An approach to the preparation and evaluation of preliminary plans for a discrete manufacturing enterprise is outlined.

Some major data processing problems arise in this type of long-range planning. Mathematical techniques applicable to the solution of these problems are discussed.

Fabrication and assembly operations

Part II Long-range planning techniques

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The basic problem confronting the planner in a manufacturing enterprise is the circularity in planning. Production planning assumes knowledge of the future capacity of the plant. This capacity is partially dependent on the projected work load in the plant. In turn, the future work load depends on production plans.

To be explored here is a method for escaping this circularity; the method projects future plant capacity with the aid of a linear programming model that observes the technological precedences occurring in the projects to be scheduled. Once an initial resource projection is available, a variety of formal methods are used for the detailed scheduling of projects. The procedures employed for detailed scheduling are based on the initial projection of plant capacity. This initial capacity is used as a restriction that is relaxed only when necessary for the adjustment of due dates of scheduled projects. Once a detailed schedule is obtained, it is possible to reallocate capacity so that the present value of cash outflow for labor is minimized; labor requirements, material costs, and income flows can be used to evaluate a schedule. Decisions to adjust schedules can be conditioned by these data in one of the scheduling procedures to be discussed.

This part of the paper proposes a model on which the scheduling function can be defined for a plant. The model includes such features as precedence relations, shop capacity, the cost of changing the resource level, due-date requirements for projects, lateness penalties, and the cost of materials. With the model, a meaningful cost evaluation can be made of a given schedule, and schedules can be compared and ranked.

basic model objects

In order to specify a model, the basic objects in the model must be defined.

A *resource*, for the purposes of this model, includes productive entities such as workers by skill class, machines, and tools. Because money does not receive the same logical treatment as the other entities listed, it is not included as a resource. A *plant*, a physical and operational section of an industry, is subdivided into more or less separated resource clusters called *shops*. At any given point in time, a shop consists of the cluster of physical resources and skilled laborers assigned to it. Both the animate and inanimate portions of a shop, and thus the shop capacity, can change as a function of time.

An *aggregate arc* is a cluster of activities necessary to complete the fabrication or assembly of some definable subassembly or end item. It is assumed that an aggregate arc, once started, can be completed without the prior completion of activities outside the aggregate arc. Associated with an aggregate arc are two quantitative measures called "length" and "weight." The *length* of an arc is the estimated duration in shifts necessary for arc completion on the basis of "normal" shop practice. The *weight* of an arc consists of the number of resource hours, including man hours by skill class, facility hours, and tool hours, needed for arc activities. Weight is a vector quantity in which each component corresponds to a different resource unit.

A *project* is a directed, connected, acyclic network composed of aggregate arcs. Associated with each project is a set of aggregate arcs which cannot have common elements with any other such set.

planning functions

Within the framework of the model, it is possible to describe the functions associated with planning. Given a collection of projects and due dates, the objectives of the *scheduling* function are (1) to produce for aggregate arcs a set of start and finish dates that do not violate the capacity restrictions of the shops and (2) to determine, with the class of possible schedules, one schedule that has reasonable economic characteristics in terms of cash flows into and out of the enterprise. Given a set of projects with due-date commitments and an incremental project, a due date for this project is determined by the *negotiation* function which maximizes the economic criteria. Given an established schedule and a set of economic inputs that govern labor costs, an optimal allocation of labor is determined by the *labor allocation* function. Finally, the *economic analysis* function tabulates and processes certain identifiable portions of the cash flows implied by the schedule. For example, direct labor costs and material purchases for project use during the schedule period are discounted and computed. To find the enterprise's cash flow, these costs, together with lateness penalties, can be balanced against receipts for goods produced.

In order to perform a scheduling function, it is necessary to acquire and maintain input/output data. The input data required are listed in Table 1 in the form of general categories of information. The types of output information to be expected are listed in Table 2. The output data can be aggregated as desired and displayed in a variety of ways.

Table 1 General categories of input data

Period length	Some defined length of time (1 month for example).
Planning horizon	The time beyond which no aggregate arc can be admissibly scheduled. In practice, it is desirable that this horizon be as large as possible.
Period of interest	The number of periods (as defined above) within the planning horizon, subject to economic analysis.
Projects and due dates	A group of projects, with each project represented by a set of aggregate arcs.
Priority	A numbering of projects in order of importance to management.
Initial commitments	The commitments on resources at the beginning of the first period of interest.
Dominating inventory schedule for raw materials	Supplied schedule for purchased goods by the purchasing department, for each period in the horizon, in the form of quantity, part number, and period. The schedule represents the maximum obtainable.
Initial resource availability	The availability of resources at the beginning of the first period of interest.
Earliest start date	The earliest date associated with each aggregate arc.
Proposal	A single project which has a range of admissible due dates rather than a specific due date.
Resource availability	The desired level of resources at the end of the last period of interest. This level should be sufficiently high so that any single aggregate arc under consideration can fit into some succession of periods after the last period of interest.
Raw material requirements	The raw material requirements for each aggregate arc.
Raw material cost	Expected cost data for raw materials.
Cost of labor	The wage rate by skill class and shift, and the cost of increasing and decreasing the labor force by skill class.
Income by project at due date	The income of each project at its due date, penalties for lateness, and bonus for earliness included.
Discount rate	The discount rate(s) for cost of incoming and outgoing money.
Labor capacity	The capacity of each shop by skill class and period.

Table 2 General categories of output data

Schedules	Expected start and finish dates for each aggregate arc.
Resource requirements	Average requirements over each time period listed by shop.
Labor allocation	Optimal staffing policy and premium labor allocation to satisfy resource requirements listed by time period.
Purchasing requirements	Input information for purchasing function.
Economic data	Expected present value of the schedule, discounted cash flow out (broken down as desired), and discounted cash flow in (broken down as desired).

arc loading

Since an aggregate arc has dimensions related to duration and resource hours, it is possible to demonstrate the loading of a single aggregate arc onto its shop during a certain time interval so that the average per-period capacity of the shop is not exceeded. The formal procedure for accomplishing this loading is now illustrated.

An aggregate arc A of length t^* is to be scheduled between time t_1 and time t_1+t^*. Let τ denote the time period index $(1,\cdots,T)$, and r_1 the total man periods of resource #1 required for A. For simplicity, we suppose that A requires only one type of resource. $R_1, \cdots, R_T$ represent the currently allocated quantities of resource #1 in periods $1, \cdots, T$. Similarly, $R_1^*, \cdots, R_T^*$ give the total capacity of the shop in time periods $1, \cdots, T$. The length of a single period is denoted by t. Note that by hypothesis, $r_1/t^* \leq R_i^*$ for all i sufficiently large. Period τ begins at $(\tau-1)t$, and ends at τt.

If R_τ^{new} denotes the resource requirements during period τ after the loading of arc A, R_τ^{new} can be determined by the following formulas:

$$R_\tau^{\text{new}} = R_\tau,$$

provided that $t_1 \geq \tau t$ or $t_1 + t^* \leq (\tau - 1)t$;

$$R_\tau^{\text{new}} = R_\tau + \frac{r_1}{t^*}\{\min[\tau t, t_1 + t^*] - \max[(\tau - 1)t, t_1]\},$$

provided that $R_\tau^{\text{new}} \leq R_\tau^*$ for all τ.

If $R_\tau^{\text{new}} > R_\tau^*$ for any τ, then A cannot be loaded contiguously in the interval t_1 to $t_1 + t^*$. In this case, $R_\tau^{\text{new}} = R_\tau$ for all τ, and A must be loaded into another time interval.

This procedure is reversible in the sense that one can remove a previously loaded arc. Moreover, the extension of this procedure to the case of a single arc with more than one resource merely requires separate arc loading for each resource, and testing for feasibility of all resources with respect to the arc loading.

initial capacity projection

The initial capacity projection is an assignment of future capacity to each shop in the plant during the period of interest. This projection is arrived at by solving the linear programming model defined below. If this model has no solution, it is an indication that there is an incompatibility between the desired project due dates and the capacities of the shop. However, a solution of the model does not necessarily guarantee that there exists a schedule of the arcs that realizes this solution. Nevertheless, the optimal solution of the model represents a good guess with regard to future work-force requirements and provides a lower bound for the cost of labor required to complete the projects specified.

The variables of Table 3 and the equations below are necessary to completely specify a linear programming formulation for the

Table 3 Definition of variables

i	Skill-class index.
τ	Time-period index $(1, \cdots, T)$.
j	Shop index.
r	Shift code; 1—first shift straight time 2—first shift overtime 3—second shift straight time 4—second shift overtime.
$x_{ij\tau}$	Number of man periods of skill class i needed in shop j during period τ.
$R_{ij\tau}$	Number of man periods of skill class i in shop j that must be expended by period τ to meet the due dates of all projects. $R_{ij\tau}$ is computed under the assumption that each aggregate arc is started at the latest time consistent with the desired completion date of the project.
$R'_{ij\tau}$	Number of man periods of skill class i that must be expended in period τ or later to satisfy start date requirements. $R'_{ij\tau}$ is computed under the assumption that each activity is started at the earliest time consistent with the project's precedence relations.
$\overline{W}_{ij\tau}$	Single shift capacity for skill class i, shop j, and period τ, expressed in number of men.
$C_{i\tau r}$	Cost per man period of skill class i in shift r, discounted with respect to period τ.
K_{ir}	Conversion of factors with the dimension "man-periods/man."
$F_i^{\min}$	Final desired minimum number of men of skill class i for period T.
$F_i^{\max}$	Final desired maximum number of men of skill class i for period T.
B_i	Present number of men in skill class i.
$\Gamma_{i\tau}^-$	Cost of laying off one worker in skill class i, discounted with respect to period τ.
$\Gamma_{i\tau}^+$	Cost of hiring one worker in skill class i, discounted with respect to period τ.
$W_{ij\tau r}$	Number of men assigned to shop j, in skill class i, during period τ, on shift r.
$W_{i\tau}^+$	Number of men hired in skill class i during period τ.
$W_{i\tau}^-$	Number of men layed off in skill class i during period τ.

most economical allocation of labor to each shop. It should be noted that this model has similarities to the economic-lot-size-model treatment developed by Dzielinski, Baker, and Manne.[1]

The variables $R_{ij\tau}$ and $R'_{ij\tau}$ are used within two types of restraints:

$$-\sum_{k=1}^{\tau} x_{ijk} \leq -R_{ij\tau}$$

$$-\sum_{k=\tau}^{T} x_{ijk} \leq -R'_{ij\tau} ,$$

where $i = 1, \cdots, n$, and $j = 1, \cdots, m$, and $\tau = 1, \cdots, T$.

The additional constraints necessary to specify the resource allocation model can be expressed in terms of the defined variables and constants. The four subscripts of W, some of which may be specified by a decimal digit, always refer to the indices i, j, τ, and r:

$$x_{ij\tau} = \sum_{r} K_{ir} W_{ij\tau r}$$

Shop capacity:

$$W_{ij\tau 1} + W_{ij\tau 2} \leq \overline{W}_{ij\tau}$$

$$W_{ij\tau 3} + W_{ij\tau 4} \leq \overline{W}_{ij\tau}$$

Final conditions:

$$-\sum_{j} (W_{ijT1} + W_{ijT3}) \leq -F_i^{\min}$$

$$\sum_{j} (W_{ijT1} + W_{ijT3}) \leq F_i^{\max}$$

Initial conditions:

$$\sum_{j} (W_{ij11} + W_{ij13}) = B_i$$

Minimize

$$\sum_{i,j,\tau,r} C_{i\tau r} W_{ij\tau r} + \sum_{i,\tau} \Gamma^{+}_{i\tau} W^{+}_{i\tau} + \sum_{i,\tau} \Gamma^{-}_{i\tau} W^{-}_{i\tau},$$

where

$$W_{ij\tau r} \geq 0, \qquad W^{+}_{i\tau} \geq 0, \qquad W^{-}_{i\tau} \geq 0,$$

and

$$\sum_{j} W_{ij\tau 1} + \sum_{j} W_{ij\tau 3}$$

$$= W^{+}_{i\tau} - W^{-}_{i\tau} + \sum_{j} W_{ij,\tau-1,1} + \sum_{j} W_{ij,\tau-1,3}$$

labor allocation

A schedule is called *aggregate feasible* if, as a result of loading the schedule arcs, the resources required in the period of interest do not exceed the per-period average capacities for these resources.

Given an aggregate feasible schedule, it becomes possible to assign manpower in such a way as to minimize the amount of money expended on a "present value" basis to meet the requirements for manpower generated in the schedule. This allocation model is similar to the initial capacity allocation model. In this case, however, the $x_{ij\tau}$ have been determined and are therefore treated as constants. Using the same symbology as the preceding model, a linear programming formulation of the labor allocation problem can be easily specified:

Requirements:

$$\sum_r K_{ir} W_{ij\tau r} \geq x_{ij\tau}$$

Shop capacity:

$$W_{ij\tau 1} + W_{ij\tau 2} \leq \overline{W}_{ij\tau}$$

$$W_{ij\tau 3} + W_{ij\tau 4} \leq \overline{W}_{ij\tau}$$

Final conditions:

$$-\sum (W_{ijT1} + W_{ijT3}) \leq -F_i^{\min}$$

$$\sum_j (W_{ijT1} + W_{ijT3}) \leq F_i^{\max}$$

Initial conditions:

$$\sum_j (W_{ij11} + W_{ij13}) = B_i$$

Minimize

$$\sum_{ij\tau r} C_{i\tau r} W_{ij\tau r} + \sum_{i\tau} \Gamma_{i\tau}^{+} W_{i\tau}^{+} + \sum_{i\tau} \Gamma_{i\tau}^{-} W_{i\tau}^{-},$$

where

$$W_{ij\tau r} \geq 0, \qquad W_{i\tau}^{+} \geq 0, \qquad W_{i\tau}^{-} \geq 0,$$

and

$$\sum_j W_{ij\tau 1} + \sum_j W_{ij\tau 3}$$

$$= W_{i\tau}^{+} - W_{i\tau}^{-} + \sum_j W_{ij,\tau-1,1} + \sum_j W_{ij,\tau-1,3}.$$

economic analysis

A basis is now presented for assigning a numerical value as the "figure of merit" of a schedule. This is done in such a way that the figure of merit reflects preference between schedules in economic terms. It would be better if the difference in the figures of merit for two schedules would represent the present value of the dollar difference between two alternate courses of action. It will become evident that it is possible to define a figure of merit that signals a preferred schedule as well as the actual difference in value.

An approach to this problem of economic measurement is based on basic definitions. Suppose that money earns interest at a rate of λ per annum, compounded continuously. The rate λ is some preset parameter based on either the marginal return for capital invested, on the interest charged for commercial loans, or on some other reasonable element of cost. A payment of p dollars, t years hence, has a present value of $pe^{-\lambda t}$. In general, if payments, viewed as a function of time, can be assigned the Stieltjes notation $dF(t)$, the total amount of money received between t_1 and t_2 is

$$\left|\int_{t_1}^{t_2} dF(t)\right|.$$

Stieltjes integration, implied here, allows us to handle discontinuities in $F(t)$.

Presuming that money is discounted at the rate λ, the *present value* of a stream of payments $dF(t)$ can now be defined as P, where

$$P = \int_0^\infty e^{-\lambda t}\, dF(t).$$

It is interesting to observe that P is just the familiar Laplace-Stieltjes transform of $F(t)$ evaluated at λ.

The basic strategy is this: if dF_1 represents a stream of payments induced by a schedule, and if dF_2 represents the costs associated with the schedule, the present value (or figure of merit) of the schedule S, which we call $M(S)$, is expressed by the formula:

$$M(S) = \int_0^\infty e^{-\lambda t}\, dF_1(t) - \int_0^\infty e^{-\lambda t}\, dF_2(t).$$

If it is desirable to have two discount rates for money, λ_{in} and λ_{out}, depending on whether the money is incoming or outgoing, the formula for $M(S)$ becomes:

$$M(S) = \int_0^\infty e^{-\lambda_{\text{in}} t}\, dF_1(t) - \int_0^\infty e^{-\lambda_{\text{out}} t}\, dF_2(t).$$

The problem now consists of breaking up and identifying the components of $dF_1(t)$ and $dF_2(t)$ in such a way that a meaningful, readily obtainable evaluation of $M(S)$ is produced. It is assumed that all items in inventory during the period of interest can be evaluated in money.

The payment picture is fairly simple to describe. Of interest is only that portion of income attributable to work completed within the period. Detail parts are assumed to have an assigned price during the period of interest and preceding periods. It must also be assumed that the price of materials and labor is known for succeeding periods of interest. The income from a portion of a project completed during a period of interest has assigned to it a definite proportion of the present value of the income from the whole project. This proportion is given by the ratio of the present value of the work and materials expended during the current period of interest, divided by the present value of the work and materials expended on the entire project. Applying this reasoning to each project or portion of a project completed in the period of interest, the present value of the work completed or partially completed during the period of interest can be accumulated.

It is worth observing that present value of income from a project is also computable when the income data for the projects are subject to uncertainty in time, if this uncertainty is representable by a probability function. In this case, the present value of income is replaced by the expected present value of income.

Figure 1 Loading logic I

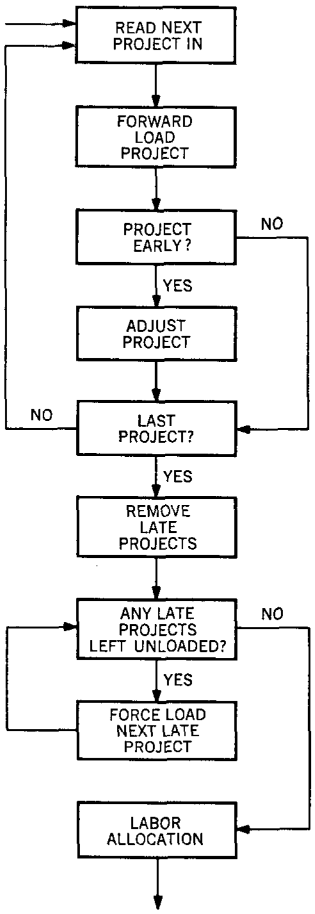
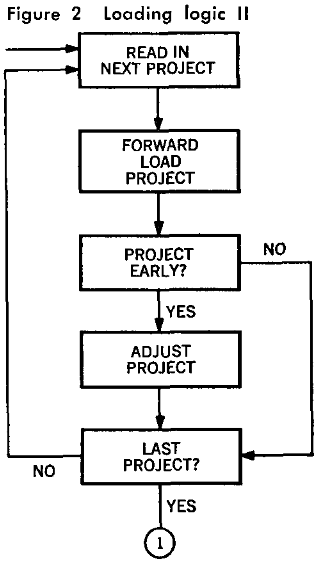


Figure 2 Loading logic II



The present value of the cost of production is broken down into components generated by labor and those generated by material. The present cost of labor appears in the solution of the labor allocation model previously described.

Scheduling of the aggregate arcs induces a schedule for the consumption of raw materials. This consumption produces a purchase schedule for the raw materials from vendors, which, in turn, originates a schedule of payments to vendors, actually a flow of money out of the enterprise. This flow of money, restricted to the period of interest, has a present value, which can be used as a statistic from which the present value of materials consumed can be estimated.

The above method of computing material costs does not include the cost of running the enterprise. These facts should be borne in mind when using this scheme for computing $M(S)$, since the exclusion of fixed costs means that $M(S)$ cannot be interpreted as profit. $M(S)$ is clearly a function of profit and discriminates for a fairly narrow range of operating levels between schedules in the sense that $M(S_1) - M(S_2)$ is the difference in profit between two schedules S_1 and S_2. The factor missing is, of course, the fixed costs induced by the respective schedules.

A collection of formal methods exists for generating aggregate feasible schedules in a constructive manner. Although there is at present no practical method for producing a schedule S that maximizes the figure of merit $M(S)$, it is felt that one of the formal methods can be used for the construction of a reasonable schedule. Following is a rough description of a programmable logic for producing an aggregate feasible loading of the plant. It is assumed that the initial capacity projection has already been accomplished.

Forward loading

1. Pick an unloaded arc which has no unloaded predecessors from the highest priority project network. Load this arc into the earliest time interval so that (*a*) the resultant loading does not exceed the initial capacity projection for its shop in any period, (*b*) the current earliest start date of the arc is not violated, and (*c*) the current dominating raw material availability schedule is consistent with the loading.
2. Recompute the earliest start dates for arcs in this project. Save the previous earliest-start dates.
3. Update the dominating raw materials schedule.
4. Repeat the first three steps until the project is loaded.

Adjust early project

This is a technique for removing an early project and reloading it with the earliest effective date adjusted toward the future. This step is repeated until the project is loaded within the initial shop capacity in such a way that it is moved closer to its due date without being late. Details for economically accomplishing this step are straightforward but involved and hence omitted here.

Force loading

1. Remove late projects.
2. Reload projects backwards from due date, using the shop capacity instead of the initial desired capacity.
3. If feasibility is violated, unload, add one period to the due date, and return to the first step.

In terms of these three steps and the labor allocation function, a loading logic may be similar to the one diagrammed in Figure 1.

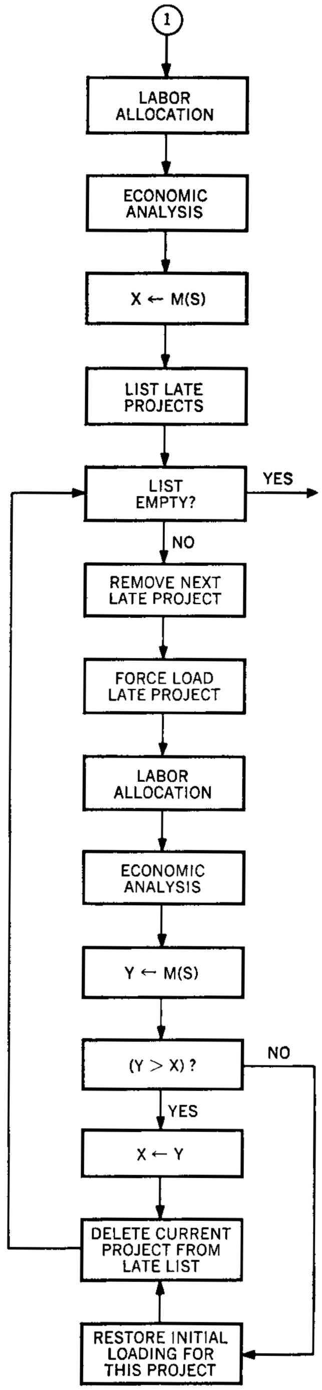
The example illustrated in Figure 1 is only a single element of a class of possible logics. Another variant, illustrated in Figure 2, utilizes the strategy of removing late projects one at a time. After the late project is removed, it is force loaded and the figure of merit of the resulting schedule is compared to the figure of merit of the preceding schedule. On the basis of this comparison, either the preceding schedule is restored or the new schedule is adopted. The flow chart of Figure 2 illustrates this procedure. The flow chart contains some additional blocks whose functions are self-explanatory. For negotiating a due date, it is feasible to use any method that results in a figure of merit of the schedule. This negotiation for a single project is accomplished by loading the project with different due dates. It is important to note that the scheduled finish date for a project is not necessarily the same or earlier than its due date.

It has been demonstrated that there is a class of methods for providing reasonable schedules, and that these schedules can be compared in a meaningful way. The problem of finding a minimum cost schedule has yet to be solved. The problem of determining a schedule with an optimal figure of merit lies, perhaps, in the direction of linear programming or, with additional restrictions, integer programming.

The problem of determining the arc length is conditioned by the characteristics of the resultant schedule and is, therefore, partially dependent on the solution to the problem. Under some circumstances, however, a preliminary estimate of the arc length can be made, for example, when the effective work force is stable and little interaction exists between arcs. It has been assumed explicitly that the arc length is invariant under changes in effective size of the work force and under changes in the composition of the load on the plant. This assumption is reasonable if arc lengths are conservatively estimated or the work force and plant load are stable. If the work force or plant load fluctuates, it would be desirable to make use of the fact in scheduling

A less ambitious tack to take might be to extend the initial labor allocation procedure. The capacity would then be increased parametrically, always along the path of least cost, within constraints on the location of the capacity in time. It is presently not clear how precisely to formulate this problem. The general idea,

Figure 2 Loading logic II continued



however, would be to have a succession of labor levels, each level fitting the requirements better, and each level being set in an optimal fashion.

A schedule feasible in aggregate is not necessarily feasible in detail. This means that scheduled start and finish dates for aggregate arcs may not be met in the plant. However, the extent to which the schedule is met is indicated in the next iteration of the scheduling progress. This paper does not consider the relevant statistical data which might be collected and used to condition the inputs for the succeeding scheduling cycles. It has been tacitly assumed that the purchasing schedule is available to, but not produced by, the system. It might be profitable to include purchasing as a system function. This can be done at the hazard of making a class of assumptions about this purchasing function. However, the assumptions would further limit the class of industries to which the model is applicable. In view of the wide difference in purchasing procedure from industry to industry or even within a single enterprise, the production of a schedule for purchasing has been excluded as a system function at this time.

The creation and implementation of a planning model depends upon the practicability of providing inputs into the system. The required definitions for aggregate arcs, shops, and resources may be difficult to provide in some application environments.

CITED REFERENCE

1. B. P. Dzielinski, C. T. Baker, and A. S. Manne, "Simulation tests of lot size programming," *Management Science* **9**, No. 2 (January 1963).