This paper discusses a control method for reducing the operating costs of a production system by continual modification of the planning operations.

The method improves resource allocations by adjusting the mathematical model of the production system to actual system performance.

The results of some preliminary experimental work with a simulated fabrication shop are presented.

Fabrication and assembly operations

Part VII Adaptive control in production planning

by S. Shapiro

Methods for allocating resources in shops with fabrication activities are generally based on a particular mathematical model of the shop activities. The methods often assume that the model is completely accurate and that the shop activities are invariant. In practice, however, one or both of these assumptions may be invalid.

The method described here continually modifies the resource allocation program on the basis of past and current system performance. This method has been experimentally implemented in the form of an IBM 7094 program used in conjunction with a resource allocation program.[1] An evaluation of the method in a simulated fabrication shop indicated a definite improvement in the optimization technique.

the production system

In allocating the resources, an economic production system should be composed of the principal functions shown in Figure 1.[2] Planning for the initial period is based on the forecasting function, which uses historic sales data to determine seasonal, trend, and smoothing factors, as well as available statistical data descriptive of the product line. For subsequent planning periods, the resource allocator compares the actual demand of the previous planning period with factors from the forecasting function to predict the demand for the current period. Data from the previous period parameters are adjusted initially to give the best performance, and no further modification of these parameters is required. If the behavior of the shops changes with time, the controller must continually modify the parameters to update the model. The modification may be accomplished either automatically by a computer or by human inspection and intervention.

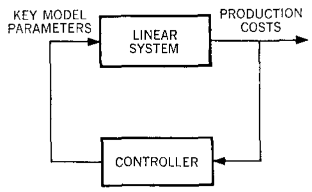
It is worthwhile to note that the choice of the parameters to be modified and the criterion of goodness for the model are somewhat arbitrary. However, once the parameters have been chosen, they are modified according to a suitable control method which is discussed below.

The main objective in adjusting the mathematical model to better approximate the shops is, of course, to reduce the actual operating costs of the entire production process as much as possible. Therefore, the most meaningful control criterion aims at model changes that will result in a reduction of production costs. When the operating costs of the system have been brought to a minimum, the closest model approximation of the shops has also been found.

control method

An adaptive control method has been designed for use with a fabrication job shop. For a given set of economic conditions (sales forecast, actual demand, etc.), it is assumed that the total production costs (labor, inventory, and backlog costs) are the output of a linear system. As inputs to this linear system, certain variable key model parameters are chosen. These parameters should be selected on the basis of their effect on production costs. Figure 3 shows how a controller provides a simple feedback loop.

Figure 3 Control loop



It should be emphasized that the assumption of a linear system is only used as a first approximation. In all probability, a highly complex process cannot be adequately described by a linear system. However, the result obtained by use of a simple linear-systems analysis can then be improved by a suitable non-linear system that provides a closer approximation of the actual production process.

In general, the number of key model parameters is determined by the number of *state variables*. These variables define the minimum number of system variables necessary to fully describe the state of the system. If the state variables are known, the entire future of the system is a function of inputs. For a free-falling body, for example, position and velocity (or some non-singular linear transformation of the two) are the state variables. In addition to position and velocity, any other variable would be completely redundant. However, one of these variables alone would be insufficient to predict the future state.

We formulate our linear system as follows:

$$\mathbf{x}(t + 1) = A\mathbf{x}(t) + B\mathbf{u}(t), \qquad t = 1, 2, 3, \cdots$$

where $\mathbf{x}(t)$ is the vector of state variables at time t, $\mathbf{u}(t)$ is the vector of the key parameters to be varied, and A and B are matrices (assumed time-invariant in our case). We also assume that the production cost $z(t)$ is a linear function of $\mathbf{x}(t)$ and $\mathbf{u}(t)$, i.e.,

$$z(t+1) = \mathbf{b}'\mathbf{x}(t) + \mathbf{e}'\mathbf{u}(t),$$

where b' and e' are constant vectors. The unknown matrices A and B and the vectors $\mathbf{b}'$ and $\mathbf{e}'$ are estimated by a least-squares technique. We assume that the parameter vector is constrained between two values, $\mathbf{u}^{\min} \leq \mathbf{u}(t) \leq \mathbf{u}^{\max}$, irrespective of time.

It is our main objective to minimize the total production cost over T periods. First, we consider the production process for the first period. If the state of the system at time $t = 1$ (the beginning of period 1) is $\mathbf{x}(1)$, and the vector of parameters is $\mathbf{u}(1)$, the state of the system at time $t = 2$, $\mathbf{x}(2)$, is given by

$$\mathbf{x}(2) = A\mathbf{x}(1) + B\mathbf{u}(1),$$

and the cost at time $t = 2$, $z(2)$, is given by

$$z(2) = \mathbf{b}'\mathbf{x}(1) + \mathbf{e}'\mathbf{u}(1).$$

Similarly,

$$\mathbf{x}(3) = A\mathbf{x}(2) + B\mathbf{u}(2),$$

and

$$z(3) = \mathbf{b}'\mathbf{x}(2) + \mathbf{e}'\mathbf{u}(2).$$

By substituting $\mathbf{x}(2)$, as given above, we get

$$\mathbf{x}(3) = A^2\mathbf{x}(1) + AB\mathbf{u}(1) + B\mathbf{u}(2)$$

$$z(3) = \mathbf{b}'A\mathbf{x}(1) + \mathbf{b}'B\mathbf{u}(1) + \mathbf{e}'\mathbf{u}(2).$$

The total cost at the end of two periods is given by

$$z(2) + z(3) = (\mathbf{b}' + \mathbf{b}'A)\mathbf{x}(1) + (\mathbf{b}'B + \mathbf{e}')\mathbf{u}(1) + \mathbf{e}'\mathbf{u}(2).$$

We now want to choose the parameter vectors $\mathbf{u}(1)$ and $\mathbf{u}(2)$ in such a manner as to minimize $z(2) + z(3)$. It is obvious that $\mathbf{u}(1)$ affects only the second term of the three terms forming the sum, and $\mathbf{u}(2)$ affects only the third term of the sum. Since the first term is a function of only the initial state of the system $\mathbf{x}(1)$, it is also clear that this term is independent of $\mathbf{u}(1)$ and $\mathbf{u}(2)$. For minimum cost, we choose $\mathbf{u}(1)$ to minimize $(\mathbf{b}'B + \mathbf{e}')\mathbf{u}(1)$ and choose $\mathbf{u}(2)$ to minimize $\mathbf{e}'\mathbf{u}(2)$. Supposing that $\mathbf{e}'$ and $\mathbf{u}(2)$ are both $1 \times m$ vectors, $(e_1, e_2, \cdots, e_m)$ and $(u_1, u_2, \cdots, u_m)$ respectively, we have

$$\mathbf{e}'\mathbf{u}(2) = e_1u_1 + e_2u_2 + \cdots + e_mu_m.$$

The problem is now to choose each u_i $(i = 1, 2, \cdots, m)$ in such a way as to minimize $\mathbf{e}'\mathbf{u}(2)$. This is achieved by taking $u_i = u_i^{\max}$ if e_i is negative, and by taking $u_i = u_i^{\min}$ if e_i is positive. We minimize the second term in the same way, using the vector $(\mathbf{b}'B + \mathbf{e}')$ instead of $\mathbf{e}'$. If the ith component of this vector is positive, we set $u_i = u_i^{\min}$, and if the ith component is negative, we set $u_i = u_i^{\max}$.

In a similar way we find:

$$\mathbf{x}(4) = A\mathbf{x}(3) + B\mathbf{u}(3)$$
$$= A^3\mathbf{x}(1) + A^2B\mathbf{u}(1) + AB\mathbf{u}(2) + B\mathbf{u}(3).$$
$$z(4) = \mathbf{b}'\mathbf{x}(3) + \mathbf{e}'\mathbf{u}(3)$$
$$= \mathbf{b}'A^2\mathbf{x}(1) + \mathbf{b}'AB\mathbf{u}(1) + \mathbf{b}'B\mathbf{u}(2) + \mathbf{e}'\mathbf{u}(3).$$

Then,

$$\begin{aligned} z(2) + z(3) + z(4) = {} & \mathbf{b}'(I + A + A^2)\mathbf{x}(1) \\ & + (\mathbf{b}'B + \mathbf{b}'AB + \mathbf{e}')\mathbf{u}(1) \\ & + (\mathbf{b}'B + \mathbf{e}')\mathbf{u}(2) + \mathbf{e}'\mathbf{u}(3), \end{aligned}$$

and minimization is achieved as above. In general, for T periods,

$$\begin{aligned} z(2) + z(3) + \cdots + z(T+1) = {} & \mathbf{b}'\left(\sum_{t=1}^{T} A^{t-1}\right)\mathbf{x}(1) \\ & + \left[\mathbf{b}'\left(\sum_{t=1}^{T-1} A^{t-1}\right)B + \mathbf{e}'\right]\mathbf{u}(1) \\ & + \left[\mathbf{b}'\left(\sum_{t=1}^{T-2} A^{t-1}\right)B + \mathbf{e}'\right]\mathbf{u}(2) \\ & \vdots \\ & + [\mathbf{b}'B + \mathbf{e}']\mathbf{u}(T) + \mathbf{e}'\mathbf{u}(T+1). \end{aligned}$$

Minimization of the production costs is performed term by term, starting with the second term of the right hand side of the above equation, in a similar manner as shown for the first few periods.

experimental results

The control method described was programmed for the 7094, and several experiments were performed by use of a simulated job shop that incorporated actual factory data. A random number generator simulated five different operating conditions of the shop. A resource allocation program was used to allocate resources over a 12-month period, and actual production costs were recorded as the simulated shop was run according to this plan. The only reason for simulating the shop was the unavailability of the facilities which otherwise could have been used directly.

Rates of labor, inventory, and backlog were selected as the key parameters for the mathematical model. First, these parameters were varied in a random manner, and the resulting production costs were recorded. Then a linear system approximation of the entire process was made, using a least-squares regression. Finally, the control strategy was formulated as described above and then programmed. To minimize the production costs over twelve monthly periods, the parameters were changed, at the beginning of each period, in accordance with our control method.

For each of the five operating conditions in the shop, the net operating costs for the resource allocations were computed. The costs included all factors affected by changes in the resource allocator, such as overtime, hiring and dismissal, inventory, etc. Fixed costs were not taken into consideration. In comparison to resource allocations based on a non-changing mathematical model, an average 9.3 percent reduction in production costs was achieved by use of the control method described. The results are shown in Table 1. It should be noted that these results were achieved with a linear systems approximation. A closer approximation of the actual production process could be provided by a suitable non-linear system.

Table 1 Experimental results

Control case	*Production (hours)*	*Fixed cost*	*Total cost*	*Net cost*	*Net improvement (dollars)*	*(percent)*
No control	26,056	$70,351	$121,548	$51,197	None	None
1	25,393	68,561	116,266	47,705	$3,492	7.3%
2	25,938	70,032	116,550	46,518	4,679	10.1
3	25,270	68,229	114,706	46,477	4,720	10.1
4	25,535	68,945	116,543	47,598	3,599	7.6
5	26,466	71,458	117,390	45,932	5,265	11.5
Average				46,846	4,351	9.3

FOOTNOTES

1. When these tests were defined, the methods of Holt, Modigliani et al. were available; see Holt, Modigliani, Muth, and Simon, *Planning Production, Inventories, and Work Force*, Prentice-Hall, Englewood Cliffs, New Jersey (1960). However, the methods described in Part IV of this paper have been developed since and may be used instead.
2. This figure is a simplified version of the control scheme of Part I of this paper and emphasizes the functions discussed here.