Fluctuations in the extrinsic conductivity of disordered metal

by S. Washburn

Random fluctuations of the electrical conductance are ubiquitous in small (typical dimension $L \lesssim 1\ \mu m$) metallic samples at low temperatures (typically $T \lesssim 1\ K \simeq 0.09$ meV). The fluctuations result from the quantum-mechanical interference of the carrier wavefunctions. The superpositions of the wavefunctions depend randomly on the placement of impurities, on magnetic field, and on the current driven through the sample. At length scale L_ϕ (the average distance over which the carriers retain phase information), the fluctuations always have amplitude $\Delta G \simeq e^2/h$, and any observations at scale larger than the coherence length yield a decreased amplitude of the fluctuations. Since the carrier wavefunctions are not classical, local objects (they extend over regions of size L_ϕ), the conductance contains nonlocal terms. For instance, the conductance is not zero far from the classical current paths through the sample and is not symmetric under the reversal of the magnetic field. In this article, the physics of the fluctuations is reviewed, and some of the experiments which illuminate the physics are described.

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Introduction

Early this decade, experiments at low temperature ($T \sim 0.1$ K), on small, localized samples revealed large fluctuations in the conductances [1]. The samples were Si-MOSFETs, and as the gate voltage V_g was changed (the chemical potential μ changed), the conductance $G(\mu)$ mapped out random fluctuations ΔG which spanned several orders of magnitude in $\Delta G/G$ and were *reproducible*. The pattern of the fluctuations $\Delta g(\mu)$ was particular to a given device, although fluctuations of similar character were observed in all devices having the same gross geometry. Subsequently, it was discovered that, even in metallic samples, similar sample-specific fluctuations were observed as a function of magnetic field [3, 4] and chemical potential [5]. At low temperatures the magnetoresistance of small wires (lithographically patterned metal films of length $L = 0.7\ \mu m$, width w, and thickness $d = 40$ nm) contained random (or at least aperiodic) fluctuations of rather smaller amplitude $\Delta R/R \sim 0.1$ percent, where the average resistance $R \sim 100\ \Omega$.

It was suggested that these random conductance fluctuations in the localized regime were the natural result of

[1] In this context [2], the words "localized" and "metallic" refer to the extent ξ of the wavefunctions of the carriers in the devices. In the localized samples, the carrier states are short compared with the length L of the sample $\xi \ll L$; in contrast, in the metallic samples, the wavefunctions span the sample $\xi \gg L$. The corollary is that on the localized side the conductance G is much less than $e^2/h \simeq 4 \times 10^{-4}/\Omega$, or equivalently, that the dimensionless conductance $g \equiv G/(e^2/h)$ is much less than unity, while on the metallic side, $g \gg 1$. Alternatively, one may view the conductance as the ratio of the dispersion W in the energy of the carrier states to the average separation ΔE between them, namely [2], $g \simeq W/\Delta E$. The term *small* indicates that the sample is shorter and narrower than some scattering length which governs the conductivity.

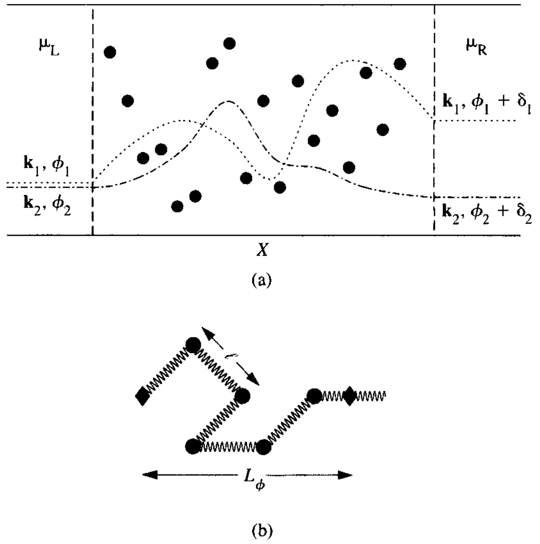


Figure 1

(a) Model of conduction through a disordered metal. The carriers enter on the left in phase at energy k^2, and after traversing the disorder (represented by impurity dots) superpose to give the total transmitted current. (b) The model which is used in the theoretical calculation of quantum-mechanical corrections to the Drude formula. The carriers are assumed to be plane waves which scatter elastically from impurities (dots) and inelastically (diamonds) from phonons, magnetic impurities, etc. The mean free path ℓ is the average free-flight distance between collisions with impurities; typically $\ell \sim 10$–100 nm in polycrystalline metal. The phase-coherence length L_ϕ is the net diffusion distance between inelastic events. At low temperature, $L_\phi \simeq 1\ \mu\text{m} \gg \ell$.

large fluctuations in the density of states $N(\mu)$ [6]. In small localized samples, $N(\mu)$ consists of a sparse random set of sharp peaks, each peak being associated with a spatially localized state in the sample. The conductance, or transmission of carriers, which proceeds by hopping between localized states [7], resonant tunneling [6], etc. is proportional to $N(\mu)$, so that it fluctuates violently as the peaks move past the chemical potential in the sample. [The fluctuations in the total density of states are not truly prerequisite, since the transport physics depends only on states within $\sim k_B T$ of μ. Fluctuations of $N(\mu)$ within this small bandwidth are sufficient to cause the large fluctuations in g.] The states are spatially isolated, so interaction between carriers in different states is irrelevant. The peaks in the g signal the presence of a percolating path (sometimes consisting of only one state) which is "aligned" (to within $\sim k_B T$) with μ so that the carriers can flow across the sample on this path. In the absence of such paths, which on average have spacing $\sim \Delta E$, the sample is a good insulator. The details of the physics of the fluctuations in the localized regime are discussed in this issue by Fowler, Wainer, and Webb [8].

In the metallic case, however, $N(\mu)$ is thought to be a rather smooth function. There is a large overlap between states, and the coupling between the carriers figures in the fluctuations. It broadens the states so that $N(\mu)$ varies on the scale of $\Delta\mu \sim W$ instead of ΔE,[2] and, in addition, the interference among the carrier wavefunctions causes local fluctuations in the diffusion constant D. In fact, in most cases the fluctuations in the diffusion constant are the larger cause of the conductance fluctuations [9]. Numerical models [10, 11], perturbation theory [9, 12–16], and direct analysis of the transmission coefficients [17, 18] lead to the conclusion that the quantum-mechanical coherence of the carriers manifests itself as fluctuations in conductance. Such fluctuations are said to be universal [16] on the grounds that they appear in any phase-coherent metallic sample, are relatively insensitive to the shape of the sample, and always have amplitude of order e^2/h.

In this paper, a brief discussion of the conduction in phase-coherent samples is followed by a discussion of experiments which reveal the random fluctuations in the conductivity of metal and Aharonov–Bohm effects. Recent advances in theory and experiment are also mentioned.

Conductivity in large samples

In order to calculate the conductivity of a bit of metal, one studies the response of the current carriers (electrons and holes) to an applied potential gradient. The carriers are described by wavefunctions $\Psi(\mathbf{x}, t) \sim C(\mathbf{x})e^{i\phi}$, where $\phi = -\omega t + \int (\mathbf{k} + \mathbf{A}) \cdot d\mathbf{x}$. [$C(\mathbf{x})$ is the amplitude of the wavefunction, which depends on position $\mathbf{x}$, $\mathbf{k}$ is the wavevector for the carrier, and $\omega = k^2$ is its energy; $\mathbf{A}$ is the vector potential of the magnetic field. For the sake of simplicity, imagine that the carrier is described by a single well-defined wavevector [19].] In **Figure 1(a)**, if the chemical potential on the left exceeds that on the right, $\mu_L > \mu_R$, current flows to the right. The carriers tend to follow the "classical paths" through the disordered material (schematically illustrated by the dotted and dash-dot lines). Conductance may be treated semiclassically for most purposes. The semiclassical approximation for residual resistance is that in the disordered region, the carriers are assumed to collide *elastically* (k^2 is unchanged) and infrequently (in several senses) with the impurities, and to fly more or less freely between such collisions. A pictorial representation of this model is displayed as **Figure 1(b)**. The conductance is obtained by studying the expectation values of the velocity operators for the carriers [19], or, equivalently, by analysis of the transmission coefficients of the disordered region [20, 21]. The calculations which

[2] Y. Imry, Department of Nuclear Physics, The Weizmann Institute of Science, Rehovot 76100, Israel; private communication, 1985. See also [2].

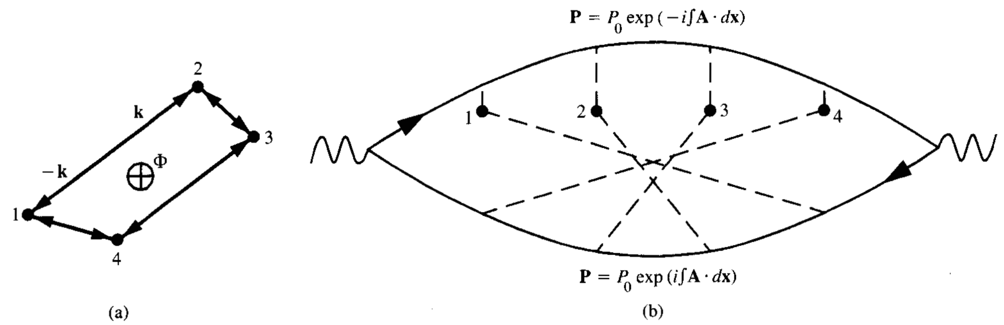


Figure 2

(a) Time-reversed paths which contribute to the "Cooperon" terms in the conductivity. The momenta $\mathbf{k}$ and $-\mathbf{k}$ are indicated by the vectors scattering elastically from the impurities (dots). Any magnetic flux threading the loop destroys the time-reversal symmetry of the paths. (b) The same pairs of paths redrawn as a Feynman graph for the two particle propagators $\mathbf{P}(\mathbf{x}, \mathbf{x}')$.

directly attack the transmission matrix, while excellent indicators of the symmetries and trends of the conductance in a magnetic field [22], are not suited to easy extraction of quantitative predictions for the amplitudes of the various effects predicted. Frequently, quantitatively accurate calculations are made by means of perturbation theory for the carrier Green functions. (In this case, the Green function measures the probability that the carrier will "move" from one point $\mathbf{x}$ in the sample to some other point $\mathbf{x}'$. It is a "wavefunction" for the motion of the carriers.) This calculation is unmanageable without certain approximations such as expanding the Green function in powers of $1/g$ and the assumption that $g = \langle g \rangle$ (where the brackets denote ensemble average of the impurity positions), or invocation of assumptions about the effects of certain averages. The well-known Drude formula [19] for the conductivity $\sigma = ne^2\ell/mv_F$ (ℓ is the mean free path length between collisions with impurities, n is the density of carriers per unit volume, m is the effective mass of the carriers, and v_F is the Fermi speed of the carriers) is obtained by assuming that the carriers emerging on the right retain none of the phase changes acquired in the collisions with the impurities—that ϕ_1 and ϕ_2 are random, so that interference between the wavefunctions does not contribute to the exiting current. In this case, the magnitude of the current is simply the sum of the squares of the amplitudes of the wavefunctions, $|C_1|^2 + |C_2|^2$.

Under most conditions, the ensemble average employed in the calculation is an excellent approximation of the physics, because the phase information is not usually retained throughout the traversal of the sample. Some inelastic mechanism such as collision with phonons, other electrons, or magnetic impurities destroys the phase of the carrier if a sufficiently long time $t \gg \tau_\phi$ (τ_ϕ is the average time between phase-randomizing collisions) has elapsed between injection and egress of the carriers. Furthermore, the ensemble average is a good approximation, because the contribution of a *particular* configuration of the impurities is to add a random component to ϕ. If ϕ is subsequently destroyed in an inelastic collision, the carrier retains just a "mobility" which depends only on the impurity concentration and cross section for collision [19].

Certain pairs of paths (called "Cooperons") can give rise to large corrections to the Drude conductivity [23]. The essential feature of these pairs of paths is that they are the time-reversal of each other: One carrier collides in sequence with impurities $1, 2, \cdots, n, 1$; the other follows the opposite path $1, n, \cdots, 2, 1$. Such a pair of paths is represented schematically in **Figure 2(a)**, where the dots represent impurities and the vectors represent the semiclassical paths of the carriers around the loop (in both directions—clockwise and counterclockwise). The resulting interference of the carrier wavefunctions is a standing wave around the loop of impurities; the magnitude of the current returning to the starting point is $|C_1|^2 + |C_2|^2 + 2C_1^\dagger C_2 = 4|C_1|^2$ († = matrix adjoint operator), which is enhanced by a factor of two over the Drude estimate. (Since the two sets of paths are the time-reversal of each other, $|C_1| = |C_2|$.) More current returning to the starting point means less current through the sample, which in turn means that the conductance decreases. The restriction is that the carriers must retain phase coherence throughout the circuit of the impurity loop. This is not a very stringent restriction at low temperatures, where inelastic mechanisms are relatively weak and infrequent, and, typically, phase coherence is retained by carriers over distances of the order of microns. The experimental work in support of this "weak localization" correction to the Drude formula is overwhelming; i.e.,

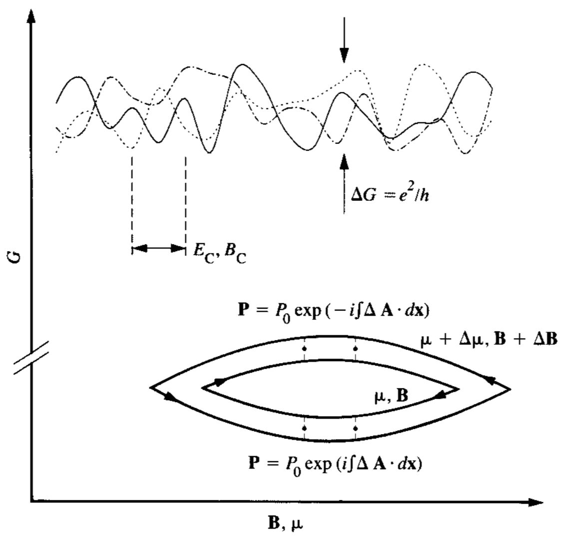


Figure 3

Illustration of the "ergodic hypothesis" used in the theory of universal conductance fluctuations (private communication, A. D. Stone). (inset) A Feynman graph associated with $F(\Delta\mu, \Delta\mathbf{B})$; the propagators depend only on the potential *difference* $\Delta\mathbf{A}$.

observations in metals and semiconductors in one, two, and three dimensions are in excellent agreement with the results of the theoretical calculation mentioned above [23].

The presence of a magnetic field tends to destroy the contributions from these time-reversed paths because it breaks the time-reversal symmetry. This leads to a negative magnetoresistance; the Drude conductance is restored when the size of the Landau orbit is comparable to or smaller than the phase-coherence length L_ϕ. This effect has been widely used to measure the phase-coherence lengths in weakly localized metals. A more elegant effect of the magnetic field occurs if the magnetic flux is threaded through the loop of time-reversed paths, as illustrated in Figure 2(a). The vector potential circulates around the flux and changes the phase of the wavefunction by $\Delta\phi = (e/\hbar)\int \mathbf{A}\cdot d\mathbf{x}$. Because of this Aharonov–Bohm effect on the carrier wavefunctions, *each* of the carriers accumulates an extra contribution to the wavefunction phase, given by $\Delta\phi = 2\pi\Phi/(h/e)$. The superposition of the time-reversed paths yields a contribution to the conductance which oscillates periodically with the enclosed flux [24], the period of oscillation being $\Delta\Phi = h/2e$. This normal-metal Aharonov–Bohm effect was convincingly demonstrated by Sharvin and Sharvin [25], who measured the conductance of a thin-walled Mg cylinder in an axial magnetic field. Exact agreement with the theory was obtained in a subsequent experiment on a Li cylinder [26]. After floundering for a couple of years, western experimentalists reproduced these results [27–30]. Similar Aharonov–Bohm oscillations (period $\Delta\Phi = h/2e$) occur in localized samples when ξ is comparable to the loop size [31–33].

The Feynman graph **[Figure 2(b)]** represents a term in the conductivity for the process of electrons circling the impurity loop [in Figure 2(a)] in the presence of the magnetic field. The Green functions $\mathbf{P}(\mathbf{x}, \mathbf{x}')$ (solid lines) include effects of impurity scattering (dashed lines). They also depend on the magnitude of the magnetic field B through the vector potential $\mathbf{A}$ ($\mathbf{B} = \nabla \times \mathbf{A}$).

Conductance in small samples

In very small samples there is the additional complication that the phase coherence may span the entire sample. Development of the theory for $L \simeq L_\phi$ by Al'tshuler et al. [9, 13, 14, 34, 35] and Lee et al. [15, 16, 36–38] established that when phase coherence is maintained across the sample, even pairs of paths which are not time-reversed give rise to corrections to g of order 1. This is to say that a variety of phase-coherent samples from the same material and having the same shape will have conductances which vary by about $\Delta G = e^2/h$. The variation in g is universal in the sense that no matter the sizes of the samples or their average conductances, the variation is always $\langle \Delta g \rangle \sim 1$. The precise value of $\langle \Delta g \rangle$ is sensitive to the shape of the samples. A variation of about 50 percent occurs between thin wires and cubes, and a similar variation is expected for other geometries.

These conclusions were reached through the study of the correlation function $F(\Delta\mu, \Delta B) = \langle \Delta g(\mu, B), \Delta g(\mu + \Delta\mu, B + \Delta B)\rangle$, where $\langle\ \rangle$ denotes an ensemble average over the impurity configuration. The inset in **Figure 3** is a Feynman graph which contributes to $F(\Delta\mu, \Delta B)$. Each factor g comes from one of the solid-line loops, and the two loops "see" the same impurity potential, but from the vantage point set by different magnetic fields (and different magnetic potentials $\mathbf{A}$). The result is that the correlation function depends only on the difference in magnetic field, not on the absolute field. Amazingly, the ensemble average, which rearranges all of the impurities, is excessive in the sense that moving so much as a single impurity also causes changes in g of order 1 (at least in one and two dimensions) [34]. The theory actually calculates fluctuations in conductance among a large ensemble of samples with the same geometry. Connection was made to the fluctuations in experiments, which were observed in particular samples as a function of chemical potential, μ, or the magnetic field, B, through the "ergodic hypothesis," which states that the ensemble average employed by the theory provides information equivalent to that obtained by sweeping B or μ [15]. The point of the hypothesis is that random changes in the carrier phases resulting from variations in B or μ will *on average* yield the same Δg as scrambling the impurity positions. Figure 3 illustrates the point—at a given value of B or μ, the sample-to-sample variations (different lines) are of the same amplitude as those swept through when μ or B varies. This intuitive claim has been given a firm theoretical underpinning [35]. The $\Delta\mu$ required to cause fluctuations is $E_C = W \simeq \hbar/t$, where W is the dispersion in the energy of the carrier and t is the time of traversal for the carriers.[2] For diffusive motion (the case appropriate to most metallic conduction), the mean free path ℓ between collisions with impurities is much less than the sample length L, and $E_C \simeq \hbar D/L^2$, where D is the diffusion coefficient for the carriers. This "correlation energy" E_C was observed directly in experiments on Si-MOSFETs [5] where $\Delta\mu$ was enforced by small changes in gate voltage. A change in $B > B_C \sim h/eLw$ will also cause variations $\Delta g \sim 1$. This field scale is consistent with the magnetoresistance results published to date [4].

Just as in the case of large samples, magnetic flux threaded through a cylinder or a ring also contributes to periodic Aharonov–Bohm oscillations. For instance, in **Figure 4,** the carriers passing on either side of the hole in the wire acquire a non-random component to their relative phase. In addition to the random terms δ_1 and δ_2, the carriers on either side of the flux threading the annulus acquire a net *relative* phase $\delta = 2\pi\Phi/(h/e)$, which yields the fundamental period of the Aharonov–Bohm oscillations $\Delta\Phi = h/e$ [22, 39]. The period $\Delta\Phi = h/2e$ remains, but it is the result of second-order processes (including Cooperons) which in most cases are weaker contributors to the interference [17]. An example of the magnetoresistance of an Sb loop (inside diameter 815 nm and outside diameter 905 nm) is displayed in **Figure 5** (inset photograph). The magnetoresistance contains the random fluctuations which are ubiquitous in small samples, but it also contains periodic oscillations which are illustrated by the inset. The fluctuations persist to high magnetic fields, unlike the effects which result from Cooperons. This persistence is sensible from the theoretical viewpoint because $F(0, \Delta B)$ depends only on the field *difference*. The theory for F, however, has no validity when B exceeds the limit B_{CL}, which marks the limit of strong fields in the classical sense—the Landau orbit becomes comparable to or smaller than ℓ (the mean free path length). Measurements in this "strong field" regime have shown that the periodic oscillations vanish soon after the limit $B = B_{CL}$ is crossed, but that the aperiodic fluctuations remain even deep in the strong-field regime (when $B \gg B_{CL}$) [41]. The Fourier transform of the magnetoresistance $R(B)$ comprises "excitations" at $1/\Delta B = 0$ which are the signature of the random fluctuations and excitations at $1/\Delta B = 140$/tesla, the signature of the h/e oscillations.

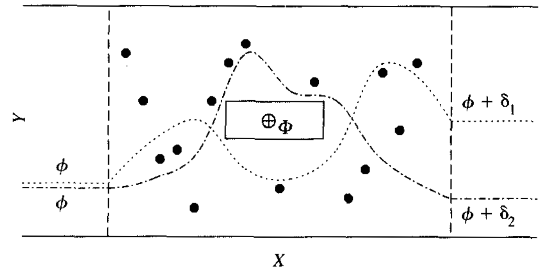


Figure 4

The same geometry as shown in Figure 1 interpreted as a loop threaded by a magnetic flux Φ.

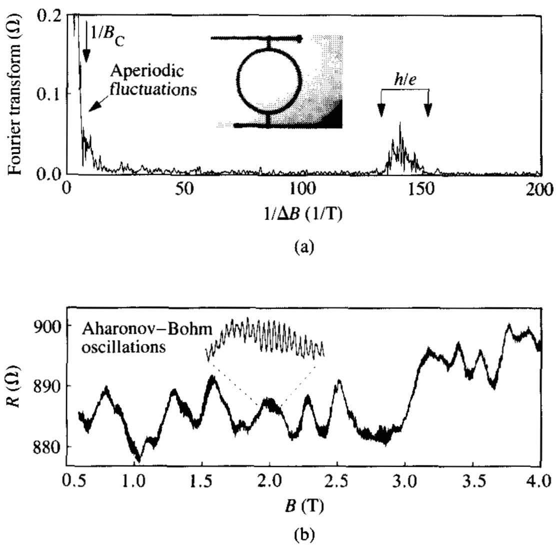


Figure 5

(a) The magnetoresistance of the Sb loop (inside diameter 815 nm and linewidth 45 nm) shown in the inset of the top figure. The inset illustrates the existence of h/e Aharonov–Bohm oscillations throughout the magnetic field range. (b) The Fourier transform of the magnetoresistance which contains structure at $1/\Delta B = 0$ from the aperiodic fluctuations and at $1/\Delta B = 140$/tesla from the h/e Aharonov–Bohm oscillations (from [40], reprinted with permission).

The conductance of any loop contains nonclassical terms such as [17, 22, 42]

$$\Delta g(B) = \sum_{n=0}^{\infty} g_n(B)\cos\left\{\left(\frac{2\pi n\Phi}{h/e}\right) + \alpha_n(B)\right\}, \quad (1)$$

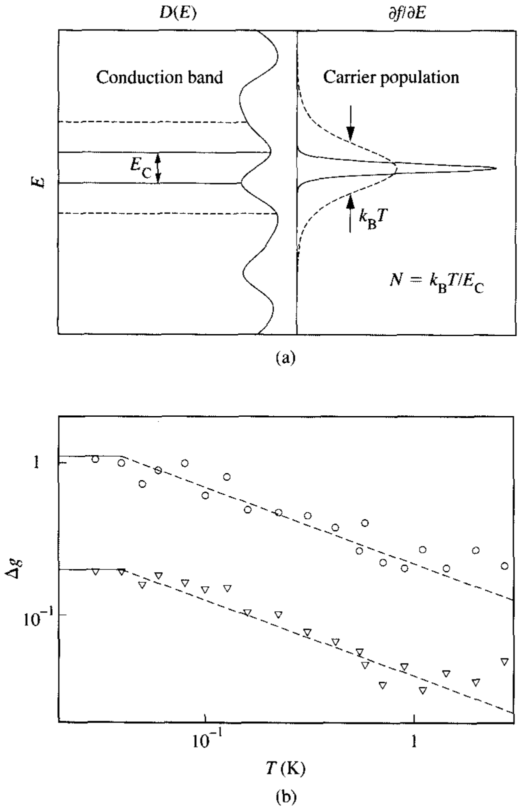


Figure 6

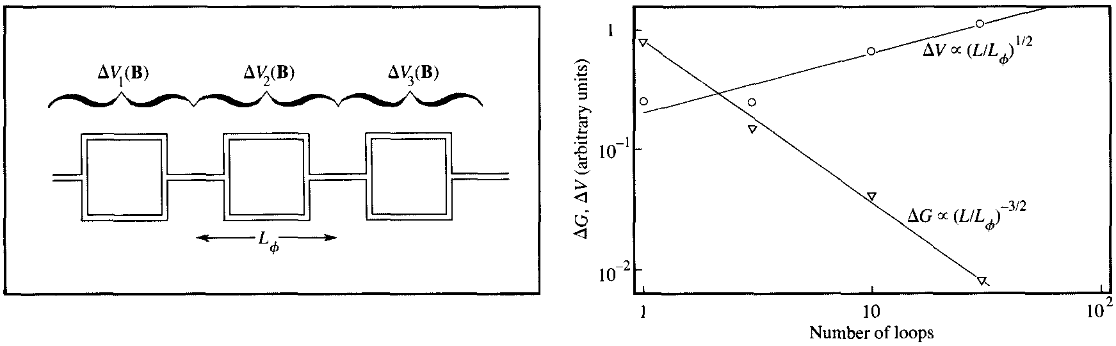
(a) Schematic illustration of the physics of energy-averaging. (b) Experimental data, aperiodic fluctuations (○) and h/e oscillations (▿), from an 830-nm-diameter loop of Au which agrees with the energy-averaging model (from [43], reprinted with permission).

where, because of the flux piercing the wires, g_n and α_n are random functions of the magnetic field. The term g_0 contributes the aperiodic fluctuations in the magnetoresistance and the peak near $1/\Delta B = 0$ in the Fourier transform. The scale on which this peak decays is $1/\Delta B \sim wL_\phi e/h$, where w is the width of the wires in the sample. The term g_1 contributes the amplitudes of the h/e periodic oscillations and the peak near $1/\Delta B = 140$/tesla in the Fourier transform, and so on. The random factors in Equation (1) imply that the magnetoresistance is not strictly periodic, but rather locally periodic within magnetic field ranges $\Delta B \lesssim B_C$. The Aharonov–Bohm oscillations in field ranges separated by more than B_C are uncorrelated—they do not share the same value of $\alpha(B)$, and the "frequency" (in inverse magnetic field) of the oscillations is slightly different, but still within the limits set by the inside and outside perimeters of the loop. (The constraint on the range of Aharonov–Bohm periods arises because the carriers must follow a path which is somewhere inside the wires forming the loop, so that the flux enclosed by the carrier paths is some value bounded by the areas enclosed by the inside and outside perimeters of the loop.) The Fourier transform of magnetoresistance contains excitations throughout the frequency range set by this geometry constraint instead of a lone delta function, which would result if the period were constant. Analogously, the density of states is locally "periodic" (the level spacings are about equal) in energy ranges $\Delta E \lesssim E_C$ [9], but uncorrelated over larger energy scales.

Decay of the fluctuations from averaging

At finite temperature ($T > 0$), the Fermi energy of the carriers varies by amounts of the order of the thermal energy $k_B T$. This smearing of the carrier energies causes "energy averaging" [10] if $k_B T > E_C$. Within correlated bands of width E_C, the $\Delta g(B)$ pattern is the same. If $k_B T < E_C$, then, as illustrated by the solid lines in **Figure 6(a),** only one band contributes to the conductivity, and $\Delta g(B)$ is independent of the temperature. If, on the other hand, $k_B T$ exceeds E_C, then [dashed lines in Figure 6(a)] several uncorrelated patterns are averaged to give the total Δg. The number of patterns is simply $N = k_B T/E_C$, and the amplitude of the total conductance fluctuation is $\propto 1/\sqrt{N}$. This decay resulting from energy averaging has been confirmed in one-dimensional samples for both the aperiodic fluctuations and the periodic oscillations [5, 43]. Samples of data from an experiment on a loop of gold are displayed in **Figure 6(b).** For temperatures less than 0.04 K, the aperiodic fluctuations (○) and the h/e oscillations (∇) are independent of temperature. When the temperature exceeds $E_C/k_B \simeq 0.03$ K, both sets of data (○ and ∇) decay as $1/\sqrt{T}$. In two and three dimensions, the physics is more subtle [16].

When the sample length $L > L_\phi$, the measured Δg is the average of the contributions from the (L/L_ϕ) phase-coherent segments of the sample [44, 45]. This has been confirmed for the Aharonov–Bohm oscillations from strings (of total length L) of Ag loops separated by $\sim L_\phi$. In these samples [see **Figure 7(a)**] the *voltage* fluctuation pattern from each loop is independent of the patterns from the other loops. These uncorrelated patterns, when added together to produce the resistance of the string, yield a voltage fluctuation whose amplitude grows according to the classical law for addition of random fluctuations, $\Delta V \propto \sqrt{N} = (L/L_\phi)^{1/2}$. The naive estimate for the conductance fluctuations, on the other hand, decays as $\Delta G \simeq \Delta V/I\langle R\rangle^2 \propto (L/L_\phi)^{-3/2}$ (since R is proportional to L). The results of the experiment are displayed in **Figure 7(b)**; the voltage fluctuation (○) increases as $(L/L_\phi)^{1/2}$ (upper line), and the conductance fluctuation (∇) decays as $(L/L_\phi)^{-3/2}$ (lower line). Upon factoring out the contribution of the segments joining the loops (which do not contribute to the Aharonov–



(a)

(b)

Figure 7

The classical averaging of the fluctuations from uncorrelated segments of a sample—in this case, a string of loops whose separation is approximately L_ϕ (a). The h/e oscillation amplitude (b) observed at T = 0.3 K from the string plotted as bare voltage fluctuations (○) and as conductance fluctuations (▽) (from [44], reprinted with permission).

Bohm oscillations), one finds that the oscillation amplitude *per loop* decays as $(L/L_\phi)^{-1/2}$, and once again the simple classical averaging for fluctuations, $\Delta g \propto 1/\sqrt{N}$, where N is the number of loops being measured [44], is confirmed. The decay of Δg_1 is the result of the random phase α_1, which varies from loop to loop. It is also the reason for the dominance of the period $h/2e$ in the experiments on cylinders and two-dimensional arrays of loops. In such large samples the effective N is 10^4 to 10^6, and the amplitude of the h/e oscillation is buried in the noise. The oscillations from the Cooperons always have the *same phase* ($\alpha_2 \equiv 0$, since the phase accumulated around the impurity loop is the same in either direction), and consequently they do not average away. Subsequent experiments on a multitude of wires formed from Si-MOSFETs have shown that the averaging law $\Delta G \propto (L/L_\phi)^{-3/2}$ is valid over several orders of magnitude in L/L_ϕ.

In the case of Aharonov–Bohm oscillations from a single loop wherein $L_\phi < L$ (the distance around the loop), the averaging is more severe. The dominant source of decay in the oscillations is the loss of phase coherence of the carriers whose interference generates the Aharonov–Bohm oscillations [40]. The carriers that do not retain phase coherence until reaching the terminus of the loop do not contribute to Aharonov–Bohm oscillations. By the definition of L_ϕ, the number of carriers retaining phase coherence is $N_\phi = \exp(-L/L_\phi)$, so that we expect $\Delta g \propto N_\phi$. This simple argument is supported by calculations for conductance fluctuations in a loop [46], and the same exponential factor appears in the theory for the oscillatory Cooperon terms discussed above [24, 26]. Experimental tests [40] of this surmise about the decay of the oscillations have essentially

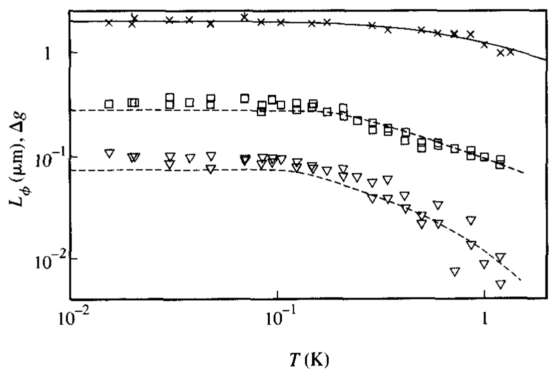


Figure 8

The temperature dependence of the phase-coherence length L_ϕ (×), the aperiodic conductance fluctuations Δg_0 (□), and the periodic Aharonov–Bohm oscillations Δg_1 (▽) for an Sb loop. The dashed lines are the predicted temperature dependences for Δg_0 and Δg_1, which result from the solid line interpolating $L_\phi(T)$ (from [40], reprinted with permission).

confirmed it. In two Sb loops, L_ϕ was measured by the conventional method; i.e., it was inferred from the magnetoresistance associated with the Cooperons [23]. The aperiodic fluctuations Δg_0 and the h/e oscillations g_1 were also measured for the same samples. For each sample, the measurements (crosses in **Figure 8**) yielded a parametric equation (solid line) for $L_\phi(T)$ which was used to predict the

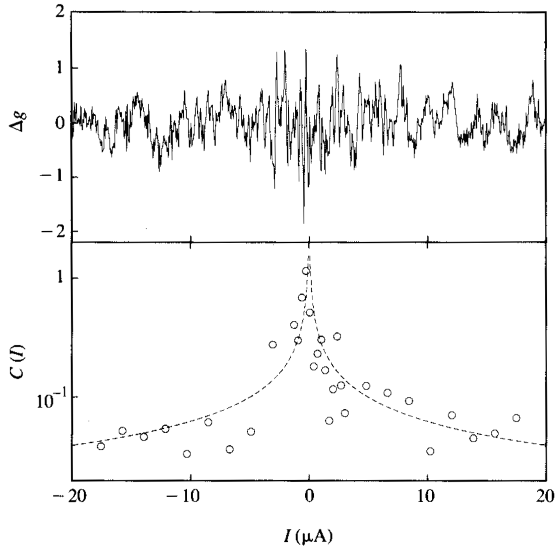


Figure 9

(a) Conductance fluctuations Δg as a function of the current through a metal wire measured at $T = 0.01$ K. (b) The local autocorrelation $C(I) = \langle (\Delta g)^2 \rangle$ as a function of the current through the wire. The dashed line is the theoretical [47] prediction (from [49], reprinted with permission).

temperature dependence of the aperiodic fluctuations $\Delta g_0 \propto [L/L_\phi(T)]^{-3/2}$ and periodic oscillations $\Delta g_1 \propto \exp[-L/L_\phi(T)]$. After accounting for the energy averaging, one finds good agreement with the predicted averaging laws [40], as illustrated by the dashed lines in Figure 8.

Nonlinearity in the conductance fluctuations

The fluctuations in the conductance also appear as the voltage across the sample ($\mu_L - \mu_R$) changes [14, 47]. This leads to a nonlinear function $g(V)$ which varies randomly on the voltage scale $V_C = E_C/e$. In contrast to the case of conductance fluctuations in a magnetic field, where the field scrambles the phases of the carrier wavefunctions, the nonlinearity in fluctuations arises because the impurity potential is tilted by the applied current $IR = V = \mu_L - \mu_R$. The precise dependence of $\Delta g(I)$ on I is an involved question which depends on the relation between the energy scales $k_B T$, eV, and $\hbar/\tau_\phi$ (τ_ϕ is the average time between inelastic collisions). According to the theory [47], when $\hbar/\tau_\phi \to 0$, $\langle \Delta g(I) \rangle$ is peaked near $k_B T/eIR_\phi$ (R_ϕ is the resistance of a wire of length L_ϕ) [49] and decays as I increases. Because the random impurity potential has no inversion center, current flowing to the right sees a different potential from that seen by current flowing to the left, and $\Delta g(I) \neq \Delta g(-I)$ [14]. These theoretical predictions [14, 47] are consistent with experiments. The study of a Si-MOSFET [48] uncovered a random $\Delta g(I)$ which was not symmetric as $I \to -I$. Somewhat more sophisticated studies of metal wires (Au and Sb) have determined that the prediction for the current dependence of $\Delta g(I)$ is substantially correct [47] and that harmonic signals generated by the nonlinear conductance have a surprising resilience against decay as the harmonic number increases [49]. Typical data for a metal wire ($L = 0.6$ μm and $w = 0.1$ μm) are displayed in **Figure 9.** As predicted [14, 47], the fluctuations Δg are of order 1, and they are not symmetric when $I \to -I$. The current dependence is obtained by studying the autocorrelation function $C(I) = \langle \Delta g(I), \Delta g(I + \Delta I) \rangle$ in narrow intervals centered at I. For $\Delta I = 0$, $C(I) = \langle (\Delta g)^2 \rangle$. The abscissa in Figure 9(a) is parsed into intervals, and $C(I)$ is calculated for each interval. Within the noise in the data, $C(I)$ agrees with the theoretical prediction (dashed line) given by [47] and [49]. Since these data were obained when the sample was at $T = 0.01$ K, the peak at $k_B T = IR$ is buried near $I = 0$. Data from higher temperatures [49] do contain a peak in the correlation function near where $IR_\phi = k_B T$.

Voltage fluctuations

The theory for conductance fluctuations was originally composed exclusively for two-probe samples, namely the geometry of Figure 1, responding to a voltage bias $\mu_L - \mu_R$. With few exceptions [5, 48, 50], the experimental situations have been rather different. For historical and technical reasons (mainly that it is easier and simpler to make accurate measurements this way), instead of establishing a potential difference between the source and sink, a constant current is forced through the sample [usually between leads 1 and 4; see **Figure 10(a)**], and the potential fluctuations ΔV are measured by leads (2 and 3, for instance) attached at various points on the device; ΔV is converted to ΔR or ΔG by the usual rules associated with ohmic samples. The voltage probes are invariably made of the same material and have the same average conductance as the rest of the sample, but they are constrained to carry no net current. The two-probe theory cannot be applied haphazardly to the analysis of data from "four-probe" measurements.

The experiments by Benoit et al. [51] revealed that the fluctuation pattern $G_{14,23}(B)$ ($G_{ij,kl} \equiv I_{i\to j}/V_{k\to l}$) was *not* a symmetric function of magnetic field. If in fact the two-probe theory were valid here, the experiment would appear to have violated fundamental time-reversal symmetries [52]. It was shown experimentally that the conductance consisted of an antisymmetric part $[G(B) = -G(-B)]$ and a symmetric part $[G(B) = G(B)]$, and that the symmetries of G are the same as for inhomogeneous classical conductors. That is, $G_{14,23}(B) = G_{23,14}(-B)$, and time-reversal symmetry ($B \to -B$ and $\mathbf{x} \to -\mathbf{x}$) is, of course, conserved [51]. Theoretical work has demonstrated that this result follows from a rigorous model of four-probe conductance [53] because of fluctuations in the Hall voltage [54].

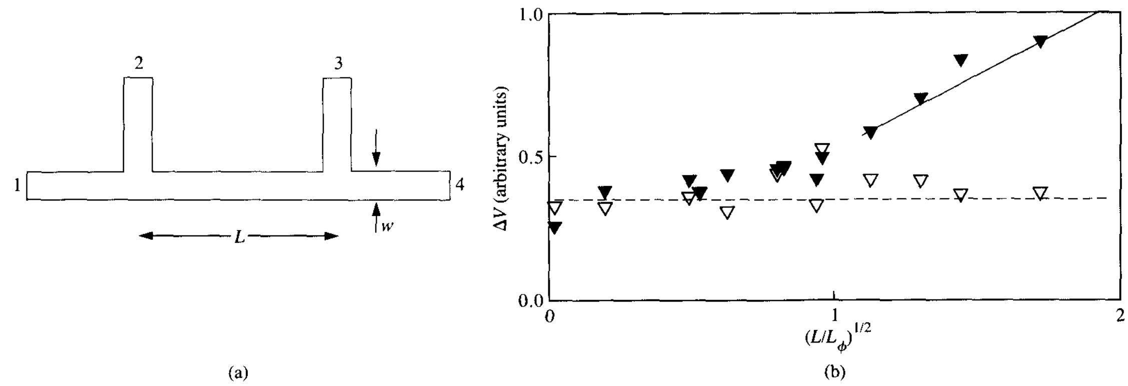


Figure 10

(a) The sample configuration for four-probe measurements of potential fluctuations. The wires have width w, and the voltage probes are separated by a distance L. (b) The voltage fluctuations ΔV measured as a function of the ratio L/L_ϕ in an Sb wire of width $w \simeq 0.1$ µm. The symmetric component ΔV_S (▼) scales according to classical averaging when $L \gg L_\phi$ and is a much weaker function of L when $L \lesssim L_\phi$. ΔV_A (▽) is nearly independent of length. The data were recorded at $T = 0.04$ K (from [55], reprinted with permission).

It also follows from analogy with classical conductors that the Hall voltage is independent of the distance L between the probes. Benoit et al. [55] have proved that the antisymmetric part of ΔV must also be independent of L. [Note: This implies that the averaging law $\Delta V \propto (L/L_\phi)^{1/2}$, which leads to $\Delta V \rightarrow 0$ as $L \rightarrow 0$, cannot hold as $L \rightarrow 0$.] The experiment [55] determined that, indeed, the antisymmetric fluctuations ΔV_A [open symbols in **Figure 10(b)**] were nearly independent of L. Measurements [55, 56] also showed that the symmetric part ΔV_S (solid symbols) was approximately independent of L when $L < L_\phi$. Of course, for $L > L_\phi$, ΔV_S followed the usual averaging law $\Delta V_S \propto (L/L_\phi)^{1/2}$, as illustrated by the solid line in Figure 10(b). The data in Figure 10(b) happen to be from a Sb line which is about 0.1 µm wide sitting at a temperature $T = 0.04$ K. The curves, however, are somewhat more universal in that data from other samples (Au wires [55], Si-MOSFETs [56]) having a range of width ($0.03\ \mu m < w < 0.25\ \mu m$) and from other temperatures ($0.01\ K < T < 1\ K$) exhibit the same trends.

A much eerier and subtler result was noticed in the four-probe samples. The measurement of $G_{12,34}(B)$ yields fluctuations as large as $G_{14,23}(B)$ when L (the distance from lead 2 to lead 3) is much less than L_ϕ [55]. From the classical point of view, this observation is truly astonishing! In the classical case, the detected voltage at lead 3 is an exponentially small function of the ratio L/w. In quantum mechanics the electron "exists" over a large area $\sim L_\phi^2$. It is in fact *impossible* for the electron to get from 1 to 2 without disturbing the potential at probe 3, and it is these "nonlocal" fluctuations which are detected in $G_{12,34}(B)$.

For $L < L_\phi$, the probes still measure potential fluctuations from a region of length $\sim L_\phi$ so that the "effective" sample length L_{eff} is never smaller than L_ϕ [38, 55, 56]. The carrier wavefunction encloses flux and fluctuates spatially (because of interaction with impurities) over the entire region where it retains coherence. The claim that $L_{\text{eff}} \geq L_\phi$ immediately yields an heuristic explanation of the near length-independence of ΔV_S when $L < L_\phi$: Even if the probe spacing is less than L_ϕ, ΔV results from flux enclosed in an area $\sim wL_\phi$ extending into the voltage probes as well as along the classical current path. Theoretical results [11, 38, 57–59] are consistent with the experimental results [55, 56] mentioned above (Figure 10(b)], but the nonlocal terms imply [58] that neither ΔV_A nor ΔV_S is ever really independent of length. The length dependence of ΔV_S is weaker than expected from classical averaging of uncorrelated segments, and ΔV_S does not go to zero as $L \rightarrow 0$.

It has also been noticed that the asymmetries mentioned above are related to the nonlocal fluctuations [58]. The antisymmetric component ΔV_A arises from fluctuations within L_ϕ of the voltage probes and so has no dependence on the separation of the probes (except for a weak dependence when $L \lesssim L_\phi$). In contrast, ΔV_S is accumulated along all of the wire between the probes and within regions of length L_ϕ around the nodes where the probes join the classically allowed current path. Crudely speaking, the effective sample length is $L_{\text{eff}} \simeq L + 4L_\phi$, where the approximate factor 4 accounts for excursions beyond the voltage probes and into them. For $L \gg L_\phi$, the effective sample length is the separation of the voltage probes, and the ensemble averaging

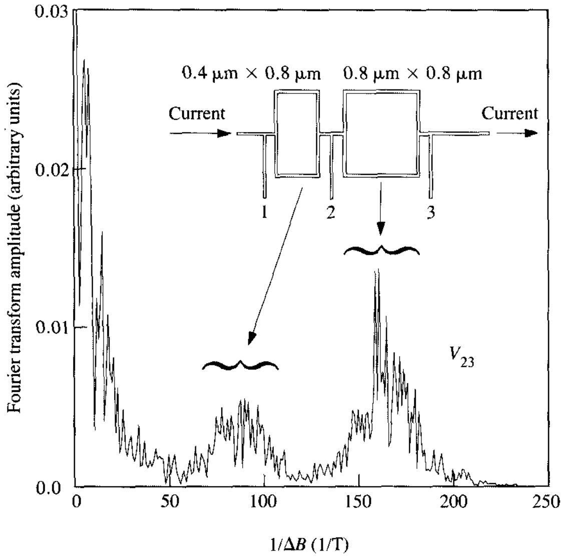


Figure 11

The Fourier transform of the voltage fluctuations measured in the two-loop Sb sample (the larger loop was 0.8 μm on a side, and the linewidth was 80 nm) at $T = 0.04$ K. Nonlocal Aharonov–Bohm oscillations from the smaller loop (0.4 μm × 0.8 μm) were detected in the experiment, which measured classically forbidden voltage fluctuations (from [60], reprinted with permission).

mentioned above [44, 45] applies. In contrast, ΔV_S depends weakly on L when $L \lesssim L_\phi$ because this averaging is inoperative; L_{eff} is dominated by L_ϕ, which does not depend on the physical probe separation.

More dramatic demonstrations of the nonlocal conductance fluctuations can be observed in the fluctuations measured in the vicinity of a phase-coherent loop [60, 61]. For instance, the sample sketched in the inset in **Figure 11** comprises two Sb loops in series. The Fourier transform of the voltage fluctuations measured between leads 2 and 3 contains the signature of Aharonov–Bohm oscillations from *both* of the loops [61]. The assumption that the carrier wavefunction is a quantity with short extent ξ, such as $\xi \sim \ell$, as might be inferred from semiclassical analysis, implies that observation of the Aharonov–Bohm oscillations from the smaller loop is forbidden. The appearance of the oscillations from the smaller loop is direct and dramatic evidence that the wavefunctions of the carriers are nonlocal quantities extending over regions of length $\xi \sim L_\phi$.

Conclusion

The theoretical and experimental study of transport of quantum-mechanically coherent conductors has revealed unexpected complexity in the physics of electrical conduction. The conductance of a particular device is spectacularly sensitive to the ambient electromagnetic fields, the temperature, the current through the sample, and the precise disposition of the impurities in the samples. The fluctuations in G provide a sensitive handle on the effects of quantum-mechanical phase coherence in disordered samples. The amplitudes of the aperiodic conductance fluctuations in wires are reduced (albeit rather slowly as power laws in temperature) by energy averaging and inelastic scattering. In contrast, the amplitude of the periodic oscillations from loops is reduced exponentially by inelastic scattering.

Owing to the experimental convenience of voltage measurements, the symmetries of the conductance, which were obscured in the original two-probe formulation of the theory for conductance fluctuations, were discovered. The conductance was found to have the same symmetries as an inhomogeneous, classical conductor. A categorically nonclassical, nonlocal character was also discovered in the conductance; the nonlocal character of the conductance is the direct signature of the nonlocal character of the carrier wavefunctions.

Other aspects of conductance fluctuations in metallic systems, such as the effects of scattering by magnetic impurities [62], are discussed in reviews of the subject [16, 24, 63] and elsewhere in this issue.

Acknowledgments

The above discussion is the result of education received variously from A. Benoit, M. Büttiker, D. DiVincenzo, C. van Haesendonck, Y. Imry, R. Landauer, P. Lee, M. Ma, D. Stone, R. Webb, and many others. I am also grateful to R. Landauer for his comments on this paper.

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Received December 20, 1987; accepted for publication January 4, 1988

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