The art of fractal landscapes

by F. K. Musgrave
B. B. Mandelbrot

Synthetic images of fractal landscapes have moved beyond science to enter the domain of "art for art's sake." We discuss some of the ramifications of this artistic aspect: improving the fractal description of terrains, adding fractal textures to surfaces, and using parallel computers. We illuminate the peculiarities of attaining artistic self-expression in representational imagery purely through formal logic, and discuss its import.

Introduction

Fractal geometry has achieved a significant role in the description of nature [1], because it offers insight into phenomena the complexity of which was previously intractable, such as the dynamics of the formation of diffusion-limited aggregates [2]. It is not surprising, since fractal geometry was developed largely with the aid of computer graphics, that computer graphics has always employed fractals for the description of natural phenomena. Realistic fractal "forgeries of nature" have been with us for 15 years, having been originally undertaken by Mandelbrot and Handelman [1] and especially Voss [3], the early images being created at the IBM Thomas J. Watson Research Center in Yorktown Heights, New York. A more recent development is the use of fractal models of nature, not for the purposes of nature, but *primarily in service of fine art*. This has been one of the main endeavors of Mandelbrot's fractals project at Yale University. The twin goals of this work have been to improve models of natural phenomena for computer graphics and to set a new standard for both realism and aesthetic quality in computer-synthesized landscape images.

Fractal models of nature

At the outset of our work in 1987, the standard fractal terrain model for computer graphics was fractional Brownian motion, extended to two dimensions. By design fBm is homogeneous, isotropic, and symmetric about the horizontal plane. Unfortunately, none of these properties holds for natural terrains on large scales. Terrain in nature tends to be horizontally asymmetric: Peaks are generally rougher than valleys. Furthermore, features due to erosion are quite salient: stream beds, river drainage systems, talus slopes, and other spatially coherent features which are not present in the naive fBm model.

A variety of approaches and algorithms, from simple modifications of fBm to elaborate physical simulations of fluvial erosion, have been developed at Yale. The first of our attempts to include erosion features included a terrain-generation-time algorithm for building a fractal terrain patch with an embedded fractal drainage network, and variations on the (usually Gaussian) distribution of the random numbers used in polygon-subdivision fBm synthesis schemes, as described by Mandelbrot [4]. Subsequently, schemes for exerting local control over the statistics of fBm have been developed [5]. These methods facilitate the creation of novel terrain models, such as impressions of ancient, heavily eroded mountain ranges, and very large scale, heterogeneous terrains including mountains, foothills, and plains. Work is underway in improving physical erosion models to generate the space-filling dendritic patterns of fluvial drainage networks [5], and to create realistic terrain with spatially coherent ridge lines.

Along with improved models, improved rendering algorithms have been developed [5, 6]. Our images involve ray-tracing terrain patches tessellated by up to 10 million triangles. More recently we have begun to employ procedural rendering techniques, wherein the terrain model is generated only when and where needed. This gives us the capability of creating renderings where the triangles tessellating the surface are always pixel-sized, and thus invisible as such.

Proceduralism and parallel computations

Proceduralism is a very important aspect of the process by which our images are produced. Procedural textures are used extensively in our images to provide visual detail where there is no geometric detail, i.e., on Euclidean surfaces. A simple, perfectly smooth sphere can be made to appear as a moon or a planet through the artful use of fractal procedural textures. This is generally easier than attempting to model, geometrically, the terrain over an entire planetoid (though Voss has done just that [3], and very nicely).

These procedural models tend to be computationally expensive. Thus, we have employed parallel computation strategies in our renderings right from the beginning. C-Linda [7] has proved especially useful for our purposes. It has allowed us to develop portable, parallel applications in the C language which can run in distributed- and shared-memory parallel architectures, while retaining their capacity to run sequentially. This is very important because new, faster machines and networks of machines are becoming available at a rate far greater than our ability to generate machine- or architecture-specific versions of our software packages. Currently, our environment of choice is a network of IBM RISC System/6000™ workstations.

Typically, the parallelism employed is very simple: Each rendering process has access to the entire model, either in shared memory or in distributed copies, and works on tasks which comprise the rendering of some relatively small subsection of the image. This subtask can be either a single scanline or, in the case of context-sensitive adaptive antialiasing, some relatively small rectangular area of the image. A supervisory process determines the task descriptions, doles out the tasks, and collects the results, which it writes to nonvolatile storage. As ray-tracing is computationally intensive, there is very little overhead associated with communication, and we generally obtain a close-to-linear speedup with the number of processors employed.

An application

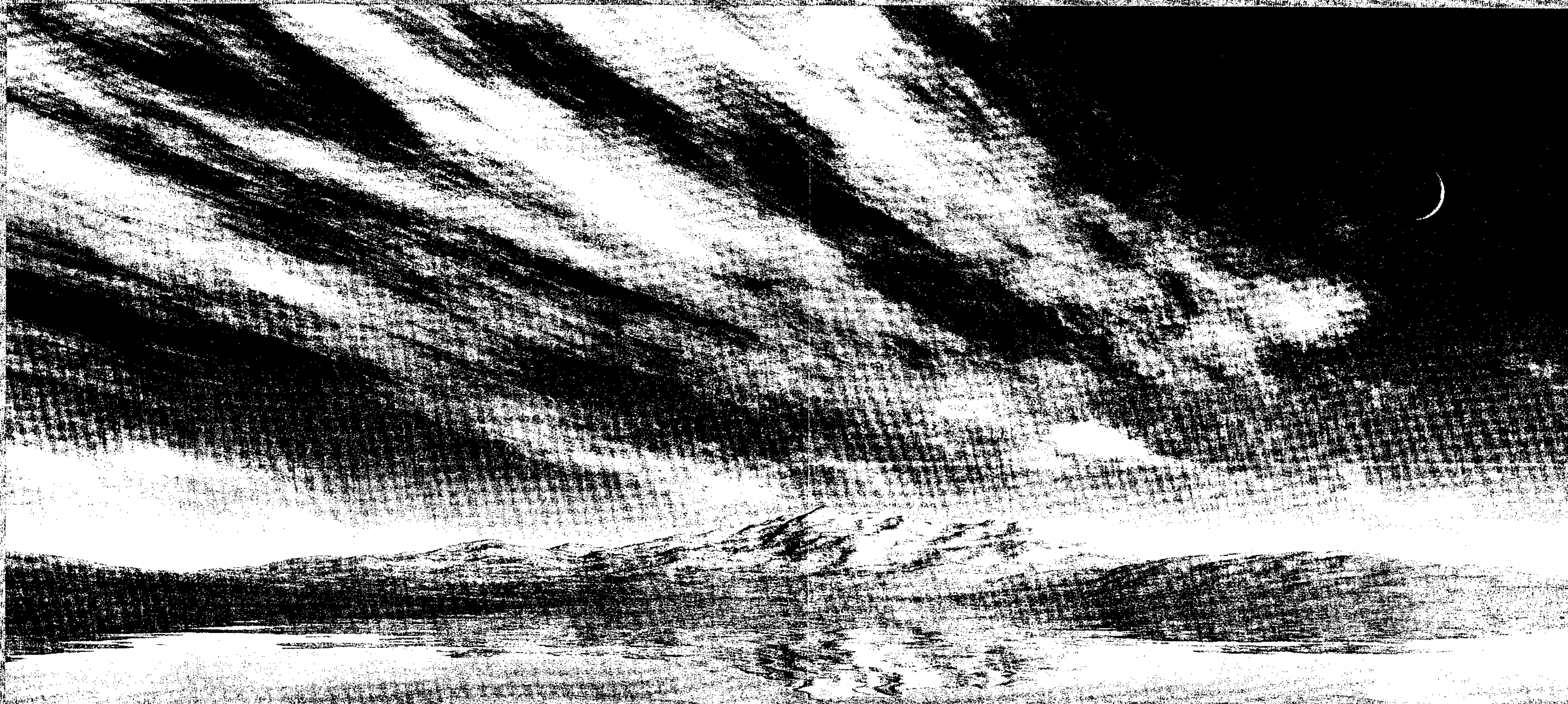
Our work in fractal models of nature has also encompassed scientific visualization. Recently we have been working with the Visualization for Planetary Exploration group at NASA Ames Research Center on improving the quality of synthetic landscapes of the planet Mars. At VPE, terrain elevation data of the planet derived from Viking orbiter images are being used in a virtual reality setting to facilitate exploration of planetary geology. Because the terrain elevation data are available only at a fixed (approximately one-kilometer) resolution, synthetic images typically lack detail in the foreground. Fractal textures have proved useful both for enhancing visual appeal and for highlighting morphological subtleties not readily apparent from shading cues alone.

Artistic process

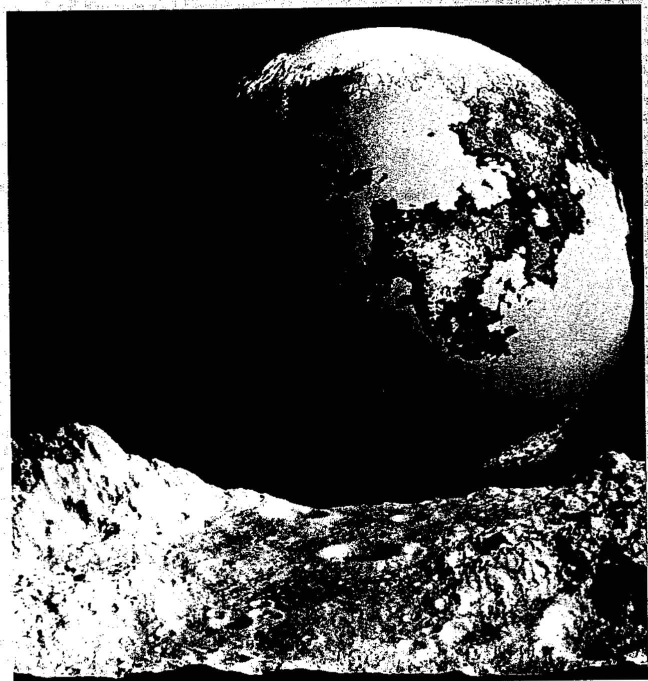
We have always maintained a certain purity in the process of making our fractal landscape images: Each image is constrained to represent the unadulterated output of the rendering program, which in turn takes unmodified input from the random fractal modeling processes. Thus, we never retouch the finished image—with the exception of the addition of the artist's signature in the corner—and we never "fiddle with" the fractal terrain models or texture functions on a local scale.

In practice, this means that we choose a visual composition much as a photographer would: We view the "world" granted by the random modeling process from a variety of points and angles, moving about actively until we choose a view and framing which provides an aesthetically pleasing visual composition. As with landscape photography, we do not seek to change the morphology of our subject matter to suit our aesthetic ends, but "photograph" the "world" as we find it. As with landscape photography, we reject most landscapes brought before us as uninteresting; we search through many instances of the random models until we find one which appeals to us.

To be sure, we have more control over our subject matter than the landscape photographer. We need not wait until the lighting is "just right"—we can set it up so. But the rules we adhere to have the side effect that any change to the input parameters will have a global effect. Thus, if we want to change the sharpness of a certain peak, and the color of a certain highlight, *all* peaks in the landscape become sharper, and *all* highlights change in color. This global, parametric control of our images distinguishes our artistic process from, for instance, landscape painting,



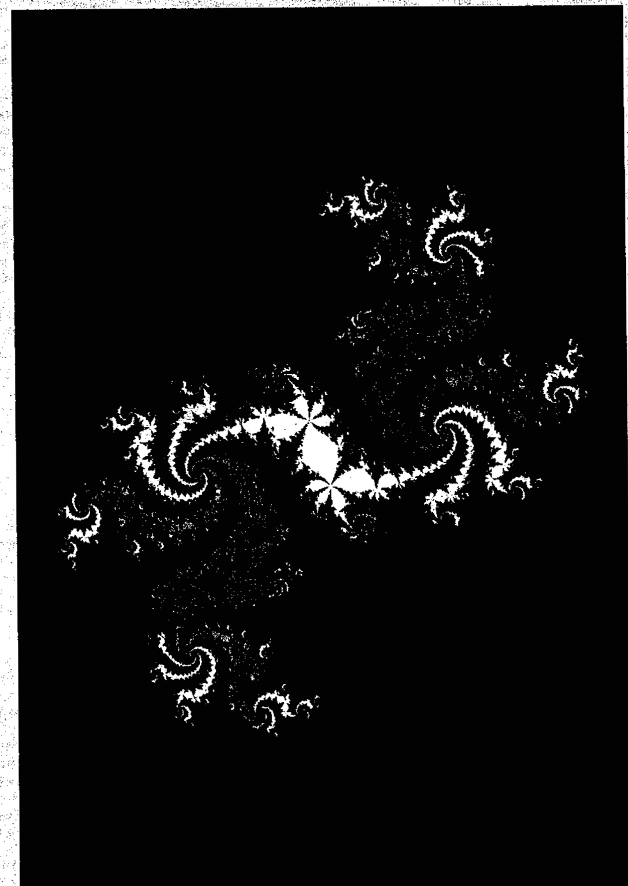
Zabriskie Point, by F. Kenton Musgrave, Craig E. Kolb, and Benoit B. Mandelbrot.
This image was conceived as an illustration for a technical paper on the model of the mirage seen in the middle foreground, yet it is also a sophisticated aesthetic statement; thus its import spans both science and art. Interestingly, the procedures used to generate the mountains, the clouds, the waviness of the mirage, the details on the surface of the moon, and the colors on the landscape are all relatively minor variations on the same underlying fBm function.



***Fractal Planetrise*, by Richard F. Voss, from *The Fractal Geometry of Nature* by Benoit B. Mandelbrot, W. H. Freeman, 1982**

***A snowy landscape that never was*, by Richard F. Voss, from *The Fractal Geometry of Nature* by Benoit B. Mandelbrot, W. H. Freeman, 1982**

LIFT, PLEASE



***A fractal dragon*, by Benoit B. Mandelbrot, from his book, *The Fractal Geometry of Nature*, W. H. Freeman, 1982**

where each brush stroke is a "local" act, not directly affecting other parts of the canvas.

Art and formal logic

Perhaps the most interesting and even significant aspect of this work is the formal character of the creative process. Our process of image production can be seen as consisting of developing a formal system and deriving theorems within that system. A formal system consists of axioms and rules of production; a "theorem" is derived by application of the rules of production to the axioms in the system. A computer program is, of course, logically equivalent to a formal system: The program embodies the rules of production, the input consists of axioms, and execution of the program represents the deterministic derivation of a "proof" for a theorem in the system. In the making of an image, we proceed by creating some rules of production, i.e., writing the computer program, then changing the axioms in the system. The latter may be seen as moving about in the n-dimensional parameter space (n being typically between about 100 and 300) seeking local aesthetic maxima.

The process is iterative; we constantly change and refine both the rules of production and the input axioms in search of a more pleasing result. As has been noted elsewhere [8], it also embodies the basic loop of scientific discovery: the formulation of a model, testing the model's predictions against observations of reality, and refinement of the model to make its predictions better match observations. In our case, the "observations" are sometimes not empirical measurements of "reality," but rather subjective judgments of aesthetic quality. Which type of measure we apply depends on whether we are currently seeking to maximize realism, as with the clouds in "Bay Fog," which appears on the front cover, or beauty, as with the colors in "Zabriskie Point." In the latter case, the "aesthetic gradient function" we seek to maximize is, by nature, not well defined. Thus, the logic involved is fuzzy yet still deterministic—even the "random" models are, in fact, only pseudorandom, and all results are therefore reproducible.

To attain artistic self-expression in representational imagery, strictly through the deterministic application of formal logic, is a strange and novel process. The peculiar way in which we go about this may be unprecedented in the history of the creative process. Our adherence to the "algorithmic purity" described above ensures that our images are, viewed from the paradigm of mathematical logic, indeed theorems proved in a formal system.

Reproduction and display

The theorem proved, of course, takes the form of an enormous string of binary digits—the image file. Normally the image file is interpreted visually through the video display hardware of the workstation monitor. "Accurate" reproduction of the image is not only difficult to define but also very hard to achieve in practice, even to subjective satisfaction. This constitutes research in progress. Unfortunately, the problem does not admit of a general solution; we can only hope to calibrate transformations between specific devices that are often unstable over time.

Problems of the reproduction of artistic images aside, we have enjoyed a modicum of exposure in the world of fine art. On April 19, 1990, "New York Notes—Music and Fractals" premiered at the Solomon R. Guggenheim Museum in New York. A reprise performance took place in April 1991 at Lincoln Center's Alice Tully Hall. The piece was designed as a part of Mary Cronson's "Works and Process" series, and was very experimental in nature. Many possible improvements, enhancements, and entirely new directions were apparent to all involved in the production; indeed, we may all feel that this represented no more than a tentative first step into an arena of unfathomed potential. For our part, we are dedicated to pushing forward the state of the art in fractal images, the capacity to display them, and the aesthetics embodied therein.

References

1. Benoit B. Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman & Co., New York, 1982.
2. Benoit B. Mandelbrot and Carl J. G. Evertsz, "The Potential Distribution Around Growing Fractal Clusters," *Nature* **348**, No. 6297, 143–145 (November 8, 1990).
3. Richard F. Voss, "Fractals in Nature: From Characterization to Simulation," *The Science of Fractal Images*, H. O. Peitgen and D. Saupe, Eds., Springer-Verlag, New York, 1988, pp. 21–70.
4. Benoit B. Mandelbrot, "Fractal Landscapes Without Creases and With Rivers," *ibid.*, pp. 243–260.
5. F. Kenton Musgrave, Craig E. Kolb, and Robert S. Mace, "The Synthesis and Rendering of Eroded Fractal Terrains," *Computer Graph.* **23**, No. 3, 41–50 (July 1989).
6. F. Kenton Musgrave, "Grid Tracing: Fast Ray Tracing for Height Fields," *Research Report RR-639*, Department of Computer Science, Yale University, New Haven, CT, July 1988.
7. S. Ahuja, N. Carriero, and D. Gelernter, "Linda and Friends," *IEEE Computer* **19**, No. 8, 26–34 (August 1986).
8. Gregory M. Nielson, "Visualization in Scientific and Engineering Computation," *IEEE Computer* **24**, No. 9, 58–66 (September 1991).

Received October 14, 1991; accepted for publication November 6, 1991

F. Kenton Musgrave *Yale University, Departments of Computer Science and Mathematics, Box 2155 Yale Station, New Haven, Connecticut 06520.* Mr. Musgrave has been a research assistant to Dr. Benoit Mandelbrot at Yale University since 1987. In that time he has worked on models of natural phenomena for computer graphics and the development of the computer as a medium for the fine arts. The fractal landscape images he has created have been published around the world in various technical and popular literature, and his technical papers have appeared in a variety of conference proceedings and journals. In 1989 Mr. Musgrave matriculated in the Ph.D. program in computer science at Yale; he expects to complete his doctorate in the spring of 1992. He received his M.S. in computer science from the University of California at Santa Cruz in 1987, and his B.A. in computer science from UCSC in 1984. Mr. Musgrave studied fine art and the physical sciences as an undergraduate at UCSC, Colgate University, and Skidmore College. He is a member of ACM SIGGRAPH and the IEEE Computer Society.

Benoit Mandelbrot *IBM Research Division, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598.* Dr. Mandelbrot is a physicist at the Thomas J. Watson Research Center of IBM, where he is an IBM Fellow, and a mathematician at Yale University, where he is Abraham Robinson Professor. For forty years, he has been seeking a measure of order in physical, mathematical, or social phenomena that are characterized by abundant data but extreme variability. The surprising aesthetic value of many of his discoveries has made him an eloquent spokesman for "the unity of knowing and feeling." Dr. Mandelbrot is best known as the founder of fractal geometry, and as the author of the books *Les Objets fractals*, 1975, 1984, and 1989 (translated into Italian, Spanish, Portuguese, and Basque) and *The Fractal Geometry of Nature*, 1982 (translated into Japanese, Chinese, and German). He is a Fellow of the American Academy of Sciences, a Foreign Associate of the U.S. National Academy of Sciences, and a Membre Titulaire de l'Académie Européenne des Sciences, des Arts et des Lettres. His awards include the 1985 F. Barnard Medal for Meritorious Service to Science ("Magna est Veritas"), granted by the U.S. National Academy of Sciences and Columbia University, the 1986 Franklin Medal for Signal and Eminent Service in Science from the Franklin Institute of Philadelphia, the 1988 Charles Proteus Steinmetz Medal from the IEEE and Union College, the 1988 Science for Art Prize from Moët-Hennessy-Louis Vuitton, the 1989 Harvey Prize for Science and Technology from the Technion in Haifa, Israel, and the 1991 Nevada Prize. He has also received a Distinguished Service Award for Outstanding Achievement from the California Institute of Technology, and a Humboldt Preis from the Alexander von Humboldt Stiftung. Dr. Mandelbrot is a graduate of the Paris Ecole Polytechnique; he holds M.S. and Ae.E. degrees in aeronautics from the California Institute of Technology and the degree of Docteur ès Sciences Mathématiques from the University of Paris. He has been recognized as a Doctor of Science, *honoris causa*, by Syracuse University, Laurentian University, Boston University, the State University of New York, and the University of Guelph, and as Doktor, *honoris causa*, by the Universität Bremen and Doctor of Humane Letters, *honoris causa*, by Pace University. Dr. Mandelbrot has had no formal teacher, but early in his work was most strongly influenced by Paul Lévy, Norbert Wiener, and John von Neumann. His early positions before joining IBM were with the CNRS in Paris, Philips Electronics, M.I.T., the Institute for Advanced Study, the University of Geneva, and the University of Lille and Ecole Polytechnique. On leave from IBM, he has been an Institute Lecturer at M.I.T. and a Visiting Professor of Economics, later of Applied Mathematics, and then of Mathematics at Harvard, of Engineering at Yale, of Physiology at the Albert Einstein College of Medicine, and of Mathematics at the Université de Paris-Sud, and a Professor of the Practice of Mathematics at Harvard.