slagle, JACM 11 (1964), 253-264.]

Page 667, change the last line of #20 to:

class of problems with the same answer, see T. S. Motzkin and E. G. Straus, Proc. Amer. Math. Soc. 7 (1956), 1014-1021.]

Page 672, line 5, +2;  $\rightarrow$  +4;

Page 672, ex. 13, delete the conjecture. (This will be replaced later by a résumé of Gary Knott's results.)

Page 674, just before Sec. 6.2.3, "2 (1972), to appear" becomes "1 (1972), 307 -310"

Page 678, line 2, add: [SIAM J. Computing 2 (1973), 33-43.]

Page 678, end of exercise 30, change "practice.)" to: "practice; but the memory distribution may be better without ROVER, since there will usually be a nice large empty region for emergencies.)

Page 678, bottom line, no 3-key  $\rightarrow$  no non-root 3-key

Page 687, end of exercise 4, add: Cf. C. F. Pinzka, AMM 67 (1960), 830.

Page 696, line 4, change  $n = N-1$ , to  $n = N$ ,

Page 698, last line of answer to 55 should be changed to:

This analysis, applied to a variety of problems, was begun by N. T. J. Bailey, J. Roy. Stat. Soc. B16 (1954), 80-87; M. Tainiter, JACM 10 (1963), 307-315; A. G. Konheim and B. Meister, JACM 19 (1972), 92-108.

Page 699, new answer:

66. Yes; and the arrangement of the records is unique. The average number of probes per unsuccessful search is reduced to  $C_{N+1}$ , although it remains  $C_N'$  when the N-th item is inserted. See Comp. J. 17 (1974), to appear.

Page 699, line -4, seed,  $\rightarrow$  seed and/or honey,

Page 700, new answer 8. See R. L. Rivest, Ph.D. thesis (Stanford University, 1974).

Page 703, line 2, change "furnished to the author by Dr. John" to: "computed by John"

Page 708, delete the entry for  $\langle X_n \rangle$  and insert a new entry just before ln :  
 $\lg x$  | binary logarithm:  $\log_2 x$  : 1.2.2

Page 708, definition of infinity: don't hyphenate "artificially"

Page 709, add a new entry at bottom:  $\approx$  | approximate equality

Page 710L, add Abel, Luther Charles, 153, 159.

Page 710L, add Amble, Ole, 549'

Page 710R, delete Armerding.

Page 710R, add Bailey, Norman Thomas John, 698.

Page 711L, add page 507 to the entry for W. W. R. Ball.

Page 711L, Best match, see Closest match.

Page 711L, add Bierce, Ambrose, 549.

Page 712L, Closest match, add page 555 and page 570.

Page 712L, full name is Coffman, Edward Grady, Jr.

Page 712L, full name is Colin, Andrew John Theodore.

Page 712R, add: Crossword puzzle dictionary, 563.

Page 712R, add: Davenport, Harold, 507.

Page 712R, add page 640 to de Bruijn.

Page 713L, insert Drysdale, Robert Lewis-(Scot), 111, 229.

Page 713R, add page 652 to entry for Ferguson.

Page 714L, add new entry: Floating point arithmetic, 41.

Page 714L, add page 644 to Gardner, Martin.

Page 714R, delete Gruenberger.

Page 715L, add Hyafil, Laurent, 377.

Page 716L, add page 698 to Konheim.

Page 716L, add page-302 to Lander.

Page 716L, delete Lehman.

Page 716L, add new entry: lg: binary logarithm, 708.

Page 717L, add new entry: Meister, Bernd, 698.

Page 717R, add page 709 to the entry for MIXAL.

Page 717R, add new entry: Motzkin, Theodor Samuel, 667.

Page 718L, add page 471 to Optimum trees, for searching,

Page 718L, add page 402 to Parkin.

Page 718R, add page 341 to 'Path length of tree' entry.

Page 718R, add new entry: Pinzka, Charles Frederick, 687.

Page 719L, Prime numbers, change '138' to '158'.

Page 719L, Priority queues, change '152' to '153'.

Page 719L, add new entry: Prusker, Francis, 377.

Page 719R, delete page 217 from Rivest.

Page 719R, full name is Roebuck, Alvah Curtis.

Page 719R, add new entry: Schönhage, Arnold, 217.

Page 720L, Searching, for closest match, add page 700.

Page 720L, full name is Sears, Richard Warren.

Page 720L, Sefer Yezirah, put a dot under the z .

Page 720L, correct spelling is Shannon, Claude Elwood.

Page 720R, add new entry: Slagle, James Robert, 666.

Page 721L, add new entry: Straus, Ernst Gabor, 667.

Page 721R, delete page 217 from Tarjan.

Page 722L, entry for Uzgalis should say: 473, 480.

Page 722R, add page 341 to 'Weighted path length' entry.

Page 722R, add new entry: Yao, Andrew, 235.

Page 722R, add new entry: Yao, Foong Frances, 215.

Change "  $\log_2$  " to " lg " in the following 305 places:

Page 91, line -1

Page 112, line 7

Page 114, line 7 (twice)

Page 116, line 24

Page 123, line 22

Page 128, lines 7 (twice), 8 (twice)

Page 134, Eqs. (47), (48)

Page 135, ex. 15, line 3

Page 143, line 5

Page 149, lines 1, 2, 6, 23, 30, 30, 31

Page 152, line 4

Page 154, line 14

Page 155, displayed formulas (four times)

Page 156, lines -3, -2

Page 157, line 4

Page 158, exs. 23 (twice), 30

Page 163, lines 13, 24, 27

Page 164, lines -11, -2

Page 168, line 8  
Page 183, lines -4, -2, -2, -1  
Page 184, lines 2, 11, 11, 11, 16, 18, 23  
Page 187, lines 15, 17, 20, 25, 28  
Page 188, lines 1, 3 . . .  
Page 190, line 5  
Page 194, lines 10, 16, 18, -7, -7, -4, -3  
Page 196, line -17  
Page 197, lines 1, 22, 26, 26, 31, 31  
Page 198, lines -10, -9  
Page 199, lines 3, 6, 6, 6, 6, 12, 14, 17, 18  
Page 203, line -7  
Page 204, line -3 (twice)  
Page 205, lines 3, 3, 11, 12  
Page 206, lines 10, 11, 12, 16  
Page 207, ex. 4  
Page 208, line 2 (thrice), ex. 15  
Page 211, lines 17, 20  
Page 212, lines 1, 9, 14, 28, -6, -2  
Page 214, (10) six times, (11), lines -8, -5  
Page 216, line 3  
Page 217, line -2  
Page 219, exs. 4, 6  
Page 220, line 9 (twice)  
Page 226, (5), (6)+1, (7), (8)+1, (9), -4  
Page 227, line 16  
Page 229, line 20 (twice)  
Page 230, (12), (13)  
Page 231, line -3  
Page 232, lines 1, 5, 7; (15)+1  
Page 233, lines 5, 8, 16  
Page 234, Theorem A  
Page 235, line 12  
Page 239, exs. 14, 18  
Page 243, exs. 42, 46

Page 248, line 7  
Page 267, lines 4, 13, -6, -2  
Page 312, line 16  
Page 355, line -7  
Page 374, lines -4, -4, -4, -1, -1  
Page 376, lines 4, 5  
Page 381, average time for merge exchange  
Page 385, line 21  
Page 410, line -4 (twice)  
Page 411, lines 4, 5, 6, 8, 9, 12, 13  
Page 412, (6)  
Page 413, (7) -4, (7), (7), -5, -3  
Page 414, line 2  
Page 420, exs. 8, 11 (twice)  
Page 422, lines 8, -7  
Page 427, line 6  
Page 445, displayed equations (8 times)  
Page 446, (25)+3 [log → lg], (25)+5  
Page 450, line -2 (twice)  
Page 451, exs. 38, 39, line -8  
Page 453, lines 5, 5, 7  
Page 460, lines -5, -3  
Page 463, lines 1, 2, 6  
Page 470, ex. 10  
Page 485, line -4 (twice)  
Page 499, lines -14, -13, -12 (twice each)  
Page 573, line -1  
Page 579, ex. 6 (twice)  
Page 603, P1  
Page 609, ex. 27  
Page 612, exs. 46, 48, 50  
Page 618, lines 14, -10, -9, -9, -2, -2, -1  
Page 619, lines 1, 19, 20  
Page 621, exs. 7, 10  
Page 622, line 4

Page 623, line 3  
 Page 629, exs. 11 (3 times), 14 (4 times), 15 (10 times)  
 Page 630, lines 7, 9, 9, 10, 10, 11, 12, 13, 13, 13, 14, 14, 14  
 Page 632, line 4  
 Page 633, exs. 15, 17 (6 times) ..  
 Page 634, lines -23, -3, -3  
 Page 635, line 10  
 Page 637, ex. 5 (3 times)  
 Page 638, ex. 18 (twice)  
 Page 641, exs. 42, 43  
 Page 668, ex. 9 (4 times)  
 Page 669, line 15 (twice)  
 Page 674, ex. 36  
 Page 677, exs. 21, 22 (twice), 23 (twice)  
 Page 678, line 5  
 Page 682, ex. 10  
 Page 686, ex. 34 (3 times)

Change the section numbers at the top of the following pages:

<u>Page</u>	<u>Says</u>	<u>Should be</u>
22	5.1.2	5.1.1
34	5.1.3	5.1.2
48	5.1.4	5.1.3
80	5.2.1	5.2
170	5.2.5	5.2.4
198	5.3.2	5.3.1
220	5.3.4	5.3.3
266	5.4.2	5.4.1
314	5.4.5	5.4.4
320	5.4.6	5.4.5
352	5.4.8	5.4.7
406	6.2.1	6.2
422	6.2.2	6.2.1
578	5.1.1	5
584	5.1.3	5.1.2
594	5.2	5.1.4
620	5.2.4	5.2.3
624	5.2.5	5.2.4
626	5.3.1	5.2.5
634	5.3.3	5.3.2
644	5.4.1	5.4
652	5.4.4	5.4.3

<u>Page</u>	<u>Says</u>	<u>Should be</u>
658	5.4.6	5.4.5
660	5.4.7	5.4.6
668	6.2.1	6.1
674	6.2.3	6.2.2
678	6.2.4	6.2.3
680	6.3	6.2.4

Also several index entries will need to change due to the repaging of material on pages 564-569.

Late additions to the corrections (volume 3)

Page 23, line 7, 100 → 300

Page 101, line -3, (11) → (12)

Also in (12), . → /2 .

Page 102, line 7, ! → .

Page 124, last line, interchange 1614 and 1601

Page 134, line 8,  $\log_2$  → lg

Page 148, line -5, scanned → promoted

Page 158, exercise 18, line 4, . → , thereby nearly cutting the average number of comparisons in half.

Page 205, lines -12 and -1, since it is not difficult . . . on n . → since we shall prove that  $H(m-1, n) < H(m-1, n+1)$  for all  $n \geq m-1$  by induction on m .

Page 206, after (19), insert a new sentence (no paragraph): This implies that  $H(m, n) \leq H(m, n+1)$  for all  $n > m$  , verifying our inductive hypothesis about step H3.

Page 220, line -8, 42(b) → 43(b)

Page 237, exercise 11, line 4, delete "separated by dotted lines"

Page 241, line 1, Fig. 48 → Fig. 49

Page 261, line -6, align the dot's and l's under the summation sign.

Page 357, line 2, was clobbered. It should be:

total time to sort is guaranteed to be  $O(N(\log N)^2)$  in the worst case; this is

Page 387, line 2, 5.3.4-6 → 5.3.4-62

Page 388, exercise 3, 47 → 43

In the last paragraph, Is there → Invent  
case, and/or on the average? → case.

Page 435, Eq. (16),  $i \leq k < j \rightarrow i < k \leq j$

Page 468, new paragraph after line 9:

For example, C. C. Foster [CACM 16 (1973), 513-517] has studied the generalized balanced trees which arise when we allow the height difference of subtrees to be greater than one, but at most four (say).

Page 476, line -11, just one less → just *l* less [italic not script *l*]

Page 547, line 13, except the leftmost, → except the one at the extreme left,

Page 560, lines -4 and -1, Pfefferneusse → Pfeffernuesse

Page 618, exercise 18, delete "(For . . .)" and in step H9' change  $K \leq K_i$  to "K  $\leq K_i$  or  $j = l$ " [italic lower case L]

line 8,  $K_j \leftarrow K$  should be  $R_j \leftarrow R$

line 9,  $K_j \leftarrow K_i$  should be  $R_j \leftarrow R_i$

Page 629, exercise 14, line 3,  $(\frac{3}{4})n \rightarrow (\frac{3}{4}n)$

Page 649, line -4, 25 → 23

Page 662, exercise 3, line 2, 5.3.3 → 5.3.3L

Page 665, Section 5.5, new answer:

3. R. B. K. Dewar has found such an algorithm of order  $N^{3/2} \log N$ , using a construction similar to that in exercise 5.2.4-18.

Page 712R, Dewar, Robert B. K., 665.

Page 714L, Foster, Caxton, add page 468.

Page 714R, Gwehenberger, Gernot, 490.

Page 715R, Incomplete gamma function, 548.

Page 716L, Lambert, Johann Heinrich, series, 619.

Page 717L, Match, search for closest, 9, 391, 405, 555, 570.

Page 721R, Szekeres, George, - 69.

Page 722L, von Mises, Richard, edler, 507.

See also the "late late corrections" on the last page of this report.

Volume 2

(This list mentions only 'high priority' changes; I also must add several dozen new references to recent literature, but these are not ready yet.)

Page 32, lines 1 and 2, change  $X$  to  $\underline{Y}$  (typewriter type) in both places.

Page 34, line 10, 3.3.2D  $\rightarrow$  3.3.2B

Page 39, Table 1 entry for  $v > 30$  the formula should be

$$v + \sqrt{2wx} + \frac{2}{3} \frac{x^2}{P} - \frac{2}{3} + O(1/\sqrt{v})$$

(delete the word 'approximately').

Page 46, line -10, 2(b).  $\rightarrow$  3(b)  
2(a)  $\rightarrow$  3(a)

Page 59, line -5, after Tingey insert "[Ann. Math. Stat. 22 (1951),  
592-5961"

Page 62, Eq. (18),  $n$ ;  $\rightarrow$   $n$ ,

Page 70, line 11, change to:

$$\frac{bx+c}{m} < k(x) \leq \frac{ax+c}{m} < \frac{bx+c}{m} + 1 .$$

Page 71, line -13, (10) and (11)  $\rightarrow$  (11) and (12)

Page 75, Eq. (32),  $j+1 \rightarrow j-1$

Page 75, change Step 1 to Step 2, and Step 2 to Step 3. Then move Step 3 from page 76 to this page and call it Step 1.

-Page 76, line 2,  $c > h \rightarrow c > h$

Page 76, line 14, steps 1 and 2  $\rightarrow$  steps 2 and 3  
line 19, step 1  $\rightarrow$  step 2

Page 77, lines 9-13, should become:

$$\sigma(10001, 100, 50) = \sigma(1, 100, 50) = -49.98.$$

$$\begin{aligned} C &\approx (-3 + 999900.01 - 97.02 - 49.98 \\ &\quad + 113.91 - 99895.60)/10^{10} \\ &= -0.00000003176 \end{aligned}$$

Page 77, line -8, should be

$$- \frac{1}{m} \sigma(m, a, c_0) + \frac{3}{m} (1 + 2 \frac{c_0}{a}) + \frac{a - 6x' - 6c}{m^2} + O(\frac{1}{2^m}),$$

Page 79, exercise 4, change rating to M19

Page 80, exercise 18, should be become:

18. [M23] (U. Dieter.) Given positive integers  $h, k, z$ , let

$$S(h, k, c, z) = \sum_{0 \leq j < z} \left( \frac{hj + c}{k} \right).$$

Show that this sum can be evaluated in "closed form" in terms of generalized Dedkind sums and the sawtooth function. [Hint: When  $z \leq k$ , we have  $\lfloor j/k \rfloor - \lfloor (j-z)/k \rfloor = 1$  if  $0 \leq j < z$ , =0 if  $z \leq j < k$ , so it is possible to introduce this factor and sum over the entire period.]

Page 88, new line in Table 1:

19 5897329 2<sup>35</sup> 3.61 4.13 5.46

Page 88, lines -2 and -1, change "in the early . . . Table 1." to:

in 1951. Line 19 was found by M. Lavaux and F. Janssens, in a computer search for spectrally good multipliers.

Page 95, line 6, S5 → S6

line 17,  $n \geq 6 \rightarrow n \leq 6$

. Page 115, line 7, E5 are → E5 are each

line 8, 2.582 → 3.582

Page 151, lines 9 and 27, Pr → Pr (not italics).

line 14, add: Conversely, there are sequences which satisfy R5 but not R4, as shown in exercise 37.

Page 153, exercise 19, HM46 → HM39

Page 154, exercise 27, change to:

27. [HM49] What is the highest possible value for  $\Pr(V_n > V_{n+1})$  in an equidistributed, white sequence? (D. Coppersmith [to appear] has constructed such a sequence achieving the value 7/8.)

Page 154, exercise 37, change to:

37. [M40] (D. Coppersmith.) Define a sequence which satisfies Definition R4 but not Definition R5. [Hint: Consider changing  $U_0, U_1, U_4, U_9, \dots$  in a truly random sequence.]

Page 168, line 8, era.  $\rightarrow$  era; indeed, octal arithmetic was even being proposed in France at the time [Aimé Mariage, Numération par huit (Paris: Le Nonnant, 1857)].

Page 168, just before the last paragraph, insert:

The book History of Binary and Other Nondecimal Numeration by Anton Glaser (privately printed, 1971), contains an informative and nearly complete discussion of the development of binary notation, including English translations of many of the works cited above.

Page 171, line 15, change to:

Negative-base systems were first considered by Vittorio Grünwald [Giornale di matematiche di Battaglini 23 (1885), 203-221, 367], who discussed the four arithmetic operations, root extraction, divisibility tests and radix conversion in such systems. However, we might safely call this a rather obscure journal, and his publication was soon forgotten. The next mention of negative-base systems in the literature was apparently by

Page 172, line -5, clockwise  $\rightarrow$  counterclockwise

Page 173, Fig. 1 is given in mirror reflection, the bottom should be the top and vice-versa!

Page 175, lines 20-21, change "So far . . . but perhaps" to  
The experimental Russian computer SETUN was based on balanced ternary notation [see CACM 3 (1960), 149-150], and perhaps

Page 179, exercise 30, (M39)  $\rightarrow$  [M39]

Page 193, lines 22-26, replace 'The concept ... scale.\* by:

On this particular machine such decimal scaling was about as easy as shifting, and the decimal approach greatly simplified input-output conversions.

Page 208, lines 2-3, see also . . . , 40. → for some useful refinements,  
see T. J. Dekker, Numer. Math. 18 (1971), 224-242.

Page 229, new exercise:

16. [M25] (R. L. Duncan.) Another way to define  $\Pr(S(n))$  is to evaluate  $\lim_{n \rightarrow \infty} \left( \sum_{k=1}^n \frac{1}{k} \right) / \ln n$ , since it can be shown that this "logarithmic probability density" exists and equals  $\Pr(S_n)$  whenever the latter exists. Prove that the logarithmic probability density of the statement " $(\log_{10} n) \bmod 1 < r$ " exists and equals  $r$ . [Thus, initial digits exactly satisfy the logarithmic law in this sense.]

Page 250, lines -11, -10, -9, change "it may have . . . (100 A.D.)." to: it is thought to be between 280 and 473 A.D. [see Joseph Needham, Science and Civilization in China 3 (Cambridge University Press, 1959), 33-34, for an interesting discussion].

Page 257, exercise 13, line 1, M22 → M25

line -2,  $cx^n \rightarrow cx$

line -1,  $c'x^{2n} \rightarrow c'x^2$

Page 270, line 20, change "algorithm." to:

algorithm [Computing 7 (1971), 281-292].

Page 334, exercise 3, put parentheses on the matrix.

Page 353, line -5, Michael E. → Michael A.

1971 → 1970

Page 354, line -13, 440. → 440].

Page 356, lines 15-16, change "In 1886, . . . is prime," to:

In 1883, I. M. Pervushin proved that  $2^{61}-1$  is prime [cf. Istoriko-Mat. Issledovaniya 6 (1953), 559],

Page 356, line 23, AMS Notices 18 (1971), 608 → Proc. Nat. Acad. Sci. 68 (1971), 2319-2320

Page 396, line 2, modulo a prime → modulo a prime (not italics)

Page 406, line -6, 487.) → 487).

Page 416, table in mid-page, rearrange so that 6 entries are given per column, with the three new entries

16	3583
17	6271
18	11231

Page 416, after line 16, insert:

E. G. Thurber [Pac. J. Math. 48 (1974)], - has proved that there are infinitely many such  $n$ .

Page 416, lines 17-18, change ", and more generally . . . may be false." to: , but even this may be false. It turns out that  $f(3.2731) < \ell(2731)$ .

Page 417, table in mid-page, rearrange so that 5 entries are given per column, with the three new entries

13	772
14	1382
15	2481

Page 417, line -7,  $1 \leq n \leq 11 \rightarrow 1 < n < 14$

Page 427, line 9, the matrices on the left should be

$$\begin{pmatrix} a & b \\ -c & -d \end{pmatrix} \begin{pmatrix} A & C \\ -B & -D \end{pmatrix}$$

Page 453, exercise 6, line 4, if  $f$  is a permutation;  $\rightarrow$  iff  $f$  is a cyclic permutation;

Page 454, exercise 12, B. Harris  $\rightarrow$  A. Rapoport, Bull. Math. Biophysics 10 (1948), 145-157, and

Page 454, exercise 13, line 1, (Solution . . . Williams.)  $\rightarrow$  [Paul Purdom and John Williams, Trans. Amer. Math. Soc. 133 (1968), 547-551.]

Page 465, exercise 18, line 4, C  $\rightarrow$  c (lower case italics)

Page 466, exercise 4, line 1,  $15\frac{1}{2}$   $\rightarrow$   $16\frac{1}{2}$   
line 2, lo-percent  $\rightarrow$  5-percent

Page 467, exercise 12, line 2

$$\sqrt{nq_s} z_s \rightarrow \sqrt{q_s/n} z_s$$

Page 468, exercise 16, middle, If we let  $\rightarrow$  If we make the notational changes  $z\sqrt{2} = x_p$ , and

Page 471, answer #4, should be:

4.  $d = 2^{10} \cdot 5$ . Note that we have "  $x_{n+1} < x_n$  with probability  $\frac{1}{2} + \epsilon$ , where

$$|\epsilon| < d/(2 \cdot 10^{10}) = 1/(2 \cdot 5^9);$$

so every potency-10 generator is respectable from the standpoint of Theorem P.

Page 473, exercise 14, line 1,  $(5 \cdot 2^{36} \dots 2) \approx 5/2^{35}$ .  $\rightarrow$

$$(2^{38} - 3 \cdot 2^{20} + 5)/(2^{70} - 1) \approx 2^{-32}.$$

Page 473, new answer 18:

18. A moment's thought shows that the formula  $S(h, k, c, z) = \sum_{0 \leq j < k} (\lfloor j/k \rfloor - \lfloor (j-z)/k \rfloor) (((hj+c)/k))$  is in fact valid for all  $z$ , not only when  $z \leq k$ . Writing

$$\lfloor j/k \rfloor - \lfloor (j-z)/k \rfloor = \frac{z}{k} + ((\frac{j-z}{k})) - ((\frac{j}{k})) + \frac{1}{2} \delta_{j0} - \frac{1}{2} \delta(\frac{j-z}{k})$$

and carrying out the sums yields  $S(h, k, c, z) = ((c/d))zd/k$

$$+ \frac{1}{12} \sigma(h, k, hz+c) - \frac{1}{12} \sigma(h, k, c) + \frac{1}{2} ((c/k)) - \frac{1}{2} (((hz+c)/k)),$$

where  $d = \gcd(h, k)$ .

Page 489, insert new answer:

19. (Solution by D. Coppersmith.) Yes, e.g. the sequence  $\langle v_n \rangle$  in exercise 26.

Page 492, new answer 37:

37. For  $n \geq 2$  replace  $u_{n/2}$  by  $\frac{1}{2} (u_{n/2} + \delta_n)$  where  $\delta_n = 0$  or 1 according as  $\{u_{n/2+1}, \dots, u_{n/2}\}$  contains an even or odd number of elements less than  $\frac{1}{2}$ .

Page 507, new answer:

16. See Fibonacci Quarterly 7 (1969), 474-475.

Page 515, line -6,  $cx^n \rightarrow cx$  (three places)  
 $x^{2n} \rightarrow x^2$  (one place)

Page 542, just before #20, insert a new line:

(Actually any matrix with determinant  $\pm 1$  would be a gcd in this case. )

Page 548, line after the program, change 05 to 05 (not italics).

Page 549, exercise 5, in step T1 set  $k \leftarrow 0$  not  $k \leftarrow 1$   
in step T2 "the  $k$ th level"  $\rightarrow$  "level  $k$ " and  
" $k = r+1$ "  $\rightarrow$  " $k = r$ "

Page 553, line before exercise 30, add:

Sharper results have been obtained by Anne Cottrell, AMS Notices (1973), A-476.

Page 556, answer 23, line 10,  $n$  odd  $\rightarrow n$  odd,

Page 560, exercise 39, after 2nd paragraph, insert a new paragraph:

But there are actually good ways to do the job in only  $O(n^3)$  operations; see, for example, J. H. Wilkinson, The Algebraic Eigenvalue Problem (Oxford University Press, 1965), 411.

Page 561, exercise 40, change to:

40. S. Winograd has shown that  $\lfloor n/2 \rfloor + 1$  is achievable for  $n = 13$  if the auxiliary coefficients are allowed to be complex.

Page 562, exercise 11, change " $(T_0, W_0) \leftarrow (V_0, U_0)$ " to " $W_0 \leftarrow U_0$ ",  
and in line 3, change " $T_0 T_j$  for  $j = n+1, \dots, N$ ." to  
" $T_n V_{j-n}$  for  $j = N, N-1, \dots, n+1$ .", and in line 4, insert a comma after "problem".

Page 577 and 578, change "record" to "block"

Page 594, line -5, delete 'The LOC field of an END card must be blank.'

Page 610, Cook, Joseph Marion, 485.  
Coppersmith, Don, 154, 489.  
Dekker, Theodorus Jozef, 208.  
Dieter, Ulrich Otto, 79, 80.

Page 611R, **Duncan**, Robert Lee, 229.

Page 614R, **Janssens**, Frank, 88.

Page 615, Kahan delete page 208

Karatsuba, **Anatolii Alekseevich**

Kesner, Oliver

Lavaux, **Michel**, 88.

Page 616L, Logarithmic density, 229.

**Mariage**, **Aimé**, 168.

Page 617R, **Needham**, Joseph, 250.

delete **Nichomachus**

Page 618L, O-notation:  $f(n) \rightarrow f(n)$ , for large n.

Page 619R, Probability over the integers, change 228 to 229

Page 620, Rapoport, Anatol, 454.

delete Record: A set . . . 578.

Page 621, delete Seelhoff

Page 622R, **Taussky Todd**, Olga, 88.

Page 623, Thurber, Edward Gerrish, 416.

add page 560 to Wilkinson

Page 624, "square" letter 0 in XOR

line -1, hyphenate North-Holland

NOTE: A number of layout changes, for consistency with other volumes, aren't listed here. (For example, on page 316 top, change 4.5.3 to 4.5.2, and below in the section heading change algorithm to Algorithm.)

Late additions to the corrections (volume 2)

Page 44, lines -8, -6, The fact . . . however, leads to  $\rightarrow$  From the fact . . . however, we can derive

Page 49, line -8,  $y_s \rightarrow y_k$

Page 51, Eq. (27), beneath the  $\Sigma$ ,  $\sqrt{ns} \rightarrow \sqrt{n} s$

Page 84, line -1,  $1 < k < n \rightarrow 1 \leq k \leq n$

Page 98, line 15,  $2\pi i \rightarrow -2\pi i$

Page 106, Fig. 9, caption line 2, frequency  $\rightarrow$  density text, line 7, altitude of  $f_j(x) \rightarrow$  altitude of  $p_j f_j(x)$

Page 166, line 16, Wiegel  $\rightarrow$  Weigel

Page 252, line -1,  $2^{e_j-1} \rightarrow 2^{e_{j-1}}$

Page 266, Eq. (19), change zero to upper case italic oh.

Page 267, line -3, This bold-face heading should be in smaller type.

Page 274, line 5, Make this equation (40).

Page 280, exercise 5, delete the Hint and raise the rating to HM35.

Page 479, exercise 10,  $\exp(2\pi i) \rightarrow \exp(-2\pi i)$  in the first four places;

line 6, change  $(\exp(2\pi i s_j \lambda) - 1)$  to  $(1 - \exp(-2\pi i s_j \lambda))$

Page 480, exercise 20,  $2\pi i \rightarrow -2\pi i$  in all four places.

Also once in exercise 21, page 481;

and in exercise 24,  $\omega = e^{-2\pi i/m}$

Page 516, line -1, (39)  $\rightarrow$  (19)

Volume 1

(This list covers only changes to the second edition of volume 1 which will be published momentarily. The number of changes between first and second edition is so huge it is impossible to list them all, so I won't list any. The reward for errors in the second edition is \$2.00.)

**Page** 134, line -15, present day → present-day

Page 151, line 9, F part → F-part

Page 164, line 5, (c ed) → (ceb)

**Page** 170, line 19, (21) → the example on page 169

Page 227, line -12, change "B. J. Loopstra's" to "B. J. Loopstra and C. S. Scholten's"

Page 304, exercise 19, line 2, Fig. 13 → Fig. 14

**Page** 313, lines -3, -2, notations and terminology which is → terminology and notations which are

Page 327, line 1, is → are

**Page** 489, exercise 62, lines 6-7, change "Dixon . . . 80." to: Dixon [Messenger of Math. 20 (1891), 79-80], who established the general case twelve years later [Proc. London Math. Soc. 35 (1903), 285-289].

Page 534, exercise 13, at end, insert the reference to Pratt's paper:

[Proc. ACM Symp. Theory of Computing 5 (1973), 268-277.]

Page 590, line 12, 507; and → 507; John Riordan and N. J. A. Sloane, J. Australian Math. Soc. 10 (1969), 278-282; and

Page 625R, King, James Cornelius, 20.

Page 626R, Loopstra, Bram Jan, 227.

Page 627L, Mirsky, Leon, 582.

Page 631L, Schorr-Kon, Jacques Jacob, 9.

Scholten, Carel Steven, 227.

Selfridge, John Lewis, 77.

Page 631R, Sloane, Neil James Alexander, 590.

See also the "late late corrections" on the last page of this report.

MIX booklet errata

Page 30, Fig. 3, step P3 should say '500 found?'

Page 34, Fig. 4, third card should say L EQU500.

Page 43, line 1, 6667 → 66667

line 2, 193,334 → 133,334

Page 44, problem 16, line 2, row . . . diagonal → row and column

line 8, 10 → 9

Change 'record' to 'block' everywhere.

LATE LATE CORRECTIONS November 1973

Volume 1

Page 33, exercise 5, series.  $\rightarrow$  series, provided that the  $a_i$  are not all zero.

Page 33, exercise 6, series.  $\rightarrow$  series, provided that any three of the four sums exist.

Page 58, Eq. (26), Delete the mark before the "."

Page 307, line 14, tree.  $\rightarrow$  tree, and speak of nodes at shallow and deep levels.

Page 313-322 lines -9, -8, change BIT 11 (1973), to appear to BIT 13 (1973),

Volume 3

Page 51, line 2,  $j.$  Set  $\rightarrow$   $j.)$  Set

Page 60, line -7, Theorem B  $\rightarrow$  Theorem C

Page 61, denominator of  $(34)_{\frac{m}{m}} \rightarrow n_{\frac{m}{m}}$

between (34) and (35), "discriminant"  $\rightarrow$  "square root of the discriminant"

Page 63, line 18,  $c \rightarrow H$

Page 188, two lines before Table 1, 4981.  $\rightarrow$  498; Elements of Combinatorial Computing (Pergamon, 1971), 213-215].

Page 336, line -3, 0.183  $\rightarrow$  0.184

Page 343, line -1, state 1.  $\rightarrow$  state 0-1.

Page 640, line -2, add sentence

The reflective property of this network was essentially given much earlier by H. E. Dudeney in one of his "frog puzzles" [Amusements in Mathematics (1917), 193].

Page 713L, add p. 640 to Dudeney

Page 713L, Elias, Peter, 570.

Page 722R, Yamada, Hisao M., 489.

