

# Small-area fault clusters and fault tolerance in VLSI circuits

by C. H. Stapper

**In previous treatments of the manufacturing yield of fault-tolerant integrated-circuit chips, fault clusters were either assumed to be absent or relatively large in area. Presented here is a treatment in which the occurrence of small-area fault clusters is assumed. Four different types of statistical distributions are considered, and a criterion is described for determining whether small-area fault clusters are present.**

## Introduction

Statistical models which have been used for predicting integrated-circuit chip yield in the presence of large-area fault clusters have been reviewed in [1]. It was shown in [2] that clusters of this type intrinsically result in a correlation between the numbers of faults per chip in adjacent chips. Here we examine the case in which such a correlation is absent, and for which the clusters are designated as *small-area* clusters. It has been believed that small-area clusters should lead to fault distributions which follow a Poisson distribution. However, this has been found to be incorrect, because such clustering leads to an increase in the variability of the frequency distribution of the number of faults per chip. A Poisson distribution has a variance that is equal to

the mean. Fault clustering results in frequency distributions of the number of faults per chip with variances that are larger than the mean.

In the yield calculations of fault-tolerant chips with large-area fault clustering, chips were subdivided into circuit blocks [1]. Usable chips were obtained by disconnecting faulty circuit blocks and by utilizing only the fault-free blocks. This same approach is assumed here, and yield formulae are derived for single circuit blocks and groups of blocks. Negative binomial, Neymann Type A, Poisson binomial, and Poisson negative binomial distributions are examined.

## A negative binomial model

The number of faults in a circuit block can be denoted by a random variable  $X_i$ , where the values of the subscript  $i$  are used to indicate the different circuit blocks. The negative binomial distribution for the random variable  $X_i$  can then be written as

$$P(X_i = k) = \frac{\Gamma(\alpha' + k)}{k! \Gamma(\alpha')} \frac{(\lambda_{CB}/\alpha')^k}{(1 + \lambda_{CB}/\alpha')^{\alpha' + k}}, \quad (1)$$

where  $k$  is equal to an integer 0, 1, 2, etc.;  $\lambda_{CB}$  is equal to the average number of faults per circuit block; and  $\alpha'$  is the cluster parameter. The prime is used here to distinguish the cluster parameter in this distribution for small clusters from the one without a prime used in the case of the large-area clusters in [1]. Note also in Equation (1) the absence of a bar over the symbols  $\lambda_{CB}$ , because this fault distribution is not the result of a Poisson compounding process. The average number of faults is therefore no longer a grand average, as it was for large-area clusters.

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The yield for an individual circuit block is equivalent to the probability of finding no faults in such a block. Using  $k = 0$  in (1) therefore results in

$$Y_{ICB} = (1 + \lambda_{CB}/\alpha')^{-\alpha'}, \quad (2)$$

where  $Y_{ICB}$  denotes the yield of an individual circuit block. We next determine an expression for the yield associated with a group of  $n$  circuit blocks.

The number of faults in a group of  $n$  identical circuit blocks is equal to the sum of the number of faults in each individual circuit block. It therefore is a random variable given by

$$X = \sum_{i=1}^n X_i. \quad (3)$$

We now must find the probability distribution function of the random variable  $X$ .

The probability distribution function of a random variable can be determined from the associated probability distribution generating function. It was shown in [3] that the distribution generating function for (1) is given by

$$G_1(s) = \{1 + (1-s)\lambda_{CB}/\alpha'\}^{-\alpha'}, \quad (4)$$

where  $s$  is a parameter. Each circuit block has such a function associated with it, and for identical blocks these functions are equal to one another. It is generally known that the probability distribution generating function associated with the sum of independent random variables is equal to the product of the probability distribution generating functions associated with these random variables. For the  $n$  identical circuit blocks, this function therefore becomes

$$G_n(s) = G_1(s)^n = \{1 + (1-s)\lambda_{CB}/\alpha'\}^{-n\alpha'}. \quad (5)$$

In this function the ratio  $\lambda_{CB}/\alpha'$  has the same value as  $n\lambda_{CB}/n\alpha'$ . Equation (5) is therefore the probability generating function of a negative binomial distribution with an average number of faults equal to  $n\lambda_{CB}$  and a cluster parameter equal to  $n\alpha'$ . This probability distribution generating function therefore leads to

$$P(X=l) = \frac{\Gamma(n\alpha' + l)}{l! \Gamma(n\alpha')} \frac{(n\lambda_{CB}/n\alpha')^l}{(1 + n\lambda_{CB}/n\alpha')^{n\alpha' + l}}. \quad (6)$$

When  $l = 0$ , this becomes the yield expression

$$Y_{nCB} = (1 + n\lambda_{CB}/n\alpha')^{-n\alpha'}. \quad (7)$$

This yield formula differs significantly from the ones used for large-area clustering, where the cluster parameter  $\alpha$  was a constant. We can write

$$\alpha'_n = n\alpha', \quad (8)$$

thus indicating that for small-area clustering the cluster parameter is directly proportional to the number of circuit blocks. The yield formula (7) can be used in formulae (15),

(28), and (29) of [1] for calculating the yield of partially good chips and chips with redundancy.

## Alternative models

Although negative binomial distributions have generally resulted in good agreement with integrated-circuit fault data, they are not the only type that can be applied to such data. For example, as has been shown, other distributions are suitable [3, 4]; one is the Neymann Type A distribution, which for a single circuit block can be expressed in the form

$$P(X=k) = \frac{e^{-\alpha'} (\lambda_{CB}/\alpha')}{k!} \sum_{i=0}^{\infty} \frac{i^k}{k!} (\alpha')^i e^{-i\lambda_{CB}/\alpha'}. \quad (9)$$

Associated with this is the yield expression

$$Y_{CB} = \exp[-\alpha' (1 - e^{-\lambda_{CB}/\alpha'})] \quad (10)$$

and the probability distribution generating function

$$G(s) = \exp\{-\alpha' [1 - e^{-(1-s)\lambda_{CB}/\alpha'}]\}. \quad (11)$$

In all of these equations  $\lambda_{CB}$  represents the average number of faults per circuit block, and  $\alpha'$  is a cluster parameter.

The above expressions pertain to single circuit blocks. When the clusters are smaller than a circuit block, we find, for groups of  $n$  blocks,

$$Y_{nCB} = \exp[-n\alpha' (1 - e^{-\lambda_{CB}/\alpha'})], \quad (12)$$

$$G_n(s) = \exp[-n\alpha' [1 - e^{-(1-s)\lambda_{CB}/\alpha'}]]. \quad (13)$$

The yield equation and the probability distribution generating function for the individual circuit blocks are obtained by using  $n = 1$  in Equations (12) and (13). Since  $n\lambda'/n\alpha' = \lambda'/\alpha'$ , it follows that  $G_n(s) = G_1(s)^n$ . Furthermore, Equation (13) is the probability distribution generating function of a Neymann Type A distribution with a mean equal to  $n\lambda'$  and a cluster parameter given by  $\alpha'_n = n\alpha'$ . We therefore also find here that this parameter is proportional to the number of circuit blocks.

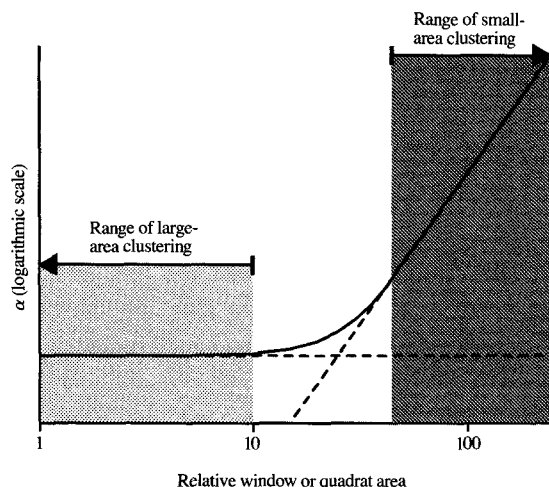
The Poisson binomial distribution has also been used for integrated-circuit yield modeling. It can be expressed in the form

$$P(X=k) = e^{-\alpha'} \sum_{i=0}^{\infty} \frac{(\alpha')^i}{i!} \cdot \frac{(\mu i)!}{k!(\mu i - k)!} \left(\frac{\lambda_{CB}}{\alpha' \mu}\right)^k \left(1 - \frac{\lambda_{CB}}{\alpha' \mu}\right)^{\mu i - k}, \quad (14)$$

where  $0 \leq \lambda_{CB}/\alpha' \mu \leq 1$  and  $\mu > 0$ . Here also,  $\lambda_{CB}$  is the mean of the distribution and  $\alpha'$  a cluster parameter. The quantity  $\mu$  is an additional parameter that has an integer value (usually 1 or 2). The yield and probability distribution generating function for this probability distribution function are given by

$$Y_{CB} = \exp\{-\alpha' [1 - (1 - \lambda_{CB}/\alpha' \mu)^\mu]\}, \quad (15)$$

$$G(s) = \exp[-\alpha' \{1 - [1 - (1 - s)\lambda_{CB}/\alpha' \mu]^\mu\}]. \quad (16)$$



**Figure 1**

Dependence of the cluster parameter  $\alpha$  on relative window or quadrat area. The range of large-area clustering is characterized by a constant value of  $\alpha$ . For small-area clustering it is proportional to the area. Note the logarithmic scale on the horizontal axis.

For clusters smaller than a circuit block, the yield and probability distribution generating function for chips with  $n$  identical circuit blocks are given by

$$Y_{nCB} = \exp\{-n\alpha' [1 - (1 - n\lambda_{CB}/n\alpha'\mu)^\mu]\}, \quad (17)$$

$$G_n(s) = \exp[-n\alpha' \{1 - [1 - (1 - s)n\lambda_{CB}/n\alpha'\mu]^\mu\}]. \quad (18)$$

It is not difficult to show that here too  $G_n(s) = G_1(s)^n$  and that Equation (18) is the probability distribution generating function of a Poisson binomial distribution with a mean  $n\lambda_{CB}$  and a cluster parameter  $\alpha'_n = n\alpha'$ . As previously, this parameter is proportional to the number of circuit blocks.

The Poisson negative binomial distribution is also potentially usable for integrated-circuit yield modeling. It can be expressed as

$$P(X=k) = e^{-\alpha'} \sum_{i=0}^{\infty} \frac{(\alpha')^i (\mu i + k - 1)!}{i! k! (\mu i - k)!} \frac{(\lambda_{CB}/\alpha'\mu)^k}{(1 + \lambda_{CB}/\alpha'\mu)^{\mu i + k}}, \quad (19)$$

where  $0 \leq \lambda_{CB}/\alpha'\mu \leq 1$  and  $\mu > 0$ . Again,  $\lambda_{CB}$  represents the mean of the distribution,  $\alpha'$  is a cluster parameter, and  $\mu$  is an additional parameter. The yield and probability distribution generating function formulas associated with this distribution are given by

$$Y_{CB} = \exp\{-\alpha' [1 - (1 + \lambda_{CB}/\alpha'\mu)^{-\mu}]\}, \quad (20)$$

$$G(s) = \exp[-\alpha' \{1 - [1 + (1 - s)\lambda_{CB}/\alpha'\mu]^{-\mu}\}]. \quad (21)$$

When the clusters are smaller than the individual circuit blocks, the formulas for the yield and probability distribution generating function take on the form

$$Y_{nCB} = \exp\{-n\alpha' [1 - (1 + n\lambda_{CB}/n\alpha'\mu)^{-\mu}]\}, \quad (22)$$

$$G_n(s) = \exp[-n\alpha' \{1 - [1 + (1 - s)n\lambda_{CB}/n\alpha'\mu]^{-\mu}\}]. \quad (23)$$

Here we also find  $G_n(s) = G_1(s)^n$ . Equation (23) represents the probability distribution generating function of a Poisson negative binomial distribution with an average equal to  $n\lambda_{CB}$  and a cluster parameter  $\alpha'_n = n\alpha'$ . Again, this parameter is proportional to the number of circuit blocks.

In all of the above formulae, cluster parameters are denoted by  $\alpha'$ ,  $n\alpha'$ , or  $\alpha'_n$ . They have values in the range  $0 \leq \alpha' \leq \infty$ . In each formula  $\alpha' = 0$  corresponds to the limiting case of maximum clustering. When this occurs, all the faults are concentrated in infinitely small clusters, while the areas surrounding them are fault-free. When  $\alpha' = \infty$ , all of the expressions reduce to the corresponding expressions associated with a Poisson distribution. This represents pure randomness, and the distinction between small and large clusters then becomes meaningless.

### Cluster parameter dependencies

The proportionality between the cluster parameter and the number of circuit blocks can be used as a criterion for the presence of small-area clustering. It can be applied by performing a window analysis or a quadrat analysis to determine the frequency distribution of the number of faults per window or quadrat [1, 4]. The object is to find the appropriate models for those distributions and determine the associated values of  $\alpha$ . If these values are directly proportional to the window or quadrat area, it is inferred that small-area clustering is present.

In general, the dependence of the cluster parameter  $\alpha$  on window or quadrat area is related to the cluster size distribution. For windows or quadrat areas larger than the largest fault clusters, the values of  $\alpha$  should increase proportionally with area. For windows or quadrat areas smaller than the smallest fault clusters, the value of  $\alpha$  should be independent of area. Therefore, if the sizes of the fault clusters are limited to a certain range, the dependence of  $\alpha$  on area should follow the curve depicted in Figure 1. The scales are logarithmic on both axes, so that  $\alpha$  is constant for small areas and increases in a straight line for large ones. The length of the curve connecting these two end conditions depends on the extent of the cluster sizes; its curvature depends on the distribution of cluster sizes.

Integrated-circuit fault data are often collected from wafers fabricated in different lots at different times. Such data are usually affected by strong wafer-to-wafer variations of fault densities [5]. These variations have the same effect as large-area clustering. The conditions for small-area clustering are therefore rarely observed in actual fabrication data. This can also be seen in Figure 2 of [1]. Although the two figures are

similar, the data on which the latter was based do not satisfy the direct proportionality criterion because of the wafer-to-wafer variation in the original data [4].

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