Each stage in multistage manufacturing processes raises a question of how much inspection is appropriate for quality assurance. Sampling procedures usually provide the least expensive way to maintain quality.

In this paper, a method for use on a computer is developed for evaluating single sampling plans on the basis of economics.

Determining economic sampling plans

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Multistage manufacturing processes are commonplace in industry. To ensure quality in the manufactured goods produced by such processes, some form of inspection is usually employed. Emphasis upon quality, while justified on its own merits, frequently makes the economic evaluation of inspection a secondary matter. The questions of where to establish inspection stations in a manufacturing process and how much inspection is necessary often are settled on a noneconomic basis. Yet, pricing and profit pictures are both influenced by answers to such questions.

Three alternatives are available at any stage of a manufacturing process: (1) no inspection at all, which passes on defective items to succeeding stages that may require expensive corrective work, (2) one hundred percent inspection, which may be more expensive than correcting defective work in later manufacturing stages, and (3) inspection of a random sample chosen from each lot, which often represents a compromise as the least expensive way to maintain quality. In utilizing the last-mentioned alternative, called sampling, questions arise in deciding how much sampling is needed and what courses of action are indicated by various sampling results. This general problem is discussed in the literature.

Relevant analytical techniques are suggested by Hald,[1-3] Hamburg,[4] Locke,[5] and Schlaifer[6] as well as others. The work of such authors served as a basis for the work discussed here. In this paper, a method is detailed for evaluating sampling plans on the basis of economics, cost functions are detailed, and numerical examples are given. Some aspects of an experimental computer program that implements the method are described briefly, and a sample output is given.

In this discussion, single sampling plans as distinguished from sequential plans are considered. It is assumed that defective items are reworked or replaced whenever they are found—whether at the inspection station or at a subsequent manufacturing stage. Excessive rework and replacement is caused when items are falsely judged to be defective. Such might be the case, for example, if product specifications include a large margin of safety.[7] This exposure to excessive rework and replacement is assumed to be present only at the first stage of manufacture. A further assumption is that rejected lots are always subjected to one hundred percent inspection.

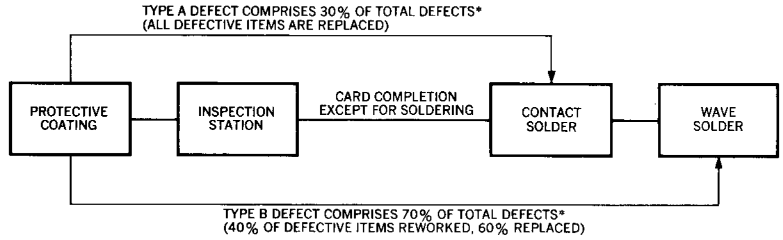
Two types of inspection are considered to accommodate applications in which a thorough inspection of a few items yields sufficient information so that abbreviated inspection (sorting) is satisfactory for all other items. Whenever any ambiguity may arise, the terms *inspection* and *sorting*, respectively, are used to distinguish these two notions.

The cost for accepting or rejecting a lot is assumed to depend on several variables, including the number of defective items in the lot (or equivalently, the lot percent defective, denoted by LPD). Given only a knowledge of the LPD for a lot, a choice among the alternatives listed in the second paragraph of this paper should be based on which alternative leads to the smallest expected (average) per lot cost for the LPD in question. It is clear that the third one of the alternatives, inspection of a random sample, encompasses two possible actions after the inspection is completed: accept the remainder of the lot or reject it. The listed alternatives indicate the four actions that are possible when each lot is presented. Comparisons among these four are facilitated by introducing the term *opportunity loss*. Here it is defined for any LPD as the difference between the expected per lot cost of the action that is taken and the minimum possible expected per lot cost. Because the LPD for a lot is not known in advance of the necessary action, the objective is to minimize the average or expected opportunity loss.

It is assumed that the probability distribution of the LPD's is known for any lot size. A weighted average of opportunity losses, using the weights determined from the LPD distribution, yields the expected (average) loss due to incorrect decisions. Thus, the expected loss for decisions made without sampling can be compared with the corresponding loss for decisions made with sampling. Comparisons between different single sampling plans are made in similar fashion.

The methods described in this paper apply when only one inspection station is considered. Procedures for optimizing sampling plans when several inspection stations are considered jointly are discussed elsewhere.[8]

Figure 1 Progress of an item through production



Cost considerations

Before discussing the particular cost considerations required in determining an optimum sampling plan, it is instructive to diagram a typical production line. The heavy lines in Figure 1 represent a production line that begins with the application of a protective coating to printed-circuit cards. Two stages of subsequent processing are illustrated: soldering contacts onto the cards and soldering components onto the cards. The problem is to determine the optimum number of items per lot to be inspected immediately after the protective coating has been applied. Costs at the inspection station are assumed to be given in terms of specific actions. It is necessary to know cost per item inspected, cost per item reworked, and cost per item replaced. Inspection and sorting costs may differ and consequently will be treated separately.

When the defective items from the protective coating operation are sent into the production line, there may be product fallout at each of the soldering operations, i.e., defective items may be removed from the production line. Figure 1 shows the amount of defect type *A* fallout at contact soldering and defect type *B* fallout at wave soldering. In these cases, we must consider the resulting costs at the soldering stations. These costs, generally, are of the same variety as are incurred at the protective coating operation. There are some exceptions:

- Because of a margin of safety in design specifications, some items would be erroneously judged defective after protective coating. That is to say, if sent into the line, these items would not fall out at any subsequent operation. (This situation is discussed further in the section on theory.)
- Inspection and/or sort costs are considered to be at zero at each soldering station. (Coating is not subject to special inspection at the soldering stations because truly defective coating becomes obvious there.)

It should be noted here that if a defective item is released for field use, the attendant costs must be considered.

Naturally, the cost of replacing a part at one of the soldering operations is higher than corresponding costs at the coating operation. (Materials and labor have been added.) In establishing the amount of inspection to be done after protective coating, the notion is to minimize the total cost of finding and correcting defects, recognizing that some corrective action may take place at inspection and at each of the soldering operations.

Per item costs may change with gross changes in sample size. For example, the number of men required to inspect 200 items per lot may not be four times the number required to inspect 50 items per lot. The cost formulas discussed herein require that such costs be entered on a per item basis for each sample size considered. Also, rejected lots sometimes entail clerical or other additional costs which must be considered.

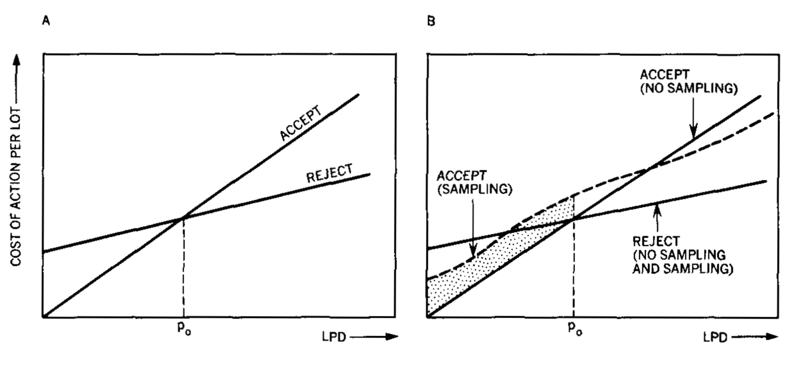
Appendix B contains a detailed set of calculations for costs based upon input numbers selected for illustrative purposes. These input numbers relate directly to the protective coating and soldering operations shown in Figure 1.

single inspection cost

Suppose, for example, that the decision to accept or reject the items is made without sampling. Corresponding expected cost plots depending on the LPD are shown in Figure 2A, with the assumptions that: (1) inspection cost per unit inspected is equal to sorting cost; (2) as previously mentioned, inspection costs at processes beyond the first stage are negligible; (3) accepted lots generate expected costs which are proportional to the LPD, depending upon the cost of necessary rework and replacement at various processes beyond the first; and (4) rejected lots receive 100 percent inspection; hence, cost is incurred even if a rejected lot contains no defective items. Costs of rework and replacement must be added for rejected lots containing some defective items. The expected amount of added cost is proportional to the LPD, depending upon rework and replacement rates at the first stage.

In the no-sampling situation, note that, at some LPD, expected costs for accepting a lot may equal those for rejecting it. That LPD value is denoted by p_0 in Figures 2A and B and is called the break-even point.

Figure 2 Cost of acceptance and rejection



Lots with LPD less than the break-even point should be accepted; others should be rejected. Each lot presents an opportunity to make the least costly decision. Ignorance of the LPD value, of course, precludes our taking advantage of the opportunity consistently. As previously stated, an opportunity loss is the difference between the expected cost of an action (acceptance or rejection) and the minimum expected cost that might have been incurred. Suppose the LPD of a lot is less than p_0, for example. If no inspection takes place and the lot is accepted, the opportunity loss is zero. If that lot is rejected, the opportunity loss is obtained as the difference between the ordinates of the two lines in Figure 2A at the LPD for the lot. Similarly, if the LPD exceeds the break-even value, rejecting it without sampling yields a zero opportunity loss; accepting it yields an opportunity loss that can be determined by taking the difference between the appropriate ordinates.

opportunity loss

The purpose of inspecting a sample is to garner evidence about the LPD. When such sampling is adopted, certain costs are unavoidable, viz., inspection costs for the sample and costs for replacing its defective items. Note, however, that the latter costs depend upon rework and replacement rates at the first process. For rejected lots, the inspection, rework, and replacement continue for the *remainder* of the lot at rates appropriate to the first process. Costs for the uninspected portion of accepted lots accrue at rates appropriate to subsequent processes.

Assumptions that we adopt later imply a nonlinear relation between LPD and the overall expected cost for lots that are accepted by sampling. This becomes evident as the formulas are developed (particularly when Equations 1 and 18 are considered jointly). The nonlinearity is reflected in the curves labeled "Accept (Sampling)" in Figures 2 and 3.

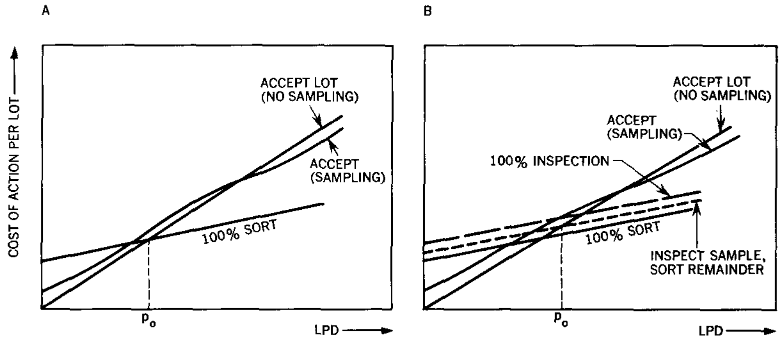
Figure 2B is based upon the same assumptions which led to Figure 2A. It illustrates the situation in terms of costs for four possible actions: accepting and rejecting without sampling as in Figure 2A, together with similar actions as the result of inspecting a sample. The intercept for the dashed curve depends upon the sample size and the cost of inspecting each item in the sample.

Again, each lot presents an opportunity to take the least costly action with the opportunity loss being the cost difference between the action taken and the least costly action. Suppose, for example, that the LPD for a lot is less than the break-even point. The lot could be accepted or rejected without sampling, as was discussed in connection with Figure 2A. If it is accepted only after inspecting a sample, the opportunity loss is obtained as the difference between the ordinates of the appropriate curves (highlighted by the shaded area in Figure 2B) determined at the LPD for the lot.

inspection and sorting

Figures 3A and B are much like Figure 2, the primary difference being that accommodation is provided for the likelihood that inspection and sorting costs are not equal. Figure 3A

Figure 3 Inspection and sorting costs



reflects a no-sampling reject line based upon sort costs only. However, it is logical to presume that some information is necessary before sorting of rejected lots can be initiated. Figure 3B illustrates such a situation; a selected number of items must be inspected prior to any sorting even if a lot is rejected without sampling. If that selected number is equal to the sample size (when sampling is employed), there is no way to distinguish between rejection without sampling and rejection with sampling. Note, however, that the break-even point (p_0) shifts in Figure 3B because some inspection is assumed to be required whenever a lot is rejected. In contrast, there is always the possibility of accepting a lot without any inspection. Opportunity losses are obtained at each LPD as before.

production considerations

Certain characteristics of the production process are assumed to be known. The first thing required is lot size. If the lot sizes under consideration do not vary greatly, it may be advantageous to use the average lot size for computing the economic sampling plan. Applications encountered by the authors have involved lot sizes that varied over a wide range. In these cases, we selected several lot sizes throughout the actual range and computed the economic sampling plan for each size selected. This provided approximate answers to the questions of optimum sampling plans for lot sizes that were not investigated.

Second, a lot percent defective table is required. This can in some cases be summarized from existing data. In other cases for which data is not available, one of two things may be done. Collection of relevant data is often possible. An appropriate form can be filled out daily by manufacturing or quality departments until sufficient information about LPD's is obtained. Whenever that procedure is impractical, the user can substitute a hypothetical distribution for the LPD. His best guess as to the LPD distribution may be good enough to determine reasonable, though admittedly not optimum, sampling plans.

The formulas in the section on theory require the user to know the percentage of defective items that are replaced and the percentage of defective items that are reworked. This is necessary so that the rework unit cost is applied only to those units reworked and the replacement cost to those units replaced. With respect to defective items, the user also must know what percentage of those defective items eventually received at subsequent processes can be reworked and what percentage must be replaced. That knowledge enables the user to apply the replacement and rework unit costs in the correct proportions. Similarly, it is important to know the probability that a defective item will fall out in each subsequent process.

A typical form suggesting the kinds of data which should be collected in this regard is included in Appendix B in Figure 7.

Theory for comparing sampling plans

In the following discussion, single sampling plans are to be compared with no sampling on the basis of economics. Lot and sample sizes are denoted by N and n respectively, and actions (or decisions) resulting from sampling are made according to the usual rules. That is, a lot is rejected if the number of defective items in the sample exceeds some agreed-upon number which is here denoted by d. Rejected lots are made defect-free (defective items are reworked or replaced) before being sent for further processing. Thus, the lot sizes remain constant. For lots that are accepted, only the sample is rendered defect-free prior to further processing. Defective items in the remainder of such lots are subject to detection at later stages of manufacture, with rework or replacement as necessary.

The cost model that we adopt accommodates two aspects of manufacturing which merit special attention. It frequently happens that defective items can be classified according to the subsequent stage of manufacture at which the item may become troublesome. In such cases, a simplifying assumption is made: classification is unique, so that no item can be troublesome at more than one stage. The symbol λ_s is used to denote the probability that, if an item fails to pass inspection at the first stage, the defect is of the type that can be troublesome at the sth stage of manufacture. It may also happen that product specifications used to define defects at the first stage contain a significant margin of safety. Thus, an item that is judged to be defective at the first stage may or may not cause quality degradation at a subsequent stage in manufacture. The symbol ν_s is used to denote the conditional probability that an item will be troublesome at the sth stage, given that inspection reveals a defect of that type. Thus, the product $\lambda_s \nu_s$, hereafter denoted p_s, is the probability that an item that would be judged defective at the first stage will be troublesome at the sth stage. We also write $q_s = 1 - p_s$.

probability model

The number of defective items in any lot is unknown prior to inspection and is regarded as a random variable, hereafter denoted by D, which may take any of the values $0, 1, 2, \cdots, N$. However, to facilitate studies in which values of D are grouped for convenience, it is expedient to write the possible values of D as $a_1, a_2, \cdots, a_k \leq N$. Each a_i is integer-valued and the relation $k \leq N + 1$ must be satisfied. Also, for reasons apparent from Equation 1, sample sizes considered by us never exceed $N - a_i (i = 1, \cdots, k)$.

Ratios $a_1/N, a_2/N, \cdots, a_k/N$ are called lot percent defectives, and the event $D = a_i$ is denoted by A_i, $i = 1, 2, \cdots, k$. The probability that $D = a_i$, i.e., that the event A_i occurs, is denoted by $P(A_i)$. Historical records[9] of production may provide estimates for $P(A_1), P(A_2), \cdots, P(A_k)$. Otherwise, special studies or "best guesses" may be necessary. Discussion here is based upon the assumption that the values are known for each of the probabilities $P(A_1), \cdots, P(A_k)$, with

$$\sum_1^k P(A_i) = 1$$

Random samples of sizes $n \leq N$ are assumed whenever sampling is employed. The number of defective items in a sample is a random variable denoted by R, and the conditional probability that $R = r$ given $D = a$ is assumed to be

$$P(R = r \mid D = a) = \binom{a}{r}\binom{N - a}{n - r} \Big/ \binom{N}{n}, \qquad a \leq N - n \qquad (1)$$

for $r = 0, 1, \cdots, \min(a, n)$ and to be zero otherwise. This is to say that the conditional distribution of R, given a, is taken to be hypergeometric with parameters N, n, and a.

acceptance and rejection

When sampling is employed, acceptance or rejection of a lot is determined by whether or not $R \leq d$ for the sample. The event $R \leq d$ is denoted by X, and immediate interest concerns the conditional probabilities

$$P(X \mid A_i) = \sum_{r=0}^{d'} P(R = r \mid D = a_i), \qquad i = 1, \cdots, k \qquad (2)$$

where $d' = \min(d, a_i)$ and the remaining notation has the obvious meaning. The joint probability that $D = a_i$ and $R \leq d$ may be written

$$P(A_i X) = P(A_i) P(X \mid A_i) \qquad (3)$$

As a matter of practice, the hypergeometric probability $P(R = r \mid D = a)$ frequently is approximated by the use of either binomial or Poisson probability tables, i.e.,

$$P(R = r \mid D = a) \cong \binom{n}{r}\left(\frac{a}{N}\right)^r \left(\frac{N - a}{N}\right)^{n-r}$$

or

$$P(R = r \mid D = a) \cong \frac{(na/N)^r}{r!} e^{-na/N}$$

For use of the capabilities of a computer, a more satisfactory procedure is to use Equation 1 without modification. We may compromise as follows for large lot sizes, with relatively small values of a and n. Compute the approximate value of

$$P(R = 0 \mid D = a) = \binom{N-a}{n} \Big/ \binom{N}{n} \tag{4}$$

by noting that

$$\begin{aligned} \log [P(R = 0 \mid D = a)] \cong {} & (N - a + 1/2) \log \left(1 - \frac{a}{N}\right) \\ & + (N - n + 1/2) \log \left(1 - \frac{n}{N}\right) \\ & - (N - a - n + 1/2) \log \left|1 - \frac{a+n}{N}\right| \end{aligned} \tag{5}$$

(which emerges after use of Stirling's approximation for the factorials in Equation 4) and applying the recursion formula

$$\begin{aligned} & P(R = r + 1 \mid D = a) \\ & = \frac{(a-r)(n-r)}{(r+1)(N-a-n+r+1)} P(R = r \mid D = a) \end{aligned} \tag{6}$$

The present version of the previously mentioned experimental computer program utilizes Equations 4 and 6.

equal repair and replacement costs

We should note that the conditional expected value for R, given a and no restriction on the number of defective items in the sample, is

$$E_a(R) = n(a/N) \tag{7}$$

where the notation on the left side of the equation has obvious meaning. The restriction $R \leq d$ holds in all lots that are accepted by sampling. For those lots, the expected value corresponding to Equation 7 is written

$$E_{a,d}(R) = \sum_{r=0}^{d_o} rP(R = r \mid D = a)/P(R \leq d_o \mid D = a) \tag{8}$$

where $d_0 = \min (d, a)$. (With $a = a_i$, the denominator in Equation 8 is $P(X \mid A_i)$.)

A major consideration here is the calculation of expected costs associated with accepted lots. The explanation at this point and in the next section is simpler if we assume tentatively that repair and replacement costs are equal. Defective items in the sample are compensated for at one rate, say c, and defective items in the uninspected part of an accepted lot involve another rate, say c'. The expected cost, given $D = a$, becomes inspection cost plus

$$E_{a,d}[cR + (a - R)c'] = \frac{\sum_{r=0}^{d_o} [cr + (a - r)c']P(R = r \mid D = a)}{\sum_{r=0}^{d_o} P(R = r \mid D = a)} \tag{9}$$

Table 1 Subscript adoption

Label in Figure 2B	*Algebraic notation*
Accept (No sampling)	$C_1(a)$
Accept (Sampling)	$C_2(a)$
Reject (No sampling or sampling)	$C_3(a)$

for defective items in accepted lots. This cost includes items found to be defective at the inspection station. Denoting inspection cost plus the right side of Equation 9 by $C(a)$, we see that the unconditional expected cost of accepted lots of type X becomes

$$\sum_{i=1}^{m} P(A_i X)C(a_i). \tag{10}$$

Various modifications to Equation 9 are considered in subsequent sections. However, we retain the notation $C(a_i)$ in all cases since no ambiguity results.

It is convenient at this juncture to again refer to Figure 2. Ordinates of the curve labeled "Accept (Sampling)" are given by $C(a)$. To distinguish these ordinates from those of other curves, subscripts are adopted, as in Table 1. Suppose for purposes of interpretation, that a lot is accepted as a result of inspecting a sample of items. Then $[C_2(a) - C_1(a)]$ gives the conditional expected opportunity loss if $(a/N) \leq p_o$ and $[C_2(a) - C_3(a)]$ gives the corresponding opportunity loss when the value of a/N exceeds p_o. The unconditional expected opportunity loss for accepted lots (based upon sampling) is computed by

$$\sum_{(a_i/N) \leq p_o} P(A_i X)[C_2(a_i) - C_1(a_i)] + \sum_{(a_i/N) > p_o} P(A_i X)[C_2(a_i) - C_3(a_i)] \tag{11}$$

Extension to cover other cases indicated in Figures 2 and 3 is completely analogous and merits no special comment.

Product design specifications having margins of safety, as previously discussed, are accommodated similarly. If a' defective items from the first stage are presented at the sth stage, some will cause quality degradation at the latter process; some will not. As a consequence of the definitions in the beginning of this section on theory, the quantity p_s may be regarded as the probability that, if a defective item is presented to the sth stage, the item will necessitate corrective action at the sth stage. If a' items are presented, the probability that r' will cause corrective actions is assumed to be

$$p(r' \mid a') = \binom{a'}{r'} p_s^{r'} q_s^{a'-r'}, \qquad r' = 0, 1, \cdots, a'$$

where $q_s = 1 - p_s$ as before. The associated random variable is denoted by R' and, accordingly, its mean value is $a'p_s$ (again from elementary probability theory). Introducing these notions into the cost considerations indicated by Equation 9, we see that the expected cost of a defective items in a lot of size N becomes

$$\frac{\sum_{r=0}^{d_o} [cr + (a - r)p_s c'] P(R = r \mid D = a)}{\sum_{r=0}^{d_o} P(R = r \mid D = a)} \tag{12}$$

when the sth stage is considered and the sample size is n.

Extension of Expression 12 to include several stages is straightforward. The expected cost corresponding to Expression 12 becomes:

$$\frac{\sum_{r=0}^{d_o} \left[cr + (a - r) \sum_{s=2}^{m} c_s p_s\right] P(R = r \mid D = a)}{\sum_{r=0}^{d_o} P(R = r \mid D = a)} \tag{13}$$

where c_s is the cost of a defective part when it necessitates corrective action at the sth stage ($s = 2, 3, \cdots, m$).

An unconditional expected cost corresponding to Expression 10 is written in an obvious way, requiring only that $C(a_i)$ be interpreted according to Expression 12 or 13, as applicable.

variations in cost formulas

It is convenient to recall that $q_s = \lambda_s \nu_s$, where λ_s is the probability that, if an item fails to pass inspection at the first stage, the defect is a type that can be troublesome at the sth stage, and ν_s is the probability that, if the defect is of that type, it will be troublesome. Thus, direct substitution into Expression 13 yields the expected cost

$$\frac{\sum_{r=0}^{d_o} \left[cr + (a - r) \sum_{s=2}^{m} \lambda_s \nu_s c_s\right] P(R = r \mid D = a)}{\sum_{r=0}^{d_o} P(R = r \mid D = a)} \tag{14}$$

when a defectives in all are generated at the first process for a lot that is accepted with sampling.

inspection and sorting costs

Introduction of inspection costs for the sample (and, when appropriate, sorting costs for the remainder of rejected lots) is straightforward. For example, consider Expression 12, which applies to accepted lots. If T denotes the inspection cost per item inspected, one merely adds nT to Expression 12 to obtain the expected cost per accepted lot. (Modifications to Expressions 13 and 14 in this case are identical.) Corresponding costs for rejected lots become

$$nT + (N - n)S + ac \tag{15}$$

where S is the sorting cost per item sorted.

It is usual at every operation to find that reworking some defective items is less expensive than replacing those same items.

However, rework is not always feasible. The distinction between the two costs is accommodated in the previous formulas by introducing new symbols.

unequal rework and replacement costs

Let α be the probability that a defective item found at the inspection station can be reworked satisfactorily, and let $1 - \alpha = \beta$. Corresponding probabilities associated with the sth operation are indicated hereafter by subscript, thus defining α_s and β_s. Similarly, let t and u indicate the respective costs of rework and replacement for each defective item found at the inspection station and adopt the same subscript notation to indicate costs at subsequent operations.

Expression 14 is rewritten as

$$\sum_{r=0}^{d_o} \left\{ \left[r(\alpha t + \beta u) + (a - r) \sum_{s=2}^{m} (\alpha_s t_s + \beta_s u_s) \lambda_s \nu_s \right] \right.$$

$$\left. \times P(R = r \mid D = a) \right\} \Big/ \sum_{r=0}^{d_o} P(R = r \mid D = a) \qquad (16)$$

to express conditional expected cost when a defective items exist in a lot prior to sampling and the lot is accepted. (Inspection cost must be added.)

expected costs of no sampling

It is possible, of course, to reject a lot without sampling. When sorting is applicable, it cannot begin immediately. Some number of items, say n', must be thoroughly inspected to determine what to look for when sorting. This leads to the conditional expected cost formula:

$$n'T + (N - n')S + a(\alpha t + \beta u) \qquad (17)$$

for lots that are rejected without sampling, given that there are a defective items in the lot.

The corresponding formula for lots that are accepted without sampling is

$$N \sum_{s=2}^{m} T_s + a \sum_{s=2}^{m} (\alpha_s t_s + \beta_s u_s) \lambda_s \nu_s$$

where T_s is the inspection cost per item at the sth process (the current computer program is based upon $T_s = 0$), and the overall expected cost of such lots is

$$\sum_{i=1}^{m} P(A_i) C'(a_i)$$

where $C'(a_i)$ is indicated by the immediately preceding cost expression. Unconditional expected cost for rejected lots is computed in the obvious way for the no-sampling case.

We now summarize the conditional expected costs, given that the defective items in the lot is a, when sampling is the adopted course of action:

Rejected lots

$$nT + (N - n)S + a(\alpha t + \beta u)$$

Accepted lots

$$nT + (N - n) \sum_{s=2}^{m} T_s$$
$$+ \sum_{r=0}^{d_o} \left\{ \left[(\alpha t + \beta u) r + (a - r) \sum_{s=2}^{m} (\alpha_s t_s + \beta_s u_s) \lambda_s \nu_s \right] \right.$$
$$\left. \times P(R = r \mid D = a) \right\} \bigg/ \sum_{r=0}^{d_o} P(R = r \mid D = a) \qquad (18)$$

Expressions for unconditional (overall) expected costs and opportunity losses are modeled after Expressions 10 and 11 respectively.

more than one action number

The previous paragraphs set forth some of the fundamentals for economic evaluation of sampling plans based upon lot size N, sample size n, and an action number d. It is convenient in some problems to examine more than one action number, say d_1 and d_2, in a single analysis. Then three events are defined, based upon the number R of defective items found in the sample. Denoting these events by X, Y, and Z:

$$\begin{aligned} 0 \leq R \leq d_1 \quad & \text{defines } X \\ d_1 < R \leq d_2 \quad & \text{defines } Y \\ d_2 < R \quad & \text{defines } Z \end{aligned} \qquad (19)$$

with

$$P(X \mid A_i) = \sum_{r=0}^{d'} P(R = r \mid D = a_i)$$
$$P(Y \mid A_i) = \sum_{r=d'+1}^{d''} P(R = r \mid D = a_i)$$
$$P(Z \mid A_i) = \sum_{r=d''+1}^{d'''} P(R = r \mid D = a_i) \qquad (20)$$

where $d' = \min(d_1, a_i)$, $d'' = \min(d_2, a_i)$, and $d''' = \min(a_i, n)$. Of course, if $a_i \leq d_1$, then $P(Y \mid A_i)$ and $P(Z \mid A_i)$ are each zero, and, if $d_1 < a_i \leq d_2$, $P(Z \mid A_i)$ is zero. Development leading to formulas for unconditional costs and opportunity loss corresponding to events Y and Z is straightforward. (If we first recall Equation 3, we see that Expressions 10 and 11 serve as models for final formulas. Modifications to Equation 9 must be compatible with Equation 20, of course, and indices for the two summations in Expression 11 should reflect, respectively, $(a_i/N) \leq p_o$ with $C_2(a_i) > C_1(a_i)$ and $(a_i/N) > p_o$ with $C_2(a_i) > C_3(a_i)$.) Figures 8 and 9 and Table 5 in Appendix B indicate calculation details of a convenient work format.

Numerical examples encountered by the authors have led to the conjecture that in all practical applications, one of these d values will be redundant. That is to say, if rejection is best when Y occurs, then rejection is best when Z occurs. Similarly, if acceptance is best when Y occurs, acceptance is best when X occurs.

Details for computing costs are cited for a particular example in Appendix B. However, the computations are quite suitable for computer calculations. Appendices C and D contain typical computer printouts for the example cited. In deference to users who wish to display details such as those that Hamburg,[4] Locke,[5] and others recommend, the program incorporates two options for the printout: one provides numbers for displaying a complete "decision tree" through the use of Bayes' theorem; the other provides only the essentials for comparing various sampling plans.

Figure 9 in Appendix B shows a work format indicating the specific computations that yield the expected opportunity losses under "sampling" and "no-sampling" assumptions.

optimum sampling plan

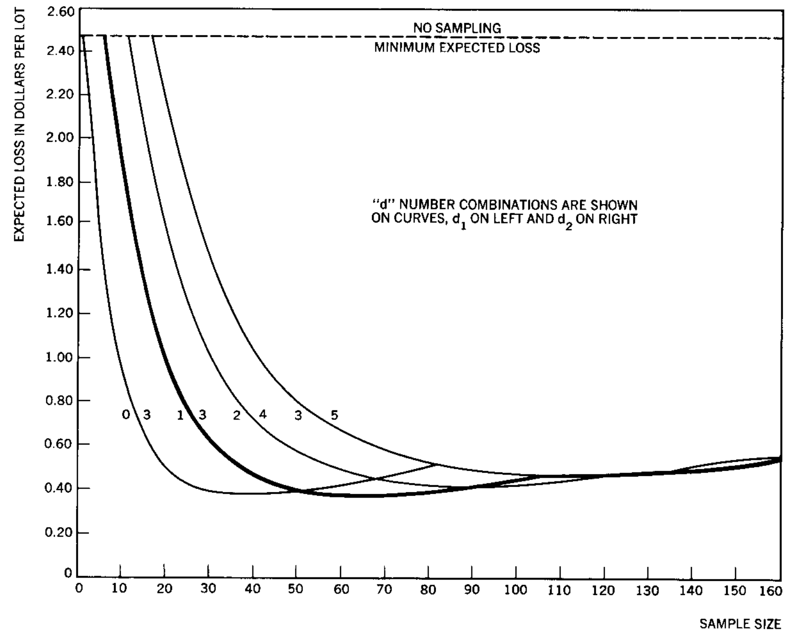
A sampling plan is determined when both the sample size (n) and the acceptance number (d) are specified. An optimum sampling plan is defined to be a plan whose expected opportunity loss is at least as small as that for any (every) other plan. Direct comparison of opportunity losses provides the only available way of finding an optimum plan.

To find the optimum sampling plan, the following procedure is adopted: holding the previously defined d_1 and d_2 values constant, the total expected opportunity losses for various sample sizes are computed. Appropriate graphic techniques are used to select the optimum sample size if d_1 and d_2 are used. Similar analyses for other d_1, d_2 values are made, and the total expected opportunity losses for the respective optimum sample sizes are easily made. (Sample size zero is considered each time by use of the "no sampling" expected losses.) Figure 4 illustrates the result. Curves for several sets of action numbers are displayed, the corresponding input information (costs, LPD distribution, etc.) being cited in Appendix B. The heavy curve of Figure 4 is associated with $d_1 = 1$, $d_2 = 3$ and is interpreted to mean (for these action numbers and the specific input information of Appendix B):

- When the sample size is less than about 110, the smallest expected opportunity loss is achieved by rejecting any lot whose sample contains two or more defective items.
- For larger sample sizes, the smallest expected loss results from rejecting items only when the number of defective items in the sample exceeds 3.
- Among the plans that were considered, the best is a plan with a sample size of about 65, and an acceptance number 1, i.e., $n = 65$ and $d = 1$. (See Appendix D, underlined row of entries.)

The right-most column in Appendix D provides necessary information for plotting the curve. Each of the other three curves in Figure 4 is interpreted in a similar way. Thus each pair of action numbers (d_1, d_2) leads to a unique "best" plan which should receive special attention in finally determining the optimum plan. Visual inspection of the various curves indicates that the optimum plan is designated by about $n = 65$ and $d = 1$. The corresponding expected opportunity loss is about 40.5 cents per lot. (The next

Figure 4 Optimum sampling



most attractive plan involving different d numbers is given by $n = 40$, $d = 0$, with expected opportunity loss 40.8 cents per lot.)

The determination of economic sampling plans can be done on a computer. Data can be evaluated more readily at a faster rate and in greater amounts. Appendix A describes some of the aspects of an experimental program corresponding to the method just discussed for evaluating single sampling plans.

ACKNOWLEDGMENT

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Appendix A: Aspects of a program for economic sampling

We now briefly describe the input requirements and output formats of the current version of an experimental program for determining economic sampling plans. Essentially the program does the calculations described in the section on theory and as outlined in Appendix B.

There are nine possible types of input cards for the program. They are listed in Table 2 with a brief description of their contents. Detailed descriptions of these cards are given in Appendix E.

problem setups

All of the card types in Table 2 are required input for the first problem in a computer run except the sort cost card. This card is omitted whenever there is no separate sorting cost. Subsequent problems in the same run require a minimum input of two title cards and a problem card. Other cards may be added optionally subject to the following restrictions.

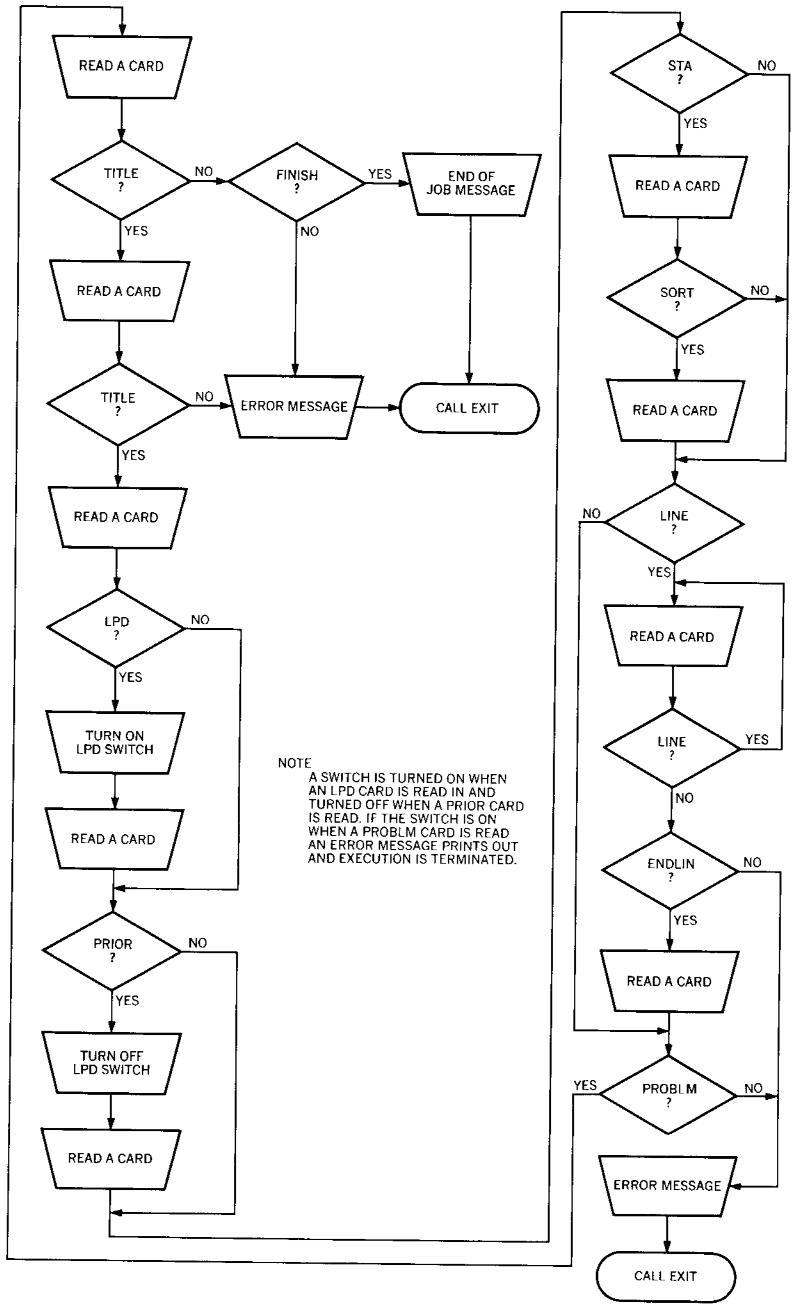
- Whenever a new lot percent defective card is added to the input, a new prior probability card must also be included in the input.
- Whenever a change is made in the line cards, all of the line cards and the ENDLIN card must be reentered.
- Whenever a new station card is part of the input and the sort cost is different from the inspection cost, a new sort cost card must also be entered.
- Whenever there is only one operation subsequent to the operation being studied, enter the subsequent operation on a line card followed by a card containing the letters ENDLIN in columns 1–6 and blanks in columns 7–80.

Figure 5 shows the order of the input cards. It also shows what action the program takes when a card is omitted or inserted. The processing initiated by the presence of a card is not shown. It should be emphasized that input cards must always be in the relative order shown in Figure 5. If an optional card is omitted, it is simply not found by the program and the card read is compared against the next card type in the sequence. If a card is out

Table 2 Possible input cards

Card type	*Description*
Title (2 cards)	Any alphanumeric information to be used to identify the output.
Lot percent defective	Tells now many LPD levels are to be considered and lists them.
Prior probabilities	Lists the probability of occurrence associated with each of the LPD levels being considered.
Station	Contains all necessary information about the operation being studied.
Sort cost	Contains the sort cost per unit if it is different from the inspection cost per unit.
Line	Contains all necessary information about a subsequent operation.
ENDLIN	Contains all necessary information about the last operation in the line.
Problem description	Tells the program what d numbers to use and what sample sizes to analyze.
Finish	This card signals the end of the input data and must be placed after the last problem card.

Figure 5 Input card order



of sequence, those following it are not found, and an error condition occurs.

There are two output formats available with the program. The output format is requested by an indicator in the problem card. If long-form output is requested, a complete set of tables (as described in Hamburg[4]) is produced for each sample size con-

output

sidered. An example of this type of output is shown in Appendix C. If the short-form output is requested, a one-line summary of the expected losses associated with the two no-sampling strategies (accept no sampling, reject no sampling) and the three sampling strategies (accept when $R \leq d_1$, accept when $R \leq d_2$, accept when $R \leq n$) is produced for each sample size considered. The minimum expected loss due to sampling is also displayed. Figure 4 can be generated by plotting minimum expected loss versus sample size; the heavy curve corresponds to the data displayed in Appendix D.

Appendix B: Numerical example description

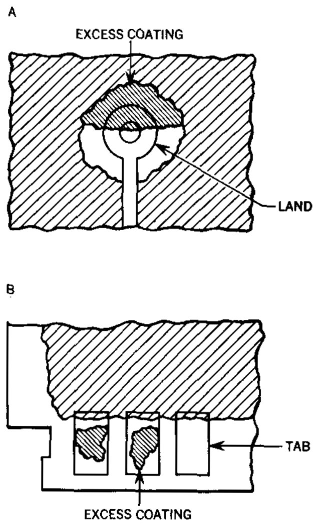
The section on cost considerations illustrates a process in the printed-circuit card production line where a polyurethane protective coating is applied to the cards. There are two major defects considered for illustration. They are illustrated in Figure 6 as (A) coating on lands and (B) coating on tabs.

In each case, excess coating can interfere with sound solder joints. In this problem, we use the flow diagram shown in the cost consideration section, along with a list of historical background data and cost figures to calculate expected costs and opportunity losses for a sampling plan.

A typical form for collecting data in this regard is shown in Figure 7. Resulting input data for determining expected losses are shown in Table 3. Figures 8 and 9 depict work formats.

Following are some computations presented for illustration.

Figure 6 Major defects in printed-circuit card production



Calculations at inspection station—100% inspection and 2% LPD

1. 100% inspection cost	800($0.005)	= $4.00 (per lot)
2. Other calculations		
a) Rework costs	800(0.02)(0.80)($0.093)	= $1.19
b) Replacement costs	800(0.02)(0.20)($0.360)	= $1.15
3. Total cost if 100% inspection is used:		$6.34

Calculations at solder operations—no inspection

1. At contact solder		
a) Inspection	800($0)	= $0
b) Rework	800(0.02)(0.30)(0.50)(0)($0)	= $0
c) Replacement	800(0.02)(0.30)(0.50)(1.00)($0.58)	= $1.39
d) Total		$1.39
2. At wave solder		
a) Inspection	800($0)	= $0
b) Rework	800(0.02)(0.70)(0.15)(0.40)($0.20)	= $0.13
c) Replacement	800(0.02)(0.70)(0.15)(0.60)($5.00)	= $5.04
d) Total		$5.17
3. Total cost if no inspection is used:		$6.56

With similar calculation for other LPD values, Table 4 is established. Table 5 describes calculation details.

Table 3A Sample input data

Data type	Indicated uses* of data		
		Processes, work stations	
	Inspection at coating	*Contact solder*	*Wave solder*
1. Unit inspection cost ($)	$T = 0.005$		
a. Sort cost (if different) ($)	$s = 0.005$		
2. Unit rework cost ($)	$t = 0.093$	$t_1 = 0$	$t_2 = 0.20$
3. Unit replacement ($)	$u = 0.360$	$u_1 = 0.58$	$u_2 = 5.00$
4. Percentage of defective units reworked/100	$\alpha = 0.80$	$\alpha_1 = 0$	$\alpha_2 = 0.40$
5. Percentage of defective units replaced/100	$\beta = 0.20$	$\beta_1 = 1.00$	$\beta_2 = 0.60$
6. Lot size	$N = 800$		
7. Lot percent defective/100 (see Table 3B)		$\lambda_1 = 0.30$	$\lambda_2 = 0.70$
8. Percentage of defective items/100 from coating that are potential defects here			Total 1.00
9. Probability that a potential defect from coating will cause a defect here		$\nu_1 = 0.50$	$\nu_2 = 0.15$

*Symbols as defined throughout the discussion

Table 3B Lot percent defective table

	(a_i/N) % *Defective*/100	$p(A_i)$ % *Lots*/100
1.	0	0.500
2.	0.010	0.250
3.	0.020	0.050
4.	0.040	0.050
5.	0.050	0.050
6.	0.100	0.050
7.	0.180	0.050
8.	...	...
9.	...	...
10.	...	...
11.	...	...
12.	...	...

Figure 7 Typical data collection form

DATE	PART NUMBER	JOB NUMBER	QUANTITY	CARD SIZE	PANEL TYPE	COATING ROOM REPLACEMENT	MANUFACTURING INSPECTION					QUALITY ASSURANCE INSPECTION			REJECTED ORDERS	
							SAMPLING DATA-DEFECT TYPE & AMOUNT	PASSED	100% INSPECTION DATA-DEFECT TYPE & AMOUNT	NUMBER REWORK & TYPE	NUMBER REPLACEMENT & TYPE	SAMPLING DATA-DEFECT TYPE & AMOUNT	PASSED	REJECTED	NUMBER REWORKED & TYPE	NUMBER REPLACED & TYPE

CARD SIZE
A
B
C
D
E

COATING ROOM REPLACEMENT
NUMBER OF PIECES REPLACED IN COATING ROOM (TAKEN FROM ROUTING)

DEFECT TYPES
COATING IN HOLES -A
COATING ON LANDS-B
COATING ON TABS -C
BUBBLES -D
INSUFFICIENT COVERAGE-E
MISCELLANEOUS -F
ADHESION -G
CONTAMINATION -H

PANEL TYPE
INNERPLANE -I
NONINNERPLANE-N

Table 4 Expected costs and opportunity losses

LPD	Expected costs (no sampling)		Expected opportunity loss (no sampling)	
	Reject lot	*Accept lot*	*Reject lot*	*Accept lot*
0.0	4.00	0	4.00	0
0.01	5.18	3.29	1.89	0
0.02*	6.34	6.56	0	0.22
0.04	8.68	13.13	0	4.45
0.05	9.86	16.42	0	6.56
0.10	15.70	32.82	0	17.12
0.18	25.07	59.09	0	34.02

* From calculations in Appendix B

Figure 8 Work format for LPD's

		(from Equation 2)	$P(A_i)\ P(X\|A_i)$, etc.
i	$(a_i/N)^*\ P(A_i)^*$	$P(X\|A_i)\ P(Y\|A_i)\ P(Z\|A_i)$	$P(A_iX)\ P(A_iY)\ P(A_iZ)$
1			
2			
3			
4			
5		(Illustration given in Appendix C)	
6			
$k = 7$			
Totals	100%		$P(X)\quad P(Y)\quad P(Z)$

* From historical records

Figure 9 Work format for costs and opportunity losses

	Costs				Opportunity Loss			
	No Sampling		Sampling size n		No Sampling		Sampling size n	
i	Reject Lot	Accept Lot	Reject Lot	Accept Lot	Reject Lot	Accept Lot	Reject Lot	Accept Lot
1								
2								
3								
4	(Table 4)		(Equation 9)		(Illustration given in Appendix C)			
5								
6								
$k = 7$					(I)*	(II)*	(III)*	(IV)*

* Labels for use in Table 5.

Table 5 Total expected opportunity loss

	Pair*
No sampling–reject lot: Inner product of columns labeled $P(A_i)$ and (I) No sampling–accept lot: Inner product of columns labeled $P(A_i)$ and (II)	1
X with sample size n–reject lot: Inner product of columns labeled $P(A_iX)$ and (III) X with sample size n–accept lot: Inner product of columns labeled $P(A_iX)$ and (IV)	2
Y with sample size n–reject lot: Inner product of columns labeled $P(A_iY)$ and (III) Y with sample size n–accept lot: Inner product of columns labeled $P(A_iY)$ and (IV)	3
Z with sample size n–reject lot: Inner product of columns labeled $P(A_iZ)$ and (III) Z with sample size n–accept lot: Inner product of columns labeled $P(A_iZ)$ and (IV)	4

* The minimum value in the first pair (no sampling) is to be compared with the sum of the minimum values from each of the three remaining pairs (sampling).

Appendix C: Example of long form of computer printout

PROTECTIVE COATING 2 HI 6
LOT SIZE # 800 D1#1 D2#3 SAMPLE SIZE=65

PAYOFF MATRIX SHOWING OPPORTUNITY LOSSES FOR ACCEPTING OR REJECTING LOTS WITHOUT SAMPLING

	PRIOR	ACTION		EXPECTED LOSSES	
LPD	PROB.	REJECT	ACCEPT	REJECT	ACCEPT
.0	.500	4.00	0.0	2.00	0.0
.010	.250	1.89	0.0	0.47	0.0
.020	.050	0.0	0.22	0.0	0.01
.040	.050	0.0	4.45	0.0	0.22
.050	.050	0.0	6.56	0.0	0.33
.100	.050	0.0	17.12	0.0	0.86
.180	.050	0.0	34.02	0.0	1.70
			TOTALS	2.47	3.12

JOINT FREQUENCY DISTRIBUTION OF SAMPLE RESULTS AND EVENTS

	SAMPLE RESULTS			
LPD	TYPE X*	TYPE Y*	TYPE Z*	TOTAL
.0	.500	.0	.0	.500
.010	.217	.033	.001	.250
.020	.031	.017	.002	.050
.040	.012	.025	.013	.050
.050	.007	.022	.021	.050
.100	.000	.004	.045	.050
.180	.000	.000	.050	.050
TOTAL	0.768	0.101	0.131	1.000

* TYPE X NUMBER OF OBSERVED DEFECTS LESS THAN OR EQUAL TO 1
TYPE Y NUMBER OF OBSERVED DEFECTS GREATER THAN 1 AND LESS THAN OR EQUAL TO 3
TYPE Z NUMBER OF OBSERVED DEFECTS GREATER THAN 3

CALCULATION OF POSTERIOR PROBABILITIES OF EVENTS GIVEN TYPE X INFORMATION

LPD	P(A)	P(X/A)	P(A)P(X/A)	P(A/X)
.0	0.500	1.000	0.500	0.651
.010	0.250	0.868	0.217	0.282
.020	0.050	0.622	0.031	0.040
.040	0.050	0.248	0.012	0.016
.050	0.050	0.146	0.007	0.010
.100	0.050	0.007	0.000	0.000
.180	0.050	0.000	0.000	0.000

PAYOFF MATRIX SHOWING OPPORTUNITY LOSSES GIVEN TYPE X INFORMATION

	POST.	ACTION		EXPECTED LOSSES	
LPD	PROB.	REJECT	ACCEPT	REJECT	ACCEPT
.0	0.651	4.00	0.32	2.60	0.21
.010	0.282	1.89	0.22	0.53	0.06
.020	0.040	0.0	0.39	0.0	0.02
.040	0.016	0.0	4.58	0.0	0.07
.050	0.010	0.0	6.68	0.0	0.06
.100	0.000	0.0	17.21	0.0	0.01
.180	0.000	0.0	34.09	0.0	0.00
			TOTALS	3.14	0.43*

CALCULATION OF POSTERIOR PROBABILITIES OF EVENTS GIVEN TYPE Y INFORMATION

LPD	P(A)	P(Y/A)	P(A)P(Y/A)	P(A/Y)
.0	0.500	0.0	0.0	0.0
.010	0.250	0.130	0.033	0.323
.020	0.050	0.344	0.017	0.171
.040	0.050	0.494	0.025	0.245
.050	0.050	0.442	0.022	0.219
.100	0.050	0.083	0.004	0.041
.180	0.050	0.001	0.000	0.000

PAYOFF MATRIX SHOWING OPPORTUNITY LOSSES GIVEN TYPE Y INFORMATION

	POST.	ACTION		EXPECTED LOSSES	
LPD	PROB.	REJECT	ACCEPT	REJECT	ACCEPT
.0	0.0	4.00	-0.0	0.0	0.0
.010	0.323	2.13	0.0	0.69	0.0
.020	0.171	0.06	0.0	0.01	0.0
.040	0.245	0.0	4.12	0.0	1.01
.050	0.219	0.0	6.22	0.0	1.36
.100	0.041	0.0	16.73	0.0	0.69
.180	0.000	0.0	33.59	0.0	0.02
			TOTALS	0.70*	3.08

CALCULATION OF POSTERIOR PROBABILITIES OF EVENTS GIVEN TYPE Z INFORMATION

LPD	P(A)	P(Z/A)	P(A)P(Z/A)	P(A/Z)
.0	0.500	0.0	0.0	0.0
.010	0.250	0.002	0.001	0.004
.020	0.050	0.034	0.002	0.013
.040	0.050	0.258	0.013	0.098
.050	0.050	0.412	0.021	0.157
.100	0.050	0.910	0.045	0.347
.180	0.050	0.999	0.050	0.381

PAYOFF MATRIX SHOWING OPPORTUNITY LOSSES GIVEN TYPE Z INFORMATION

	POST.	ACTION		EXPECTED LOSSES	
LPD	PROB.	REJECT	ACCEPT	REJECT	ACCEPT
.0	0.0	4.00	-0.0	0.0	0.0
.010	0.004	2.64	0.0	0.01	0.0
.020	0.013	0.57	0.0	0.01	0.0
.040	0.098	0.0	3.55	0.0	0.35
.050	0.157	0.0	5.59	0.0	0.88
.100	0.347	0.0	15.63	0.0	5.42
.180	0.381	0.0	31.25	0.0	11.90
			TOTALS	0.02*	18.54

* MINIMUM VALUES ARE TO BE MULTIPLIED BY CORRESPONDING PROBABILITIES FROM THE FIRST TWO GROUPS ABOVE THE DASHED LINE IN THIS APPENDIX, I.E., EXPECTED OPPORTUNITY LOSS FOR THIS SAMPLING PLAN IS

(0.768)(0.43) + (0.101)(0.70) + (0.131)(0.02) = 0.4036

Appendix D: Example of short form of computer printout

```
LIST INPUT DATA
  PROTECTIVE COATING  2 HI 6

LPD      0.0     0.010   0.020   0.040   0.050   0.100   0.180

PRIOR    0.500   0.250   0.050   0.050   0.050   0.050   0.050

STA     800      .005     .093     .36     0.0  .80  .20

LINE                      0.0      .58     0.0  0.0 1.00  .30  .50

ENDLIN                    .20     5.00     0.0  .40  .60  .70  .15

PROBLM    1    3    5  200    5
```

LOTSIZE	SAMPLE SIZE	PERCENT SAMPLF	NO SAMPLING EXP. LOSS REJECT	NO SAMPLING EXP. LOSS ACCEPT	SAMPLING EXP. LOSS TO ACCEPT R LE. 1	SAMPLING EXP. LOSS TO ACCEPT R LE. 3	SAMPLING EXP. LOSS TO ACCEPT R LE. SS	MIN EXP. LOSS DUE TO SAMP.
800.	5.	0.01	2.472	3.118	2.667	3.107	3.114	2.667
800.	10.	0.01	2.472	3.118	1.920	2.957	3.110	1.920
800.	15.	0.02	2.472	3.118	1.362	2.606	3.106	1.362
800.	20.	0.02	2.472	3.118	1.003	2.175	3.102	1.003
800.	25.	0.03	2.472	3.118	0.779	1.776	3.098	0.779
800.	30.	0.04	2.472	3.118	0.638	1.457	3.094	0.638
800.	35.	0.04	2.472	3.118	0.547	1.218	3.090	0.547
800.	40.	0.05	2.472	3.118	0.488	1.042	3.086	0.488
800.	45.	0.06	2.472	3.118	0.450	0.912	3.082	0.450
800.	50.	0.06	2.472	3.118	0.426	0.815	3.078	0.426
800.	55.	0.07	2.472	3.118	0.412	0.740	3.074	0.412
800.	60.	0.07	2.472	3.118	0.406	0.682	3.070	0.406
800.	65.	0.08	2.472	3.118	0.406	0.637	3.066	0.406
800.	70.	0.09	2.472	3.118	0.410	0.601	3.062	0.410
800.	75.	0.09	2.472	3.118	0.417	0.573	3.058	0.417
800.	80.	0.10	2.472	3.118	0.428	0.551	3.054	0.428
800.	85.	0.11	2.472	3.118	0.441	0.534	3.050	0.441
800.	90.	0.11	2.472	3.118	0.455	0.521	3.046	0.455
800.	95.	0.12	2.472	3.118	0.471	0.511	3.042	0.471
800.	100.	0.13	2.472	3.118	0.488	0.504	3.038	0.488
800.	105.	0.13	2.472	3.118	0.506	0.501	3.034	0.501
800.	110.	0.14	2.472	3.118	0.524	0.499	3.030	0.499
800.	115.	0.14	2.472	3.118	0.543	0.500	3.025	0.500
800.	120.	0.15	2.472	3.118	0.562	0.503	3.021	0.503
800.	125.	0.16	2.472	3.118	0.583	0.507	3.017	0.507
800.	130.	0.16	2.472	3.118	0.604	0.513	3.013	0.513
800.	135.	0.17	2.472	3.118	0.625	0.521	3.009	0.521
800.	140.	0.17	2.472	3.118	0.646	0.530	3.005	0.530
800.	145.	0.18	2.472	3.118	0.667	0.540	3.001	0.540
800.	150.	0.19	2.472	3.118	0.688	0.551	2.997	0.551
800.	155.	0.19	2.472	3.118	0.709	0.563	2.993	0.563
800.	160.	0.20	2.472	3.118	0.730	0.576	2.989	0.576
800.	165.	0.21	2.472	3.118	0.750	0.590	2.985	0.590
800.	170.	0.21	2.472	3.118	0.770	0.605	2.981	0.605
800.	175.	0.22	2.472	3.118	0.790	0.620	2.977	0.620
800.	180.	0.22	2.472	3.118	0.809	0.635	2.973	0.635
800.	185.	0.23	2.472	3.118	0.829	0.651	2.969	0.651
800.	190.	0.24	2.472	3.118	0.847	0.668	2.965	0.668
800.	195.	0.24	2.472	3.118	0.866	0.684	2.961	0.684
800.	200.	0.25	2.472	3.118	0.884	0.701	2.957	0.701

END OF JOB

Appendix E: Detailed input card descriptions

Columns	*Contents*
Title card format	
1 − 6	The characters TITLEƀ.
	Note: ƀ indicates blank.
7 − 80	Any alphanumeric information.
Lot percent defective card format	
1 − 6	The characters LPDƀƀƀ.
7 − 10	The number of lot percent defective levels listed on this card (max 12).
	Units digit must be in column 10.

Columns	*Contents*
11 – 15 16 – 20 21 – 25 26 – 30 31 – 35 36 – 40 . . . 66 – 70	Lot percent defective levels to be considered. Numbers must be entered with a decimal point in consecutive fields. For example, if you have three LPD levels, 10%, 20%, 30%, you would enter .1 in columns 14 – 15, .2 in columns 19 – 20, .3 in columns 24 – 25.

Prior probabilities card format

1 – 6	The characters PRIORb̸.
7 – 10	Left blank.
11 – 15 16 – 20 21 – 25 26 – 30 31 – 35 36 – 40 . . . 66 – 70	The probability of occurrence associated with each of the LPD levels in the preceding card. Numbers must be entered, with a decimal point, in consecutive fields. There must be one entry on this card for each entry on the LPD card.

Station card format

1 – 6	The characters STAb̸b̸b̸.
7 – 11	Lot size. Units digit must be in column 11.
12 – 21	Inspection cost per unit. Must be entered with a decimal point.
22 – 31	Rework cost per unit. Must be entered with a decimal point.
32 – 41	Replacement cost per unit. Must be entered with a decimal point.
42 – 51	Other costs (see section on cost considerations). Must be entered with a decimal point.
52 – 56	Percentage of defective units reworked. Must be entered with a decimal point. For example, 75% is entered as .75.
57 – 61	Percentage of defective units replaced. Must be entered with a decimal point.

Sort cost card format

1 – 6	The characters SORTb̸b̸.
7 – 16	Sort cost per unit. Must be entered with a decimal point.

Line card or ENDLIN *card format*

1 – 6	The characters LINEb̸b̸ or ENDLIN.
7 – 21	Left blank.
22 – 31	Rework cost per unit. Must be entered with a decimal point.
32 – 41	Replacement cost per unit. Must be entered with a decimal point.

Columns	Contents
42 – 51	Other costs (see section on cost considerations). Must be entered with a decimal point.
52 – 56	Percentage of defective units reworked. Must be entered with a decimal point.
57 – 61	Percentage of defective units replaced.
62 – 66	Percentage of defective parts from the operation being studied that are potential defect causes at this operation.
67 – 71	Probability that a potential defect from the operation being studied will cause a defect at this operation.

Problem card format

1 – 6	The characters PROBLM.
7 – 11	d_i (see section on theory for explanation). Units digit must be in column 11.
12 – 16	d_2 (see theory section). Units digit must be in column 16.
17 – 21	First sample size to be considered. Units digit must be in column 21.
22 – 26	Last sample size to be considered. Units digit must be in column 26.
27 – 31	Value by which the sample size is to be incremented over the range between first and last. Units digit must be in column 31.
32 – 35	Left blank.
36	Enter a zero (0) if short-form output is desired. Enter a one (1) if long-form output is desired.

Finish card format

1 – 6	The characters FINISH.
7 – 80	Left blank.

CITED REFERENCES AND FOOTNOTE

1. D. R. Cox, H. Hald, and G. B. Wetherill, various papers, *Technometrics* **2**, No. 3, 275–372 (August 1960).
2. A. Hald, "Single sampling inspection plans with specified acceptance probability and minimum average costs," (in English), *Skandinavish Akluarietidskrift*, 22–64, 145–183 (1965).
3. A. Hald, "Asymptotic properties of Bayesian single sampling plans," *Journal of the Royal Statistical Society*, Series B, **29**, No. 1, 162–173 (1967).
4. M. Hamburg, "Bayesian decision theory and statistical quality control," *Industrial Quality Control* **XIX**, No. 6, 10–14 (December 1962).
5. L. G. Locke, "Bayesian statistics," *Industrial Quality Control* **XX**, No. 10, 18–22 (April 1964).
6. R. Schlaifer, *Probability and Statistics for Business Decisions*, McGraw-Hill Book Company, New York, New York, Chapter 36 (1959).
7. E. L. Grant, *Statistical Quality Control*, McGraw-Hill Book Company, New York, New York, 515–534 (1964).
8. D. M. McNamara, *Optimal Allocation of Test and Inspection Resources to a Multistage Manufacturing Process*, (Unpublished M.S. Thesis) School of Engineering and Applied Science, The George Washington University, February 22, 1968.

9. The reader may note that this approach differs from the usual Bayesian treatment. In the latter, $P(A_i)$ may be defined by a binomial probability distribution with parameters N and p, where p is a random variable whose distribution is specified.
10. S. S. Wilks, *Mathematical Statistics*, John Wiley & Sons, New York, New York, Chapter 6 (1964).