Geometric tolerancing: I. Virtual boundary requirements

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We examine the representation of geometric tolerances in solid-geometric models from the perspective of two classes of functional requirements. The first class deals with positioning of parts with respect to one another in an assembly, and the second with maintaining material bulk in critical portions of parts. Both are directly relatable to the geometry of the parts. Through examples, we demonstrate that these functional requirements can be captured in a specific form of tolerances designated as virtual boundary requirements (VBRs). We further demonstrate that the only proposed theory of tolerances in solid models, and the current dimensioning and tolerancing standards in industrial practice, are both inadequate for dealing with VBRs. Accordingly, we develop a theoretical basis for the rigorous statement and interpretation of VBRs.

Introduction

The goal of mechanical product design is to synthesize a set of mechanical parts having specific characteristics in order to fulfill certain functional requirements. The design specifications include the geometric and the material characteristics for each part, the spatial relationships among the parts, and their internal state in an assembly. The design process seldom leads to a unique set of parts, or a set of unique characteristics for a given set of parts. Instead, one finds that a range of characteristics is always permissible. A part with characteristics that are within the allowable range is readily interchangeable with another such part, so far as the functional requirements of the product assembly are concerned.

The product design specification is a key piece of information that flows from the Design function to the Manufacturing function in an enterprise. Both Design and Manufacturing functions use this information for a variety of purposes such as design analysis, process planning, and production system design. It is important to note that the primary design specifications may not be in the most appropriate form for a given purpose. It may therefore be necessary to derive equivalent alternative specifications which may be more suitable.

Today, the predominant medium for communication of mechanical design specifications is a set of engineering drawings. A typical drawing may show a number of two-dimensional projected views of the nominal geometry of a mechanical part, along with extensive annotations. These annotations characterize the nominal geometry of the part with dimensions, and the allowable variations from the nominal geometry with tolerances. In addition, they usually indicate material properties and other design specifications such as internal stress state. There are various corporate, national, and international standards that seek to provide guidelines for uniform application and interpretation of such annotations (see, e.g., [1–3]).

The current practice, however, suffers from a number of limitations. It is very difficult to ensure that an appropriate set of dimensions has been specified to completely characterize the nominal geometry of a part. Standards have largely evolved from special-case considerations and lack generality. The definitions of the terminology they contain are highly informal. There are significant gaps and ambiguities in their interpretation. In any case, engineering drawings are primarily meant to be interpreted by experienced humans, who make up for incomplete, ambiguous, and inconsistent information often contained in the drawings. With the increasing use of computers in virtually all forms of industrial activity, whether for the complete automation of the activity or to assist humans in performing the activity, computer representations of product design specifications are increasingly necessary. The representations must be complete, unambiguous, consistent, and meaningful in order to ensure that various application algorithms using the representations will in turn produce other meaningful information. Such informationally complete representation schemes for mechanical design specifications are unknown at this time. This work is part of our exploration of the geometrical aspects of such specifications.

A multitude of computer-aided mechanical design systems are commercially available, embodying a number of geometric representation schemes. Some of these schemes are merely extensions of the engineering drawing practice to the realm of computers. Others, known as solid-geometric modeling schemes, are sharp deviations from current practice. Solid-geometric models are expected to be the future medium of communication for geometrical specifications of mechanical product designs, gradually replacing the current two-dimensional orthographic projections in engineering drawings [4, 5]. But the solid-geometric modeling systems available today are for the most part capable of representing only the nominal geometry of parts. Furthermore, these representations do not lend themselves readily to design modifications or optimization using dimensions as the variables. At present, two-dimensional projections are derived from a solid model, and these views are annotated with dimensions and tolerances in the conventional manner. Providing means for annotating solid models directly would eliminate the need to deal with a separate drafting system, but would not contribute to the completeness of geometrical specifications or ease of computer interpretations. Thus, the meaningful integration of geometric tolerance information into solid-geometric models is clearly of importance.

The problem of geometric variations has been studied to some extent. Hillyard and Braid [6] have dealt with characterizing the nominal geometry (restricted to polyhedral solids represented by their boundaries) by a set of geometric constraints (distance and/or angle) among the boundary elements (vertices, edges, and faces). By invoking an analogy to engineering frame structures, a given dimensioning scheme (i.e., the given set of constraints) could be checked for minimality and completeness in defining the geometry. Tolerances were included by specifying allowable variations in the parameters (dimensions) associated with the constraints. These tolerances implied a family of perfect-form geometries, such as planar and cylindrical surfaces, and did not account for the imperfect nature of manufactured parts. Hillyard and Braid [7] have also done some preliminary explorations of extending this approach to include curved surfaces and modification of the nominal geometry through alteration of dimensional parameters. The latter idea has been pursued further by Gopin and Gossard [8], Lin et al. [9], Fitzgerald [10], and Light and Gossard [11]. A scheme for incorporating current industrial dimensioning and tolerancing practices into boundary representations of geometry has been described by Yu et al. [12].

None of these papers addresses the problem of an adequate theoretical basis for computer representation of and reasoning about geometric tolerances. For the reasons mentioned earlier and others, current tolerancing standards are not adequate either. The lack of a formal theory of dimensioning and tolerancing, and the dire need for one, were first identified by Requicha [13], who examined the current dimensioning and tolerancing practices in industry with a view toward uncovering the theoretical basis for such practices. Orthogonal dimensioning schemes and both conventional (plus or minus) and geometric tolerances were addressed, although how to deal with both types of tolerances simultaneously was not.

Refining his earlier ideas, Requicha then proposed an approach in which a tolerance specification is a set of geometric assertions on the surface features (two-dimensional subsets) of an object's boundary [14]. An object is deemed acceptable if its surface features lie within tolerance zones, which are regions of space constructed by offsetting the object's nominal boundary. Conventional tolerances are considered a special case of geometric tolerances, and no attempt is made to characterize the nominal geometry using dimensions. This work is the most relevant to our work, and we discuss it further later in this paper. A review of key issues in the representation of tolerances in solid models, alternative theoretical approaches, and their implications can be found in [15]; a scheme for representing features and associating tolerances and other attributes with features in a constructive solid-geometric model is described in [16].

One of the primary goals of any representation scheme for geometric tolerances is to enable the designer to express the

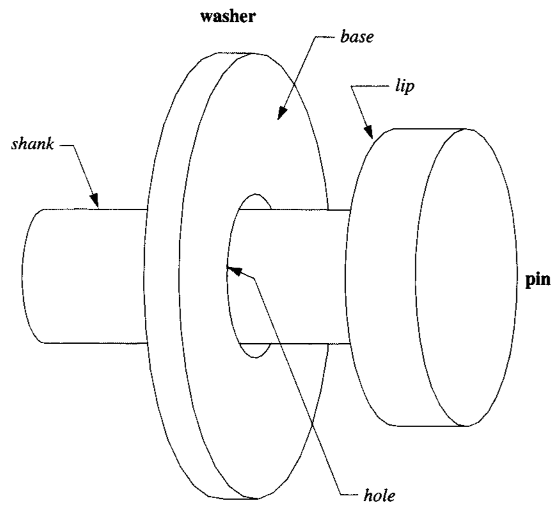


Figure 1

Illustration of **pin** and **washer**.

largest class of mechanical parts that are acceptable for a given set of functional requirements. By focusing on expressing the actual functional intent as much as possible in the design specifications, the designer provides greater latitude in the choice of the manufacturing processes to produce the product, subject to different manufacturing objectives and policies (e.g., lowest cost or shortest time). None of the papers cited approaches the tolerance representation problem from the perspective of functional requirements. Further, the problem of deriving equivalent alternative tolerance specifications from primary tolerance specifications has received very little attention in the literature (see [17, 18]). Our earlier work [19] raised these issues as topics to be studied in detail.

In this paper, we examine the representation of geometric tolerances in solid-geometric models. The related issue of representation conversions is the subject of a companion paper [20]. We study the problem from the perspective of two classes of functional requirements, directly relatable to the geometry of mechanical parts. The first involves positioning parts with respect to one another in an assembly; the second, maintaining material bulk in critical portions of parts. Through examples, we demonstrate that these requirements can be captured as *virtual boundary requirements* (VBRs). We further show that Requicha's theory [14] and the current dimensioning and tolerancing standards [2] in industrial practice are both inadequate for dealing with VBRs, and develop a theoretical basis for the rigorous statement and interpretation of VBRs. Finally, we discuss some open research issues in geometric tolerancing for functional requirements.

Assembly and material bulk requirements

In this section we consider two classes of functional requirements that can be directly related to the geometry of mechanical parts. The first pertains to the spatial relationships among parts in an assembly, the second to maintaining material bulk in critical portions of parts. We present an example of each class and show that it is possible to provide a unified representation for these requirements in the form of VBRs.

• *Examples*

The first example illustrates spatial relationships among parts in an assembly.

Example 1

Consider an assembly of two parts, referred to as **pin** and **washer** and shown in **Figure 1**. The functional requirement here is that it should be possible to establish the specified spatial relationship between the two parts. We restrict our attention to the specification of the required spatial relationship in geometrical terms and proceed as follows. We start by identifying relevant *surface features* (that are portions of the boundary) of each part. Thus *lip* refers to the planar face in the form of an annulus, and *shank* refers to the long cylindrical face; each are relevant surface features of **pin**; *base* and *hole* are the corresponding surface features of **washer**. Next we identify a suitable set of pairs of surface features from the set of identified features of the parts. A pair should not contain surface features from the same part. For our example, the pairs are (*lip*, *base*) and (*shank*, *hole*). We can then express the positional relationship requirement between the parts in the assembly in terms of the positional relationship requirements on each pair of surface features, namely, (1) *lip* should be in "close contact" with *base*, and (2) *hole* should "surround" *shank*. □

Note that the first requirement reduces the number of degrees of freedom of one part with respect to the other (from six to three), provided that *lip* and *base* are perfectly planar surface patches. In contrast, the second requirement serves only to limit the ranges of variations of the remaining degrees of freedom. Requirements such as the close contact requirement reduce the minimum number of independent parameters necessary to specify the positional relationship between the parts, if the features were perfect. We refer to such requirements as *datum* requirements. The features on which datum requirements are asserted are referred to as datum features. Thus, in the present example, both *lip* and *base* are datum features, while *hole* and *shank* are not. The meaning of the close contact requirement when the features are not perfect planar patches (as would be the case with any actual part) is explained informally in current tolerance standards [2]. Essentially, the requirement stipulates that *lip* and *base* should be in as close a contact as possible without being restricted by *hole* and *shank.*

Hereafter, we refer to a specification of the positional relationships among the parts in an assembly as an *assembly* requirement. In the present example, given actual physical instances of **pin** and **washer**, we need to verify that it is possible to position one part relative to the other such that their surface features have the spatial relationships stated above. We can then conclude that the parts are in compliance with the assembly requirement. Implicit in this verification using physical parts is the impossibility of establishing a configuration where there is any volumetric interference between the parts. Such is not the case, however, when we deal with nonphysical models of the parts (presumably solid-geometric models in a computer, generated from the actual physical part by a suitable sensory system). Thus, we need to add an additional spatial noninterference requirement: namely, (3) it should be possible to establish the required positional relationship between **pin** and **washer** without causing any volumetric interference in the process. It is beyond the scope of this paper to consider this noninterference requirement as a part of the assembly requirement. Thus, for our purposes, the existence of a spatial configuration (for the models of a set of actual parts) in which the desired feature-to-feature relationships hold implies that the assembly requirement is satisfied. See [21] for additional assembly examples.

The next example is concerned with maintaining material bulk in critical portions of parts.

Example 2

Consider a part referred to as **tank**, shown in **Figure 2**. The requirement here is one of ensuring that the volume occupied by the material in the cylindrical wall of **tank** encloses at least a hollow cylindrical volume of specified diameter and thickness. Here we identify only relevant surface features on **tank**, namely, *inner_cylinder* and *outer_cylinder*, and defer the expression of this material bulk requirement in terms of these features to the section on virtual boundary requirements. □

Henceforth, we refer to a specification of the geometric characteristics of critical material portions of a part as a *material bulk* requirement. In the present example, the bulk requirement is concerned only with the volume of material and not with its relationship to other portions of the part. See [21] for examples that illustrate cases where such relationships also are important.

Thus far we have presented representative examples of assembly and material bulk requirements. We now show that both these classes of requirements are expressible in a unified manner.

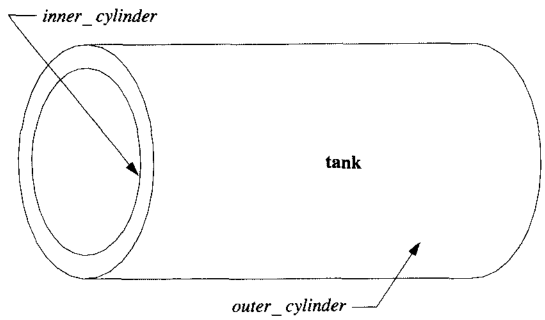


Figure 2

Illustration of **tank**.

• *Virtual boundary requirements*

Note that the assembly requirement in Example 1 has been stated in terms of the spatial relational requirements among surface features of two *different* parts (models). From the assembly requirement, we would like to derive a set of requirements that can be stated individually for each part. We proceed as follows. We start with the desired assembly configuration, and with each pair of features identified earlier, we associate an appropriate virtual surface. For example, *vs_1* (a planar surface) is associated with (*lip*, *base*) and *vs_2* (a cylindrical surface) with (*shank*, *hole*) (see **Figure 3**). To the extent warranted by the assembly requirement, we identify geometrical requirements to be satisfied by the virtual surfaces. In our example, the requirements are that *vs_2* be of a specified size and that it be perpendicular to *vs_1*. We imagine the virtual surfaces to be members of a collection of geometric entities that do not move with respect to one another. We can then restate the functional requirement in terms of the spatial relationships to be satisfied by each part with respect to such a *rigid collection* of virtual surfaces. Thus we demand that it should be possible to position **pin** relative to the virtual surfaces in such a manner that (1) *vs_1* lies to the nonmaterial side of *lip*, (2) *lip* is in close contact with *vs_1*, and (3) *vs_2* surrounds *shank*. Similarly, we require that it should be possible to position **washer** relative to the virtual surfaces such that (1) *vs_1* lies to the nonmaterial side of *base*, (2) *base* is in close contact with *vs_1*, and (3) *hole* surrounds *vs_2*. Note that the rigid collection of virtual surfaces associated with each part is in some sense an abstraction of the relevant surface features of the mating part.

We refer to functional requirements expressed through a rigid collection of virtual surfaces as *virtual surface*

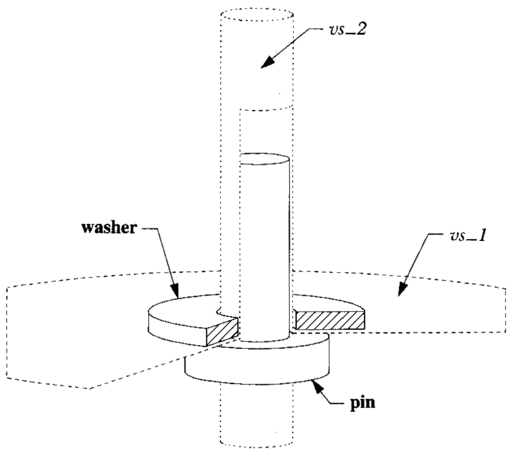


Figure 3

Virtual surfaces for the **pin/washer** assembly.

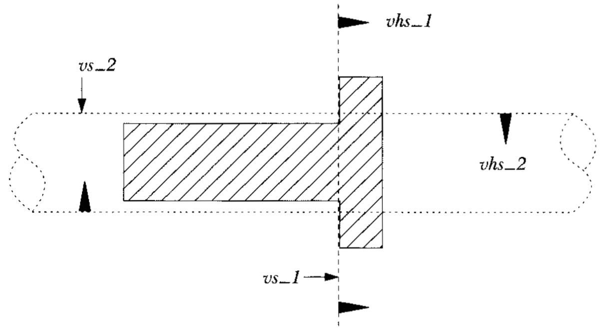


Figure 4

Virtual half-spaces for **pin**.

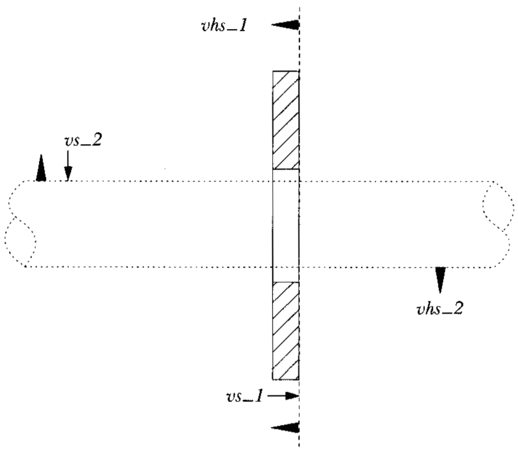


Figure 5

Virtual half-spaces for **washer**.

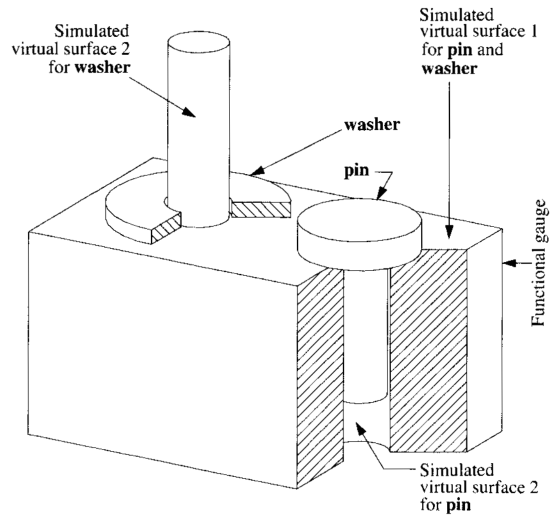


Figure 6

A functional gauge for **pin** and **washer**.

requirements. Equivalently, we can associate a (possibly unbounded) virtual half-space with each virtual surface such that the virtual surface is the boundary of the virtual half-space. The primary motivation for doing this is to ease the formalization of notions such as "being on the nonmaterial side" and "surrounding" in the next section. We refer to functional requirements expressed through a rigid collection of (boundaries of) virtual half-spaces as *virtual boundary requirements.*

The choice of the virtual half-space associated with a virtual surface is dependent on the part for which we wish to state the virtual surface requirement. **Figure 4** shows the virtual half-spaces associated with **pin**, and **Figure 5** shows the same for **washer**. The material sides of the virtual half-spaces are indicated by solid triangles in the figures. Note that the virtual half-space associated with a given virtual surface for **washer** is the complement of the virtual half-space associated with the same virtual surface for **pin**.

There is an additional reason for associating virtual half-spaces with virtual surfaces. By using the virtual half-spaces, it is possible to construct a model of a physical solid such that a suitable subset of its boundary is a close (and finite) approximation to the virtual surfaces. It is often possible to use the physical solid to verify directly the compliance of a physical part with the associated virtual surface requirement. In industry, such a physical solid is referred to as a *functional gauge*. See **Figure 6** for an illustrative functional gauge for **pin** and **washer**.

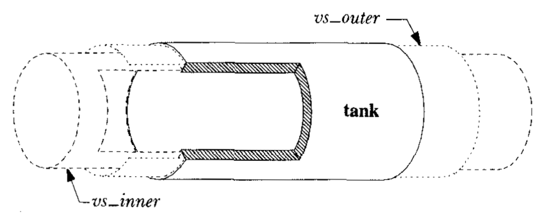


Figure 7

Virtual surfaces for **tank**.

We next consider our material bulk example and associate a virtual surface with each of the relevant features. Thus *vs_inner* (a cylindrical surface) is associated with *inner_cylinder*, and *vs_outer* (another cylindrical surface) is associated with *outer_cylinder* (see **Figures 7** and **8**). We require that the virtual surfaces be coaxial and of specified sizes. The material bulk requirement can now be expressed in terms of the spatial relationship to be obtained between the part and the rigid collection of virtual surfaces. In other words, it should be possible to position **tank** (model) with respect to the virtual surfaces such that (1) *vs_inner* lies to the material side of *inner_cylinder* and surrounds *inner_cylinder* and (2) *vs_outer* lies to the material side of *outer_cylinder* and is surrounded by *outer_cylinder*. In this case, although it is possible to come up with a model of a physical solid using the virtual half-spaces such that a

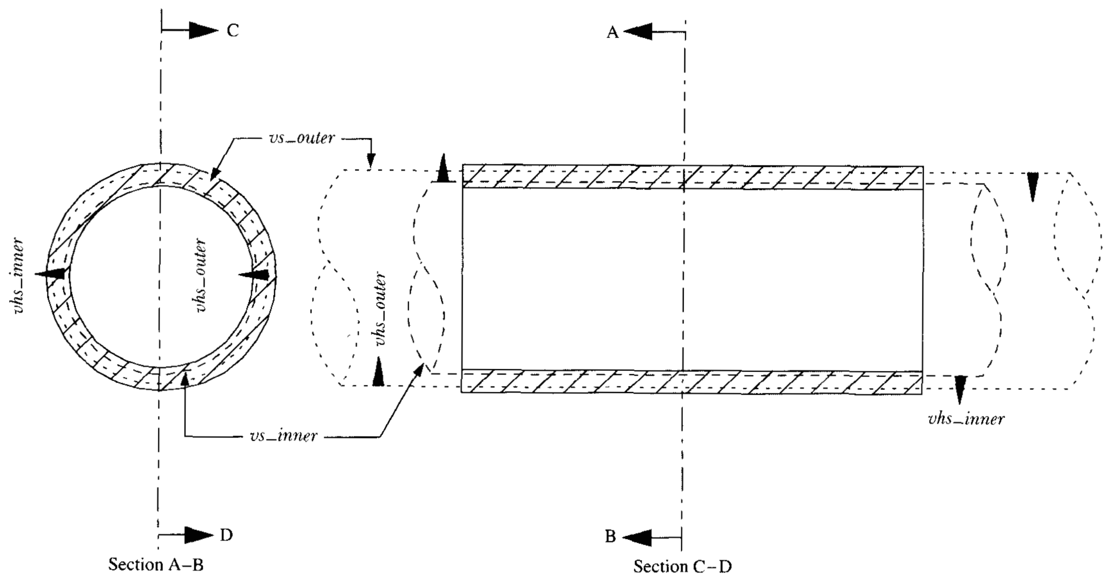


Figure 8

Virtual half-spaces for **tank**.

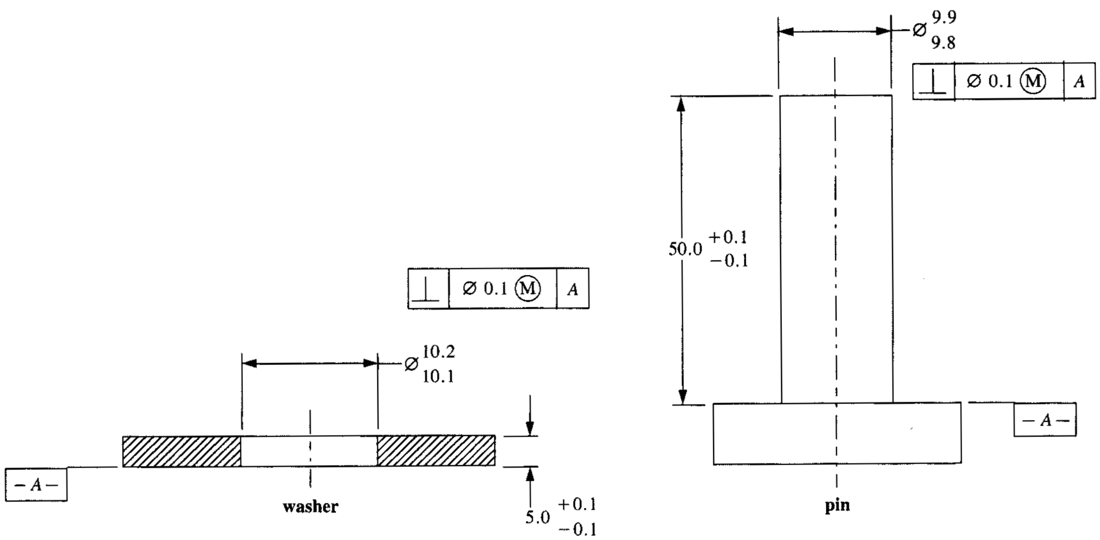


Figure 9

Partial drafting specification of **washer** and **pin**.

suitable subset of the physical solid's boundary is a close and finite approximation to the virtual surfaces, it is not possible to use that physical solid to directly verify the compliance of a physical part with the virtual surface requirement. Thus there exists no functional gauge for this example.

We have seen that VBRs are useful for capturing assembly and material bulk requirements, both of which are directly relatable to the geometry of mechanical parts. Thus, we can think of VBRs as a specific form of geometric tolerance specifications. We complete this section with some observations regarding current industrial tolerancing practices [2] related to VBRs.

Figures 9 and **10** show the manner in which the parts in our illustrative examples are specified, in conformance with ANSI standards [2]. In current practice, virtual surfaces on which contact requirements are imposed are specified by associating datums with the corresponding features on the part. Thus, in Figure 9 the virtual surface associated with *lip* in **pin** and *base* in **washer** is indicated by the datum A. Unfortunately, some of the virtual surfaces with no contact requirements are also indicated using datums. For example, the virtual surface associated with *outer_cylinder* in **tank** is indicated in Figure 10 by the datum A and the *Least Material Condition* (LMC) modifier on its reference in the positional tolerance. The majority of noncontact virtual surfaces are specified in current practice using *Maximum Material Condition* (MMC) (Figure 9) and LMC (Figure 10) types of tolerances on the position (Figure 10) or on the orientation (Figure 9) of features of symmetry (axes and median planes) associated with so-called features of size (cylinders and pairs of parallel planes). The sizes of such virtual surfaces are derived from those specifications by adding or subtracting the specified tolerances from the maximum or minimum sizes of the features, depending upon the type of features ("solid" or "hole") and the indicated material conditions (see [2]). The positions of the virtual surfaces are the same as those of the nominal features. Since the MMC and the LMC tolerances are associated only with the so-called features of size, current practice is inadequate for specifying virtual surfaces associated with features that are not features of size. See [21] for such examples. One can conclude from these observations that VBRs are a generalization of current practices in datum-referenced MMC and LMC tolerancing.

We have introduced a number of concepts in this section through representative examples and discussed them informally to gain an intuitive appreciation of their relevance and importance. In the next section we examine formal aspects of these concepts.

Formalization

The objective of this section is to provide a formal basis for many of the concepts introduced in the previous section. We start with some general concepts before addressing VBRs.

Wherever appropriate, we compare and contrast our approach with the theory proposed by Requicha [14], and with current industrial tolerancing practices [2].

• *General concepts*

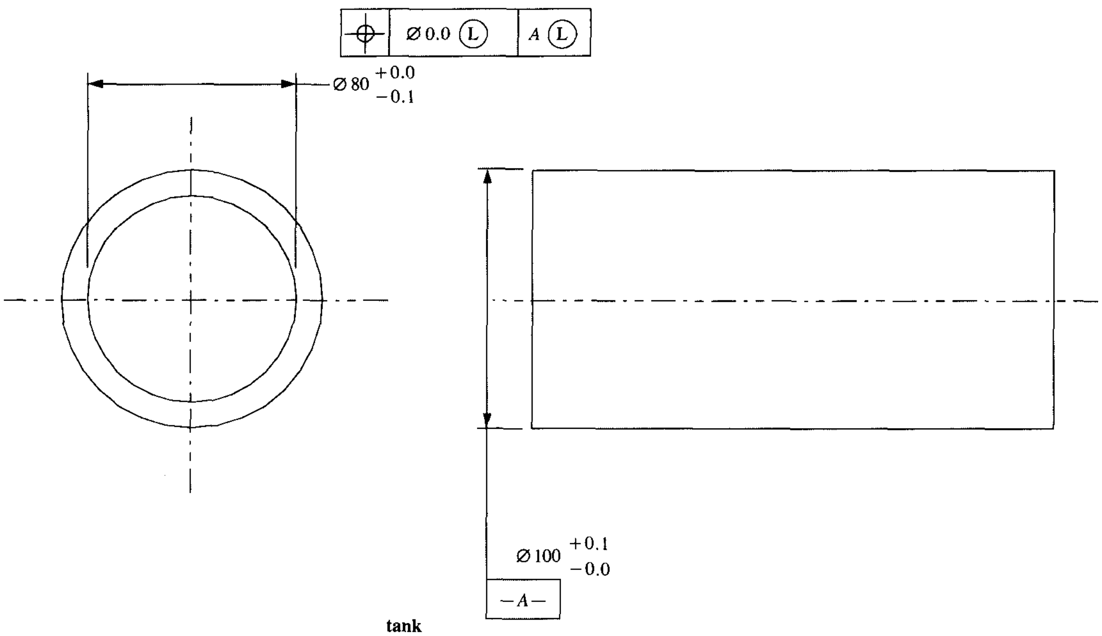
Here, we present several definitions and notations for later use, and introduce the concept of rigid collections of geometric entities.

We deal with the topological space $(E^3, \mathcal{T})$ and its subspaces $(A, \mathcal{T}')$, where E^3 is the three-dimensional Euclidean space, $\mathcal{T}$ is the usual topology on E^3, A is a subset of E^3, and $\mathcal{T}'$ is the relative topology induced on A by $\mathcal{T}$ [22]. We invoke the notions of interior, boundary, and closure of subsets of E^3 (denoted respectively by i, ∂, and cl followed by the subset of interest) with respect to $\mathcal{T}$ or a specific $\mathcal{T}'$. We refer explicitly to the applicable topology, wherever it is not obvious from the context.

A point in E^3 is denoted by $\mathbf{p}$. A subset A of E^3 is a *regular subset* if it equals the closure of its interior [23]. A regular half-space, simply called a *half-space*, is any subset H of E^3 that satisfies $H = \text{cl}\ \{\mathbf{p} : f(\mathbf{p}) < 0\}$ for some analytic function f of E^3 [24]. The function f is chosen such that the boundary of H is $\partial H = \{\mathbf{p} : f(\mathbf{p}) = 0\}$. It is interesting to note that not all analytic functions satisfy this equality (see [24] for an example). We assume the existence of a set of *primitive half-spaces*, such as plane, cylinder, cone, sphere, and toroid, in our discussions. A *primitive surface* is the boundary of one of the primitive half-spaces.

A *nominal solid* S_N is a bounded and regular subset of E^3, whose boundary consists of surface patches that are subsets of primitive surfaces. An *actual solid* S_A is a bounded and regular subset of E^3 and is a valid and unambiguous solid model of a manufactured part. A *solid*, denoted by S, is any nominal or actual solid, or any half-space. In a constructive solid-geometric representation, a solid is defined through regularized Boolean operations and rigid motions of primitive half-spaces. In our discussions of solids, however, we do not restrict ourselves to any particular representation. We require only a concept of the boundary of a solid, and the ability to refer to all subsets of this boundary.

In what follows, we denote by $\mathcal{L}(\mathbf{p}_1, \mathbf{p}_2)$ the open line segment connecting points $\mathbf{p}_1$ and $\mathbf{p}_2$; by $d(\mathbf{p}_1, \mathbf{p}_2)$ the Euclidean distance between $\mathbf{p}_1$ and $\mathbf{p}_2$; by $d(\mathbf{p}, A)$ the Euclidean distance between point $\mathbf{p}$ and A; by $\mathcal{B}(\mathbf{p}; r)$ the open ball of radius $r > 0$ centered at $\mathbf{p}$; and by $\mathcal{N}(\mathbf{p}, S; r)$ [defined as $\mathcal{B}(\mathbf{p}; r) \cap S$] the neighborhood of $\mathbf{p}$ with respect



tank

Figure 10

Partial drafting specification of **tank**.

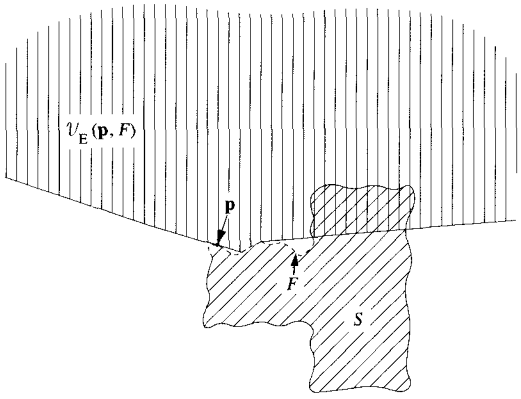


Figure 11

A two-dimensional external visible region.

to a solid S. The projection of $\mathbf{p}$ onto a subset A of E^3, denoted as $\mathcal{P}(\mathbf{p}, A)$, is defined as all points of cl A that are closest to $\mathbf{p}$. In other words, $\mathcal{P}(\mathbf{p}, A) = \{\mathbf{q} : \mathbf{q} \in \text{cl } A \wedge d(\mathbf{p}, \mathbf{q}) = d(\mathbf{p}, A)\}$. We also refer to $\mathbf{p}$ as a *pre-image* of any point $\mathbf{q} \in \mathcal{P}(\mathbf{p}, A)$.

Offset solids (see [25, 26]) play a vital role in the interpretation of VBRs. Given a solid S, a *regularized growing* of S by a scalar $a \geq 0$ is defined as $S \uparrow^* a \equiv \{\mathbf{p} : d(\mathbf{p}, S) \leq a\}$. A *regularized shrinking* of S by a scalar $a \geq 0$ is defined as $S \downarrow^* a \equiv \overline{\overline{S}^* \uparrow^* a}^*$, where $\overline{}^*$ denotes regularized complementation. A *regularized offset* of S by an arbitrary scalar a is defined as

$$O(S; a) \equiv \begin{cases} S \uparrow^* a & \text{if } a \geq 0, \\ S \downarrow^* |a| & \text{if } a < 0. \end{cases}$$

A point set P in E^3 is called a *geometric entity.* A geometric entity P subjected to a rigid-body transformation $\mathcal{M}$ is called a *congruent instance* of P and denoted as $\mathcal{M}P$. Collections of geometric entities whose members are moved around as a whole without causing any relative movement within are needed later and are defined as below.

Definition 1

A *rigid collection* of geometrical entities $\mathbf{P} \equiv \{P_1, \cdots, P_k\}$ is a nonempty set of geometrical entities such that $\forall \mathbf{p} \in P_i$ and $\forall \mathbf{q} \in P_j$, $d(\mathbf{p}, \mathbf{q})$ is invariant under a rigid-body transformation $\mathcal{M}$ applied to the whole set $\mathbf{P}$. □

We indicate a congruent instance of $\mathbf{P}$ by $\mathcal{M}\mathbf{P}$.

• *Features*

Surface features are essential elements of any theory of geometric tolerances. Here, we define surface features and introduce the concepts of *visible regions* and *fitting entities* for actual surface features. Using these concepts, we then define *datum systems* associated with actual solids.

A *nominal surface feature* F_N of a nominal solid S_N is a regular subset in the relative topology of the boundary of S_N, such that F_N is a subset of only one primitive surface and has an associated half-space denoted as H_{F_N} satisfying the following properties:

1. $F_N \subset \partial H_{F_N}$.
2. $\forall \mathbf{p} \in \text{i} F_N$, $\mathcal{N}(\mathbf{p}, S_N; r) = \mathcal{N}(\mathbf{p}, H_{F_N}; r)$ for sufficiently small r. This simply means that the material sides of H_{F_N} and F_N must be identical.

A nominal surface feature can consist of disjoint patches provided a unique half-space can be associated with it as described above. We now define sets of nominal surface features with associated rigid collections of half-spaces.

Definition 2

Let $T_N \equiv \{F_{N1}, F_{N2}, \cdots, F_{Nk}\}$ be a set of nominal surface features. Associated with T_N is a *rigid collection of half-spaces,* $\mathbf{H}_{T_N} \equiv \{H_{F_{N1}}, H_{F_{N2}}, \cdots, H_{F_{Nk}}\}$ such that the spatial relationship between any two $H_{F_{Ni}}$ and $H_{F_{Nj}}$ for $i \neq j$ is completely determined by the spatial relationship between F_{Ni} and F_{Nj}. In addition, given a set of scalars $\mathbf{a} \equiv \{a_1, \cdots, a_k\}$, a *rigid collection of offset half-spaces* denoted as $O(T_N; \mathbf{a})$ is defined as $O(T_N; \mathbf{a}) \equiv \{O(H_{F_{N1}}; a_1), \cdots, O(H_{F_{Nk}}; a_k)\}$. □

We note here that our nominal surface features are equivalent to the *simple* nominal features defined by Requicha [14]. In addition, Requicha has defined *composite* nominal features that are unions of simple ones. A nominal feature in his scheme is required to have an associated *extended* feature, defined as a Boolean composition of a set of half-spaces that satisfies a number of conditions (see [14] for details). Under this requirement, certain collections of simple nominal features cannot be combined to form composite nominal features. See Figure 11 in [14] for an example of an inadmissible composite nominal feature.

The tolerancing semantics provided by Requicha is based on offset solids generated from extended features. This seems to be the primary reason for demanding the existence of an extended feature associated with a nominal feature in [14]. We believe that to capture functional requirements through virtual boundary specifications, it may be necessary to deal with sets of simple nominal surface features which cannot be combined into composite nominal features. Hence, we chose to address collections of simple nominal features using the notion of rigid collections and tie the tolerancing semantics to rigid collections of offset half-spaces.

An *actual surface feature* F_A is a regular subset in the relative topology of the boundary of an actual solid S_A such that it *corresponds* to a nominal surface feature F_N. We say more about sets of nominal surface features and corresponding sets of actual surface features later. We can associate primitive surfaces and primitive half-spaces with actual surface features through *fitting* procedures. Such perfect-form geometric entities associated with actual surface features are needed for addressing concepts such as datums and for a formal statement of VBRs.

Fitting surfaces may lie either to the material side or to the nonmaterial side of actual surface features. We need both types of fitting surfaces. Precise statement of the side of an actual feature in which a fitting surface should lie requires the concept of visible regions defined below.

Definition 3

Let F be a surface feature of a solid S, and $\mathbf{p}$ be a point such that $\mathbf{p} \in F$. The *external visible region* of $\mathbf{p}$ in F is

$$\mathcal{V}_{\mathrm{E}}(\mathbf{p}, F) \equiv \mathrm{cl}\ \{\mathbf{q} : \mathcal{L}(\mathbf{p}, \mathbf{q}) \cap F = \varnothing \wedge \mathcal{L}(\mathbf{p}, \mathbf{q}) \cap \mathcal{N}(\mathbf{p}, S; r) = \varnothing\}$$

for sufficiently small r. The *internal visible region* of $\mathbf{p}$ in F is

$$\mathcal{V}_{\mathrm{I}}(\mathbf{p}, F) \equiv \mathrm{cl}\ \{\mathbf{q} : \mathcal{L}(\mathbf{p}, \mathbf{q}) \cap F = \varnothing \wedge \mathcal{L}(\mathbf{p}, \mathbf{q}) \cap \mathcal{N}(\mathbf{p}, S; r) \neq \varnothing\}$$

for sufficiently small r. □

Figures 11 and **12** show examples of external and internal visible regions in two dimensions. Note that if $\mathbf{q} \in \mathcal{V}_{\mathrm{E}}(\mathbf{p}, F)$, then $\mathcal{L}(\mathbf{p}, \mathbf{q}) \subset \mathcal{V}_{\mathrm{E}}(\mathbf{p}, F)$, and a similar property holds for internal visible regions.

Using the notion of visible regions, we can now state a generalized fitting methodology and define fitting entities for a set of surface features.

Definition 4

Let $T_{\mathrm{A}} \equiv \{F_{\mathrm{A}1}, \cdots, F_{\mathrm{A}k}\}$ be a set of actual surface features on an actual solid S_{A}, and $T_{\mathrm{N}} \equiv \{F_{\mathrm{N}1}, \cdots, F_{\mathrm{N}k}\}$ be the corresponding set of nominal surface features. Find a set of scalars $\mathbf{a} \equiv \{a_1, \cdots, a_k\}$, and a rigid-body transformation $\mathcal{M}$ such that

- *Containment condition:* for all i, $F_{\mathrm{A}i} \subset \mathcal{M}O(H_{F_{\mathrm{N}i}}; a_i)$.
- *Material-side condition:* for all i and for all $\mathbf{p} \in F_{\mathrm{A}i}$, there exist a point $\mathbf{q} \in \mathcal{P}[\mathbf{p}, \partial\mathcal{M}O(H_{F_{\mathrm{N}i}}; a_i)]$ and a point $\mathbf{r} \in [\mathrm{cl}\ \mathcal{L}(\mathbf{p}, \mathbf{q})] \cap F_{\mathrm{A}i}$ such that $\mathbf{q} \in \mathcal{V}_{\mathrm{E}}(\mathbf{r}, F_{\mathrm{A}i})$.
- *Closeness condition:* $\partial[\mathcal{M}O(T_{\mathrm{N}}; \mathbf{a})]$ is *closest* to T_{A} in some appropriate sense.

We then define $\mathbf{H}_{T_{\mathrm{A}}} \equiv \{H_{F_{\mathrm{A}1}}, \cdots, H_{F_{\mathrm{A}k}}\} = \mathcal{M}O(T_{\mathrm{N}}; \mathbf{a})$ as a *rigid collection of externally fitting half-spaces* for T_{A}. □

Essentially, the material-side condition states that for every point $\mathbf{q}$ in the projection of $F_{\mathrm{A}i}$ onto its fitting surface, there exists a pre-image $\mathbf{r}$ in $F_{\mathrm{A}i}$ such that $\mathbf{q} \in \mathcal{V}_{\mathrm{E}}(\mathbf{r}, F_{\mathrm{A}i})$. The fitting half-spaces may not be unique. A similar definition can be given for a *rigid collection of internally fitting half-spaces* for T_{A} by using $\overline{\mathcal{M}O(H_{F_{\mathrm{N}i}}; a_i)}^{*}$ in place of $\mathcal{M}O(H_{F_{\mathrm{N}i}}; a_i)$ in the containment condition and $\mathcal{V}_{\mathrm{I}}(\mathbf{r}, F_{\mathrm{A}i})$ instead of $\mathcal{V}_{\mathrm{E}}(\mathbf{r}, F_{\mathrm{A}i})$ in the material-side condition.

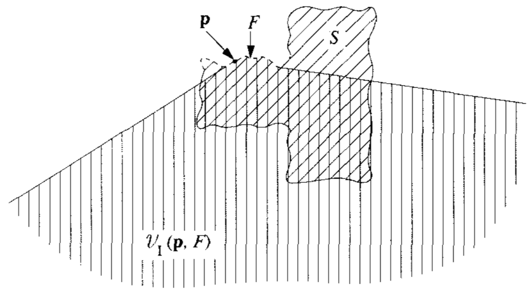


Figure 12

A two-dimensional internal visible region.

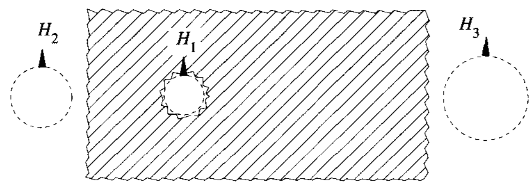


Figure 13

Illustration of need for the material-side condition in fitting a hole.

Note that the definitions of *measured* entities (equivalent to our fitting entities) provided by Requicha [14] are incomplete. For symmetric features such as sets of two parallel planes and cylindrical surfaces, Requicha uses the containment condition above, and a *scaling* process in place of the closeness condition. The material-side condition is not used at all. This does not cause any problem for features bounding convex shapes (such as tabs and bosses); for features bounding nonconvex shapes (such as slots and holes), however, this can clearly lead to wrong results. **Figure 13** illustrates this point for a hole feature in two dimensions. Here, the desired externally fitting half-space for the hole is $H_1 = \mathcal{M}_1O(H_{F_{\mathrm{N}}}; b_1)$. However, both $H_2 = \mathcal{M}_2O(H_{F_{\mathrm{N}}}; b_2)$ and $H_3 = \mathcal{M}_3O(H_{F_{\mathrm{N}}}; b_3)$ have larger diameters than H_1 and

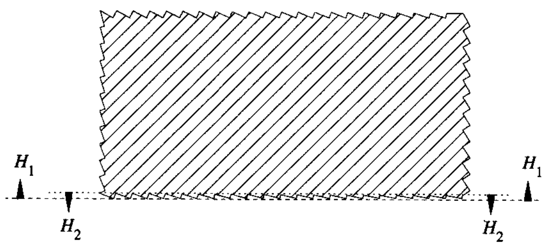


Figure 14

Illustration of need for the material-side condition in fitting a planar feature.

enclose the hole. According to the definition of fitting by Requicha [14], H_2 is preferred over H_1, H_3 over H_2, and so on without convergence. For planar features, Requicha uses the containment and the closeness conditions, again without invoking the material-side condition. As illustrated in **Figure 14**, this can lead to ambiguity in whether the boundary of the fitting half-space lies to the material or nonmaterial side of the actual feature. In this case, the desired externally fitting half-space is $H_1 = \mathcal{M}_1 O(H_{F_N}; b_1)$. But $H_2 = \mathcal{M}_2 O(H_{F_N}; b_2)$ may be as close to the actual feature as H_1 or closer. The material-side condition is needed in the definition of fitting to select H_1 over H_2. Rigorous statement of the material-side condition is the primary reason for our introducing the concept of visible regions.

The overall measure of closeness between a set of actual surface features and a rigid collection of corresponding primitive surfaces, denoted as $\mathcal{C}[T_A, \partial \mathcal{M}O(T_N, \mathbf{a})]$ is defined to be the sum of the individual measures of closeness, denoted as $\mathcal{C}[F_{Ai}, \partial \mathcal{M}O(H_{F_{Ni}}; a_i)]$, between each actual surface feature and the corresponding primitive surface. Different criteria of closeness such as the maximum distance between the primitive surface and the actual feature and the integral sum of distance between the actual feature and the primitive surface can be applied in estimating individual measures of closeness. We require that any closeness criterion for estimating individual measures of closeness satisfy the following relationships:

1. $\mathcal{C}[F_{Ai}, \partial \mathcal{M}O(H_{F_{Ni}}; a_i)] \geq 0$.
2. If $\forall \mathbf{p} \in F_{Ai}$, $d(\mathbf{p}, \partial H_1) > d(\mathbf{p}, \partial H_2)$, where $H_1 = \mathcal{M}_1 O(H_{F_{Ni}}; b_1)$ and $H_2 = \mathcal{M}_2 O(H_{F_{Ni}}; b_2)$ are candidate half-spaces that are offset and/or congruent versions of each other, then $\mathcal{C}(F_{Ai}, \partial H_1) > \mathcal{C}(F_{Ai}, \partial H_2)$.

We use these requirements in our companion paper [20] to establish an important property of fitting entities.

The concept of fitting is a key concept in our theory of geometric tolerances for a number of reasons. First, it is useful in determining the acceptability (for certain functional requirements) of actual surface features. An actual feature may be fitted with a half-space whose position is fully unconstrained, partially constrained, or fully constrained with respect to the actual solid. In the latter two cases, a fitting half-space may or may not exist for an actual feature. This can determine whether or not the feature is acceptable. In the former two cases, the actual position of the fitting half-space with respect to the actual solid provides information about the position of the actual feature, which may be useful in determining the acceptability of the actual feature. Second, a well-chosen set of actual features fitted externally can be used to establish a coordinate frame of reference on an actual solid, which is useful in tolerance specifications and verifications. This leads us to datum systems.

A *datum feature* is a set of surface features, *so designated.* A *datum* is a set of half-spaces rigidly associated one-to-one with the members of a datum feature on an actual solid. Given an actual solid and a datum feature on it, a datum associated with the datum feature is simply a rigid collection of externally fitting half-spaces for the datum feature, established by the fitting methodology of Definition 4. The issues of the uniqueness of the datum established and the role of the closeness criterion in ensuring the uniqueness require further study. Here we merely note that a datum may not be unique. In interpreting tolerance assertions we take this to mean *any* datum associated with a datum feature, even though we refer to datums as if they were unique. The most common datum features are single planar patches, single cylindrical patches (holes and bosses), and sets of two nominally parallel planar patches (slots and tabs). The datum semantics we have discussed pertain to *Regardless of Feature Size* (RFS) datums in current practice.

A *datum system* is an ordered set of at most three datums. A datum system can be referred to in controlling spatial relationships among features, as explained later. Given an actual solid and its datum features for a datum system, the datum system is established as follows: (1) The first member of the datum system (*primary datum*) is established using the fitting methodology of Definition 4. (2) If the datum system has a second member (*secondary datum*), that datum is also established using the same fitting methodology, but with an additional constraint. The constraint is that the spatial relationship between the secondary and the primary datums must be the same as that between the datums established on the nominal solid (e.g., the axis of the secondary datum cylinder must be perpendicular to the primary datum plane). (3) The third member of the datum system (*tertiary datum*), if any, is established similarly, subject to the additional requirement that the spatial relationships among the datums should be the same as those among the datums established on the nominal solid (e.g., the tertiary datum plane should be perpendicular to the primary datum plane and coplanar with the axis of the secondary datum cylinder). Note that the specific geometric constraints imposed on the secondary and the tertiary datums are determined by the specific geometric relationships among the primitive half-spaces associated with the nominal datum features. See ANSI standards [2] for examples of typical datum systems.

The datum establishment methodology described above implies (see [20] for a proof) that the actual datum feature is in contact with the datum boundary. The standards [2] refer to another type of datum called the *floating* or MMC datum, which is applicable only to features of size. The sizes of floating datums are not determined by the actual datum features; the standards state that the so-called *virtual condition sizes* (i.e., the sizes of virtual surfaces) are the relevant sizes. Thus, floating datums do not imply any contact requirements. In other words, in the fitting methodology given earlier, only the containment and the material-side conditions are invoked in establishing the datum for an actual solid. Such datums are simulated by fixed size features on gauges and other manufacturing equipment. We feel that such surface features are more appropriately handled through associated virtual surfaces. Therefore, we do not consider such datums.

- *Tolerance specification and verification*

For the purpose of specifying allowable geometric variations from the nominal solid, we require that a set $\mathbf{F}_N \equiv \{F_{N1}, \cdots, F_{Ni}, \cdots\}$ of nominal surface features be identified such that the union of all nominal surface features is the boundary of the nominal solid. We refer to such a set as a *complete set of nominal surface features.* $\mathbf{F}_N$ is not unique, and the regularized (in the relative topology of ∂S_N) intersection of any two distinct nominal surface features in it need not be empty. For tolerancing purposes, we need a set of subsets of $\mathbf{F}_N$ defined as follows.

Definition 5

Let $\mathbf{F}_N$ be a complete set of nominal surface features of a nominal solid. A *nominal tolerance set* $\mathbf{T}_N \equiv \{T_{N1}, \cdots, T_{Ni}, \cdots\}$ is a subset of the power set of $\mathbf{F}_N$ such that the null set is not a member of $\mathbf{T}_N$ and every member of $\mathbf{F}_N$ is a singleton member of $\mathbf{T}_N$. □

Note that since $\mathbf{T}_N$ is dependent on the functionality of the part, it is not defined uniquely.

We next define tolerance specifications, and what is meant by compliance with such specifications.

Definition 6

Given a nominal solid S_N, and a nominal tolerance set $\mathbf{T}_N$, a *tolerance specification* $\mathbf{A}$ is a set of a collection of geometric assertions $\mathcal{A}_i$ associated with each member T_{Ni} of $\mathbf{T}_N$. □

This is essentially the same as the notion of tolerance specifications provided in [14]. The differences are in the definitions of surface features (as discussed previously) and nominal tolerance sets. Tolerance assertions $\mathcal{A}_i$ on a member T_{Ni} of a tolerance set $\mathbf{T}_N$ fall broadly under the following two categories: (1) $\mathcal{A}_i$ control the *geometric form*, and, where applicable, the *intrinsic size(s)* of the feature in a singleton member of $\mathbf{T}_N$; (2) $\mathcal{A}_i$ control the *spatial relationships* between the feature in a singleton member of $\mathbf{T}_N$ and a datum system, or among the features in a nonsingleton member of $\mathbf{T}_N$ and possibly a datum system. Note that a spatial relationship tolerance assertion invoked on a singleton member of T_N must refer to a datum system. An actual solid S_A satisfies a tolerance specification $\mathbf{A}$ if and only if there exists a decomposition of ∂S_A into a set of actual surface features $\mathbf{F}_A \equiv \{F_{A1}, \cdots, F_{Ai}, \cdots\}$ such that

1. $\cup F_{Ai} = \partial S_A$.
2. There exists a *one-to-one correspondence* between $\mathbf{F}_A$ and $\mathbf{F}_N$. We can then define an actual tolerance set $\mathbf{T}_A$, each member of which has a corresponding member in $\mathbf{T}_N$.
3. Each member T_{Ai} of $\mathbf{T}_A$ *satisfies* the set of assertions $\mathcal{A}_i$ associated with the corresponding member T_{Ni} of $\mathbf{T}_N$. We denote this by $T_{Ai} © \mathcal{A}_i$.

We have adopted this tolerance semantics entirely from the *theoretical inspection procedure* given in [14]. Note that the one-to-one correspondence requirement does not prescribe or guarantee a unique segmentation of the boundary of the actual solid.

A general theory of tolerancing is concerned with different types of tolerance assertions and what it means for a set of features to satisfy them (see [14]). Here, we restrict our attention to VBRs.

- *Virtual boundary requirements*

VBRs and their verification can be formally stated as follows.

Definition 7

A *virtual boundary requirement* is a type of tolerance assertion $\mathcal{A}$ made on a member $T_N \equiv \{F_{N1}, \cdots, F_{Nk}\}$ of a tolerance set $\mathbf{T}_N$. It is characterized by a set of scalars, $\mathbf{a} \equiv \{a_1, \cdots, a_k\}$. For an *assembly* requirement each $a_i \geq 0$; for a *material bulk* requirement each $a_i \leq 0$. The member $T_A \equiv \{F_{A1}, \cdots, F_{Ak}\}$ of the actual tolerance set $\mathbf{T}_A$ corresponding to T_N satisfies the virtual boundary requirement if and only if there exists a set of scalars $\mathbf{b} \equiv \{b_1, \cdots, b_k\}$ and a rigid-body transformation $\mathcal{M}$ such that the following conditions hold:

For assembly,

- *Position condition:* $\mathcal{M}O(T_N; \mathbf{b})$ is a rigid collection of *externally* fitting half-spaces for T_A (subject to appropriate

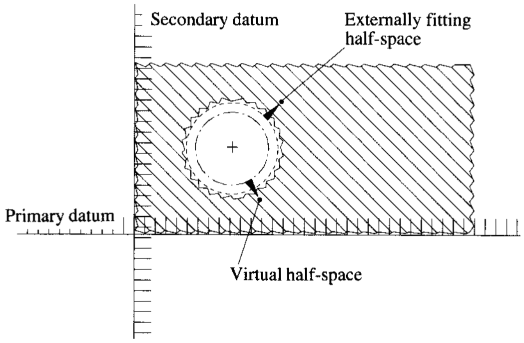


Figure 15

A part satisfying the assembly VBR.

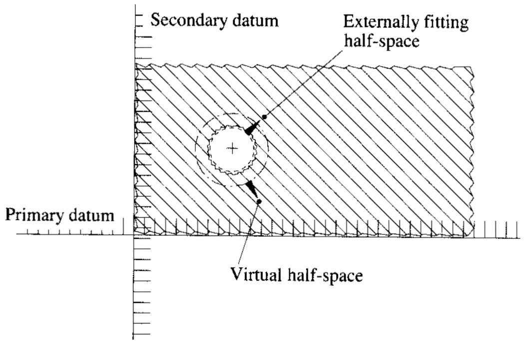


Figure 16

A part failing the containment condition of the assembly VBR.

relationships to the datum system, if any, referred to in $\mathcal{A}$ and established on the actual solid).

- *Containment condition:* $\forall i, F_{Ai} \subset \mathcal{MO}(H_{F_{Ni}}; a_i)$.

For material bulk,

- *Position condition:* $\mathcal{MO}(T_N; \mathbf{b})$ is a rigid collection of *internally* fitting half-spaces for T_A (subject to appropriate relationships to the datum system, if any, referred to in $\mathcal{A}$ and established on the actual solid).
- *Containment condition:* $\forall i, F_{Ai} \subset \overline{\mathcal{MO}(H_{F_{Ni}}; a_i)}^{*}$. □

The exact geometric relationships to be satisfied by $\mathcal{MO}(T_N; \mathbf{b})$ with respect to the datum system referred to in the VBR are determined by the geometric relationships between $O(T_N; \mathbf{b})$ and the datum system established on the nominal part. They may or may not *rigidly* constrain $\mathcal{MO}(T_N; \mathbf{b})$ with respect to the datum system. Generally, reference to a datum system with just a single datum implies only an orientational constraint, whereas a reference to a datum system with three datums implies a complete positional (both location and orientation) constraint.

We illustrate VBRs using a simple example in two dimensions. Imagine a rectangular plate with a hole where an assembly VBR is invoked on the hole with respect to two of the sides of the plate. **Figure 15** shows an instance of the plate that satisfies the VBR. The two sides of the plate are used to establish a datum system with a primary datum and a secondary datum that is perpendicular to the primary datum. In this example, the fitting half-space and the virtual half-space are rigidly constrained with respect to the datum system. **Figure 16** shows an instance of the plate with too small a hole that fails the containment condition of the VBR. **Figure 17** provides an instance with the hole that is grossly out of place in the plate. In this case, we are unable to find a fitting half-space located correctly with respect to the datum system, thus failing the position condition.

We note here that the *MMC Position Tolerance* (which is just an instance of a VBR) semantics given by Requicha [14] is incorrect. Requicha's theory includes only the containment condition above, which means that some actual solids that should be rejected are in fact accepted (see Figure 17 for an example). A necessary and sufficient condition for the satisfaction of VBRs can be established in terms of the offset parameters a_i and b_i. This is addressed in our companion paper [20] and forms the basis for converting VBRs to another representation.

Concluding remarks

We have addressed the problem of representing geometric tolerances that arise from certain functional requirements. It has been shown that assembly and material bulk requirements can be specified as VBRs, which are generalizations of current MMC and LMC tolerances. A theoretical basis has been developed for the rigorous statement and interpretation of VBRs. This theoretical basis is further extended in the companion paper [20] to derive an alternative tolerance specification from VBRs.

The development of a comprehensive and powerful theory of tolerancing in mechanical design requires a detailed study of a variety of functional requirements. Such a study may identify the limitations of VBRs as we have currently defined them and may suggest refinements and modifications. For example, suppose that we wanted to specify the thickness of the **tank** wall and its average diameter independently. This could not be represented as a material bulk requirement because the volume is not the quantity of interest. The following simple generalization of the present formalism would, however, enable us to capture the thickness requirement. The set of scalars a_i that characterizes a VBR must currently be fully specified. We can relax this to the requirement that a_i should satisfy a set of specified constraints, and that in verifying a VBR, a suitable set of a_i must be found in addition to b_i and $\mathcal{M}$. Similarly, our preliminary exploration of kinematic requirements suggests the need for allowing a_i of *different* polarities (i.e., there is at least one pair of scalars a_i and a_j within **a** such that $a_i a_j < 0$) to characterize the VBR and modifying the verification of the requirement suitably.

Our study of assembly functional requirements and their geometric representations covers only a subset of all assembly requirements that arise in practice. Further work needs to be done to extend this coverage. An investigation should be carried out concerning the adequacy of VBRs for different classes of assembly requirements, such as those involving more than two parts and those that establish more or less rigid associations among parts (e.g., assembly using threaded fasteners). An important aspect which should be studied is the relationships among different features of the same part, each of which has primary close contact with another part in an assembly. See [21] for an example for which this is an issue.

VBRs are concerned only with the final configurations of parts in an assembly. They do not address whether, starting from an arbitrary spatial configuration in which the parts are suitably separated from one another, it is possible to achieve the desired final configuration without causing any volumetric interference in the assembly process. This question is an example of the validity of VBRs and needs to be studied further.

Finally, some of the concepts developed in this work (e.g., visible regions, rigid collections of half-spaces, fitting) are applicable to the formalization of general tolerance specifications such as size and form. We suspect that they may even be necessary in some cases, for example in approximating an actual surface feature with a perfect form entity (refer to the discussions related to fitting of a set of actual features in this paper and in [14]). The applicability of our concepts to a general tolerancing theory, the general properties of visible regions, the uniqueness of fitting entities, and the role of measures of closeness in ensuring such uniqueness need further exploration.

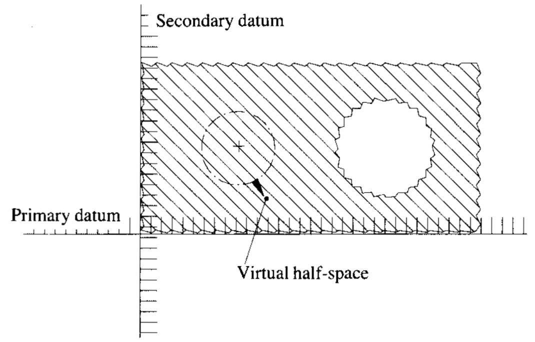


Figure 17

A part failing the position condition of the assembly VBR.

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