Spatial variation of currents and fields due to localized scatterers in metallic conduction

by R. Landauer

Volume 1 of this journal, thirty-one years ago, included a paper with the above title. Studies of small samples, in recent years, as well as earlier work on disordered samples, have caused some of the content of the earlier work to become widely understood. The aspects stressed in the title, however, relating to the spatial variations in the vicinity of a localized scattering center, have received little attention, except in electromigration theory debates. Here, we return to these aspects of the earlier paper, and emphasize that the transport field associated with a point-defect scattering center is a highly localized dipole field. The nonlinearity of resistance in terms of scattering cross section is discussed. A theory of these effects, which does justice to the coherent multiple-scattering effects present at low temperatures, does not yet exist. Such a theory is likely to modify the effects, but it is unlikely to cause them to disappear. We also discuss closed loops, without leads; the persistent currents expected in these; and a possible method of detecting the persistent currents.

1. Introduction, residual resistivity dipoles

This paper intentionally repeats an earlier title [1]. Reference [1] remained obscure for well over two decades, until its viewpoint was revisited by Anderson et al. [2]. Much of the content of [1] is still unnoticed, and we return to it here. In the meantime, an emphasis on small samples, Aharonov–Bohm effects, universal fluctuations, and localization has brought attention to a number of related topics. It would overwhelm this brief discussion, and this author's skills, to attempt to make *all* the possible cross-connections. That is best done by the reader via other papers in this issue, and with the help of earlier reviews [3–8].

Reference [1] studied the spatial variation of electron transport currents, and the associated transport fields, in the vicinity of spatially localized scatterers, including both point defects and reflecting planes. Reference [1] pointed out that there was, in fact, pronounced spatial variation. The principal deficiency of [1]: It did not allow for the coherence between successive elastic scattering events which can be expected at low temperatures. This oversimplification is repeated here to some extent. We offer two justifications. First of all, there is a regime of validity where, typically, only a few elastic scattering events occur before inelastic scattering takes place. More significantly, much of what we have to say can be expected to reappear in a more sophisticated form in treatments which do allow for coherence. This paper's return to earlier themes is not intended to repeat the content of [1].

Consider **Figure 1.** Current is maintained through a conductor containing localized scatterers and/or phonon scattering. One additional scattering center is introduced. Most likely the resistance will increase. We use the qualification "most likely," because in the presence of coherent multiple scattering the resistance can decrease. (For example, if in a periodic array of scatterers one is missing, supplying the missing scattering center will reduce the resistance.) Additionally, in the presence of inhomogeneity, it is possible, even under classical macroscopic conductive behavior, to have "backwards" flow in places, flow opposed to the direction of the overall flow. We ignore this and invoke a mean field view under which the extra scatterer introduced in Figure 1 is exposed to an average incident carrier flux.

If a fixed current is maintained at the boundaries of the sample, and the resistance is increased by the extra scattering center, the voltage across the sample increases. But the field cannot increase uniformly throughout the sample; the field increase must occur near the extra scatterer. How is the spatial variation of field and current flow disturbed by a localized scatterer? This is a question which we might expect to find discussed in every solid-state text, and certainly in more specialized review articles. But that is not the case. If instead of a point scatterer we had introduced a macroscopic cavity, the answer would be obvious. There would be charges on the surfaces of the cavity, constituting the sources of a dipole field which causes the current to detour around the cavity. As we shrink the cavity, where can we expect a transition? The answer given in [1] and elaborated subsequently [9–11]: In some ways, there is no transition. The additional transport field associated with the scatterer is a dipole field whose sources lie within a screening length (modified by Friedel oscillations) of the scattering potential. The detour current pattern, taking the scattered current around the scattering center, is more complex than in the case of the macroscopic inhomogeneity, and is shown in **Figure 2,** taken from [1]. The incident carriers, after scattering by the defect, move ballistically for about one mean free path. At greater distances from the scattering center, the detour current resembles that due to a macroscopic cavity. The localized dipole field established in the presence of current flow can be made plausible in an alternative way. Consider an interstitial hydrogen in a symmetrical lattice site. The proton will be screened by a charge which has the symmetry of the surrounding lattice. In the presence of electronic transport, we can expect this symmetry to be broken, resulting in the residual resistivity dipole we have been discussing.

Spatial variations are not only of conceptual interest. If we are interested in electromigration of the defect shown in Figure 1, i.e., the defect motion in the presence of fields and currents, then the exact conditions at the location of the

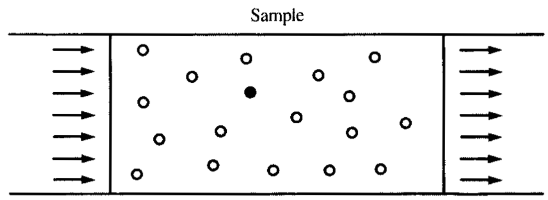

Figure 1

Constant total current flow is maintained across the sample boundaries. An additional point-scattering center, shown in black, is introduced.

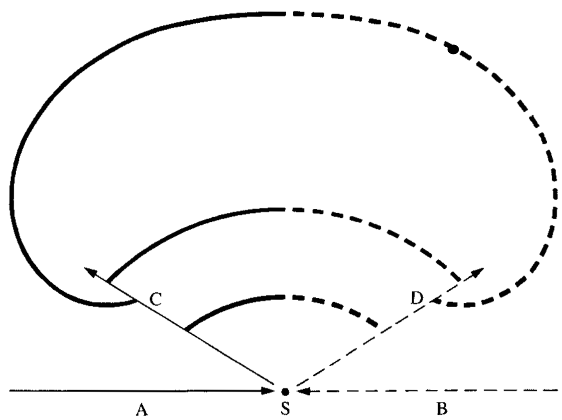

Figure 2

Electrons in excess numbers are incident along A, then are scattered to C, then scattered by the background. The number of electrons incident along B is less than the equilibrium number. The deficit is scattered to D, then scattered by the background. The excess and deficit diffuse together and recombine along the arcs.

defect must matter. Electromigration is a field beset by controversy; but all modern participants admit to the existence of strong spatial variations at a lattice defect. As we cannot here describe electromigration theory, we cite two recent items to lead to the citation trail [12].

Spatial variations, and the resulting electric field concentration, are also likely to be relevant for nonlinear effects, regardless of the exact microscopic mechanism leading to nonlinearity. This is suggested by inelastic point contact spectroscopy, which utilizes the high fields present at a very small area contact between two conductors [13]. Spatial variations can generate nonlinearities not only because the transport fields are nonuniform and spatially concentrated. Spatial variations are also associated with changes in the local carrier densities; after all, a spatially varying field must be generated by localized charges. Nonlinearity in bulk samples, or in mesoscopic samples, has not yet received very detailed attention; some discussion can be found in [14–18]. We return to the subject of nonlinearity in Section 5.

Spatial variations need not be mentioned or understood explicitly. Sufficiently sophisticated diagrammatic techniques can handle their effects without explicit allusion, as shown in parts of the electromigration literature. But simpler semiclassical techniques make it easier to become aware of spatial variations and their potential effects.

Spatial variations are not limited to dense degenerate Fermi gases. The densely populated systems, however, permit the spatial variations to become more striking because

a. Only in a dense electron gas can we pile up enough carriers to permit rapid spatial changes of the transport field.
b. A dense electron gas provides enough screening of Coulomb fields so that all point defects give a highly localized scattering potential, if we neglect the long-range elastic distortions they generate.

2. $R/(1-R)$

In the one-dimensional case, [1] found a resistance proportional to $R/(1-R)$, where R is the reflection probability. At that time, this was still a result considered applicable to a single barrier, and not yet understood to apply to a more complex entity, e.g., a disordered array of sequential barriers. Furthermore, the details of the analysis in [1] concentrate on a localized plane barrier in a three-dimensional medium. The strictly one-dimensional case was understood, but considered too playful to be worth presenting. The $R/(1-R)$ result had two obvious limiting forms; in neither case was the result really new. For $R \ll 1$ the result simply repeated the fact explained in every text: Resistance is proportional to scattering probability. In the opposite case, where $T = (1-R) \ll 1$, the result was also known. This is the case of the resistance of a tunneling barrier which permits little carrier penetration. In 1957 solid-state tunneling barriers, as reproducible and quantitatively characterizable entities, did not exist. Nevertheless, their theory had been discussed at least as early as 1933 [19]. Thus, the $R/(1-R)$ result could be considered to be a trivial interpolation of known answers. It was not, however, regarded that way [20].

Let us, here, make the $R/(1-R)$ result plausible. Assume that a current is flowing and is *maintained* as a barrier is introduced. Let us, furthermore, consider a diffusive problem in which we have noninteracting carriers, and later invoke the Einstein relation to make the transition to electrical behavior. If we have *thermal equilibrium* and insert the barrier, we generate only very localized disturbances at the barrier location. In the presence of transport, we can think of an excess number of carriers, in addition to that present in equilibrium, arriving from the left. Similarly, we can assume a deficit arriving from the right. Consider the excess carriers incident on the barrier from the left. A fraction R is reflected, adding to the concentration on the left. But this serves to diminish the current, and the current is supposedly maintained. Therefore, an identical additional incident influx must be brought in from the left, thus doubling the concentration change, making it $2R$ (in our somewhat arbitrary units). The extra incident flux, R, is also reflected in part. This adds further to the concentration on the left, first through the reflected stream, then through the need to send in extra carriers for the resupply. This gives us an additional density contribution $R \cdot (2R) = 2R^2$. But some of this resupply is also scattered, leading to a further addition of $2R^3$. Thus we see a net density change on the left-hand side of the obstacle, after summing over all orders of reflection, given by

$$2(R + R^2 + R^3 + \cdots) = 2R/(1-R). \tag{1}$$

This is, of course, clearly a result which assumes that the successive orders of Equation (1) have no coherent phase relationship; it is the probabilities that have been added. For subsequent use we give the exact one-dimensional result for the resistance, given in [7, 8, 11]:

$$R_{\mathrm{el}} = \frac{\pi\hbar}{e^2}\frac{R}{(1-R)}. \tag{2}$$

The nonlinearity in scattering probability displayed in Equation (1) is most pronounced in one dimension. In that case the reflected incident carriers *must* be resupplied and *must* pass through the barrier. In higher dimensions the scattered incident carriers can detour around the scatterer in question, and the nonlinearity will be less pronounced. These nonlinearities were mentioned in [1] and taken up in more detail in [9] and [11].

There is a separate source of nonlinearity in both scattering cross section and in scatterer density, which does not exist in the one-dimensional case and is rooted in the interaction between scattering centers [10]. We have already pointed to the existence of a detour current, taking the scattered incident current around the obstacle. Note that the existence of the "detour" current does not require a repulsive potential or backscattering. It exists for attractive potentials, and for scatterers with a high degree of small angle-scattering. After all, any scattering action will reduce the incident current flow, and if the current is maintained, this current, at least in part (and if we are not in one dimension), goes elsewhere. Thus, the space average of the current is not changed; the current through the obstacle is replaced, in part, by the detour current. If obstacles were uncorrelated in their positions, then one obstacle would not change the average current incident on another obstacle. Obstacles are, however, not uncorrelated; *in most cases, they are guaranteed not to overlap*. Thus, an obstacle is never exposed to the diminution of the current that occurs within the volume of another scatterer, but only to the detour current of the other obstacle. This situation is exactly the same as the one that leads to $4\pi P/3$, or the Lorentz correction, in dielectric theory. The reason a polarizable molecule does not see the space average electric field, but a different effective field, is that a molecule is guaranteed to be outside the other molecules. Unfortunately, this obvious physical explanation of the internal field, due to Bragg and Pippard [21], is not widely appreciated. A detailed discussion of it, and of the shortcomings of the ordinary textbook viewpoint, has been provided [22].

As an example, consider the case of impenetrable obstacles with a scattering cross section equal to, or close to, their geometrical cross section. For example, take cubes, and take them large compared to the Fermi wavelength, so that the scattering is classical. If we fill space with such blocks, the conductor must become impenetrable. Indeed, we know from percolation theory that in the case of a random placement of cubes on a lattice, the conductor becomes impenetrable at some filling factor less than unity. On the other hand, if we took the resistance to be proportional to both obstacle density and scattering cross section, the resistance would not show this required divergence. It is the nonlinearity we have discussed which permits the resistance to become infinite.

3. Reservoirs, interfaces, two probes, four probes

The method we have invoked has been developed into a systematic approach to the calculation of resistance, shown in **Figure 3.** Two reservoirs, maintained at different electrochemical potentials, act as sources of carriers. The reservoirs feed the carriers into sections of ideal conductor. The sample is characterized by its scattering action, describing the pattern of emerging carriers in terms of the incident carriers. The potential difference, or electrochemical potential difference, can then be calculated, either between the ideal conductors or between points deep inside the reservoirs. The method is, essentially, the method of Figure 2. We follow the incident carriers and calculate how they emerge. This results in the generation of space charges. We then let this charge be screened self-consistently, as for any other imbedded charge. The additional screening charge is not associated with any transport. The effect of the screening charge on the transmission behavior of the sample is a second-order or nonlinear effect. This method has been applied to a wide variety of situations, which are listed in **Table 1.** Citations may be found in [7] and [8]. Admittedly, some of the thrusts listed in Table 1 represent rather symbolic attempts, or existence theorems, indicating that *something* can be done in the desired direction.

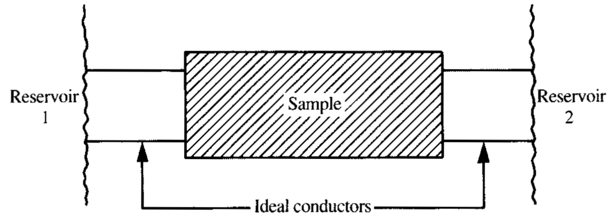

Figure 3

Standard geometry for calculation of resistance from the scattering matrix of the sample.

Table 1 Situations allowing resistance calculation from sample scattering specification.

Many-dimensional; with or without magnetic field
Many-body interactions in sample
Boltzmann statistics
Classical particle diffusion
Boson transmission and diffusion
Inelastic scattering in sample
Device configurations
Nonlinear conductance

The use of the ideal conductor, without scattering, as shown in Figure 3, is partly pedagogical. We can, instead, evaluate the potentials just inside the reservoirs, at points where the carriers coming from the sample have not yet suffered scattering within the reservoir. The *ideal conductor* is also a mild concession to experimental reality; it would be hard to attach measurement probes to a point *just inside* the reservoir. Finally, the ideal conductor serves a purpose unrelated to our subsequent discussion in this paper. The transmitted carriers emerging from a sample will, typically, not be distributed uniformly across the end of the sample. The ideal conductor permits a smoothing of the potential as we move away from the sample interface and toward the reservoir. More detailed discussions related to this sort of question can be found in [23].

The recent literature has characterized the arrangement in which potential differences between ideal conductors are measured as four-probe measurements, and the case where the potential differences are measured between reservoirs as two-probe measurements. This leaves an impression that this simple dichotomy covers all the real possibilities, and that is misleading. First of all, *reservoirs* are not exactly a typical laboratory household item. Real systems involve *circuits*, usually including electronics which is hard to characterize on a fundamental basis. In general, it is the exact way that current is fed into a sample that matters, the incident carrier distribution in real space as well as the distribution in momentum space. Connection to a reservoir permits a particularly simple evaluation of the incident distribution; it is the Fermi distribution from the reservoir in question. More generally, however, it is the kinetics in the leads to the sample that matters. If the preferred carrier distribution for conductivity in the leads (in space or in momentum) differs from that in the sample, there will be an interface resistance, as described in [7] and [8]. This interface resistance is distinguished in the recent theoretical literature (aside from a few papers cited in this paper) by its complete invisibility; the topic is not mentioned in passing. The incident carrier distribution will, in general, have to be found self-consistently. It is not determined immediately and directly by the kinetics in the leads. After all, the carriers incident on the sample include those that have left the sample, have been scattered within the leads, and have then returned to the sample. This point is now well understood in connection with quantum-mechanically coherent behavior; the paths in a sample, or between two probes, can include intervening portions outside of the sample [24, 25]. It is, however, a point which is also manifested in relatively incoherent systems. The need to consider incident velocity distributions, determined in a self-consistent way, was discussed in Section 8 of [1] in 1957.

A brief discussion about measurement probes may also be in order. We can measure electrochemical potential by equilibrating with the electron distribution in the sample, through a lead drawing no net current [25–27]. In principle, we can envision a lead which couples very loosely to the sample's electrons and does not appreciably disturb the motion in the sample.[1] We can also, in principle, envision a measurement probe which couples equally to all directions of motion in the sample. The loose coupling may be obtainable via a scanning tunneling microscope probe. It is less obvious how to couple uniformly to the various possible directions of motion; an STM probe heavily weights electrons moving perpendicularly to the surface; carrier energy due to motion transverse to the interface is essentially wasted in the tunneling process [28, 29]. The prevailing experimental method, using leads made by electron-beam contamination lithography, does not provide loose coupling, nor equal weighting of the velocity classes. Should one call the resulting measurement an electrochemical potential related to the original structure without the measurement leads? This is a matter of taste; generally, it is the practice in physics to search for minimally perturbative measurement methods. In recent years there has been growing awareness that real measurement leads, with a geometry defined by electron-beam resist exposure, contribute to the overall kinetics of the system being measured. A wire with a stub attached to it is not the same as the wire without the stub. This is apparent from the transmission line analogy. It was understood in [30], and is much more explicit in [31]. But stubs are not measurement leads; they are only lateral extensions of the conductor. A clearer understanding of the phase-breaking role of the measurement apparatus at the end of the measurement lead came with Büttiker's work in [26]. Note that if we are concerned with measurements on an ideal conductor, inserted between a sample and a reservoir, then we can (in principle, probably not in reality) use a whole array of loosely coupled identical probes to achieve a variety of measurements. We can, for example, use the equivalent of a phased array to measure the carriers present in a particular "channel," moving in a specified direction (toward reservoir or toward sample). Alternatively, we can use a random array to eliminate the effect of interference oscillations, and thereby measure an average carrier population in the ideal conductor.

In fact, the very definition of electrochemical potential, away from thermal equilibrium, permits some exercise of taste and choice. The definition which seems to have received some acceptance in this field: The electrochemical potential, or quasi-Fermi level, is that which in the equilibrium system, without transport, would take us to the same average electron occupation. In other words, it measures the electron density. This was invoked implicitly in [32] and much more explicitly by Engquist and Anderson [33]; for further details see [34].

The electrochemical potential is not the only measurable potential. The voltage, or electrostatic potential, is also measurable. To measure the potential difference between two points on the surface of a conductor we can couple to each point capacitively, then drive the capacitive probes piezoelectrically to and from the sample. The resulting alternating current between the connected oscillating

[1] Private communication: Y. Imry, Department of Nuclear Physics, The Weizmann Institute of Science, Rehovot 76100, Israel.

capacitive probes measures the voltage difference. For simplicity of interpretation, the oscillatory period should be long compared to the RC relaxation time.

While an STM tip can be used for capacitance measurement, we cannot expect to get the highly localized coupling provided in tunneling experiments. In tunneling, the exponential dependence on gap thickness eliminates tunneling from all but the portion of the tip closest to the sample. Capacitance varies inversely with the separation, and the portions far from the tip will swamp the capacitance. But even in the case of tunneling measurements, it will be hard to bring *two* tips near each other.

It is now understood that two probes, attached to a sample with their separation short compared to the inelastic scattering length, tie into the sample's pattern of coherent multiple scattering in very different ways [24, 25, 35]. Thus, two adjacent probes can give very different values of electrochemical potential, and can give a potential difference which is counter to that expected from the direction of current flow. While the combination of coherent multiple scattering and the effect of the probes on the carrier behavior make this a much more likely effect, *uphill voltages* can occur even in inhomogeneous classical conductors. We are here referring to *voltage* measurements and to a conductor controlled by $i = \sigma(r)E(r)$, where σ has a pronounced spatial variation. In that case, a point near the surface can be connected by a conducting channel, or tube, to a region at some distance from it, and can be relatively decoupled from the conductor in its immediate neighborhood. This easily permits the appearance of *uphill voltages*. Uphill *electrochemical potentials* can, of course, be made to appear by the same pattern of inhomogeneous conductivity. We return to the subject of uphill voltages in the next section.

We have already made the point that reservoirs are an unlikely laboratory object. There is, however, one case in which the result which gives the resistance between reservoirs seems applicable. Gimzewski and Möller [36] have measured the resistance between two metallic surfaces bridged by a single atom, and Lang [37] has provided a more detailed theory. The situation is illustrated in **Figure 4.** P and Q are points just inside the metal; the potential difference between them is given by Equation (2), or more specifically by its many-dimensional generalization [7, 38]. At P, for example, the carriers which originated from the other side (METAL 2) and were transmitted by the bridging atom have not suffered an inelastic collision. The electron population at P, therefore, is determined in part by μ_2. Deep inside METAL 1, say, on the dashed circle, it is determined by μ_1, and is unrelated to the events in METAL 2. Between P and the dashed circle on side 1, and between Q and the dashed circle on side 2, there is a spreading resistance. This is the *Sharvin* resistance, discussed in detail in [3, 8, 13] and [39]. It is a nonclassical form of spreading resistance because the dimensions at the "bridge" are small compared to a mean free path. As explained in [3] and [8], this additional Sharvin resistance gives us a potential drop between M and N which is characteristic of reservoirs. The resulting expressions for conductance exhibit only a sum over suitably weighted transmission probabilities, without complicating denominators. Now if the bridging atom were attached to relatively *narrow* leads, the equivalent of the dashed circles through M and N would be hard to locate. If we went too far into the narrow leads we would add additional series resistance. The specific geometry of Figure 4, however, saves us from that. The three-dimensional spreading resistance from M to ∞, $\sim\rho/r$ (where r is the radius of the dashed hemisphere), is much less than the Sharvin resistance, $\sim\rho\ell/a^2$, where ℓ is the mean free path and a the dimension of the bonding atom. Thus, the drop beyond the dashed hemispheres is negligible.

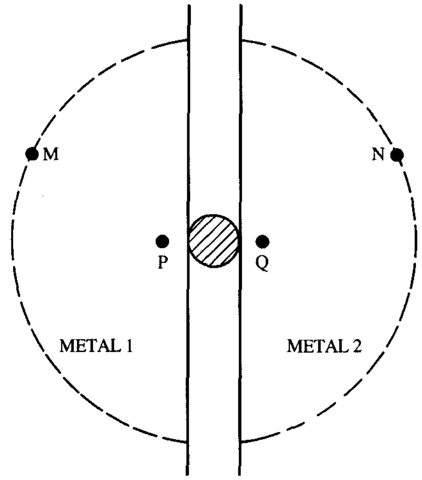

Figure 4

An atom bridging two large metallic contacts. M and N are far enough into the metallic contacts so that the electrochemical potential of the other contact has little influence on the carrier distribution. The dashed circles can be presumed to be a mean free path away from the bridging atom, but the exact choice for this radius is not important.

4. Spatial voltage variation

In an inhomogeneous medium there are places where transport is easy, and others where it is hard. That is why residual resistivity dipoles come into existence. That is why a tunneling barrier has a voltage drop right across the barrier, in the presence of current. If, instead of considering a particular sample, we average over an ensemble of all possible spatial redistributions of scattering centers, the

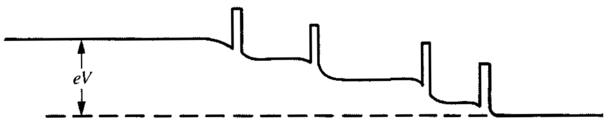

Figure 5

Random obstacle chain with applied voltage.

resulting voltage distribution will be uniform for a rectangular parallelopiped with current introduced at opposing faces. At an early stage in the development of solid-state physics, the convenience of treating an ensemble average was recognized. All too often, the fact that it was a mathematical device, unrelated to the behavior of a specific sample, was forgotten.

Reference [35] tells us:

> "... the current density depends only on the voltages at the leads, and not on the precise electric field configuration. We can therefore, without loss of generality, write the current density in terms of any potential which has the correct values at the leads. It is convenient to choose the classical potential $V^{cl}(r)$ such that $\nabla^2 V^{cl}(r) = 0 \ldots$."

We have intentionally selected a statement from one of the more perceptive and significant recent papers. The quotation represents a real improvement over the earlier common presumption that the voltage *actually is* $V^{cl}(r)$. The notation V^{cl} may represent an unwarranted implication that the only reason for spatial field variations comes from quantum-mechanical interferences in multiple scattering. We still, however, find the voltage distribution treated as a matter that does not *have to be understood*; it is not a matter of interest in its own right. Reference [35] is correct within its assumptions; the precise voltage distribution does not matter in the case of noninteracting electrons. Crudely speaking, it is the total driving force including both concentration gradients and electric field that matters. We introduce the qualifier, "crudely speaking," because we are dealing with localized scatterers and rapid microscopic variations. We are far from the regime where the local transport coefficients ($i = \sigma E$, $i = -D\nabla n$) can be invoked. In the case of noninteracting carriers an incorrect field variation is compensated by carrier concentration gradients, together providing the necessary spatially varying driving force and thus ensuring the continuity of current. If, on the other hand, we start from a theory allowing for the Coulomb interaction between electrons, then the large space charges associated with the carrier pileup needed for diffusive currents become impossible. In that case a much smaller carrier pileup will produce the required inhomogeneous field distribution needed to maintain the current continuity. Allowing for Coulomb interaction between carriers will automatically give us the correct internal field distribution.

Consider a one-dimensional array of obstacles presumed to be disordered (**Figure 5**). A voltage is applied, and Figure 5 shows the spatial variation of electronic potential energy due to the applied voltage added to the scattering potential. Reference [17] discusses the nonlinear effects implicit in Figure 5; here we stress its more elementary implications, assuming that the separation between obstacles is large compared to both the Fermi-level wavelength and the screening length. Let us follow the approach stressed in Section 3, and elsewhere in this author's work: We assume current flow incident on the sample and calculate the resulting diffusive carrier pileup, assuming noninteracting carriers. Then we let voltages be established as a result of the self-consistent screening caused by the diffusive carrier concentration.

Extra carriers incident on the sample, say from the left, will give rise to a complex pattern of multiply scattered waves. Between two adjacent barriers, however, we will have some combination of right-moving waves and left-moving waves. Thus, ignoring oscillations related to the Fermi wavelength, we will have a fixed density between two adjacent barriers. As a result of Fabry–Perot resonances, the density need not vary monotonically as we move along the chain; there can be "uphill" density changes. Now consider the screening. Long wavelengths are screened most effectively; uniform charge densities extending over distances large compared to the screening length will be totally compensated by the screening charge. Thus, deviations from neutrality will be confined to the regions near the barriers where the fields and potential drops must occur. Note that the screening calculation is somewhat complex. It involves the density of states, and this is the density of states in the potential which includes the barriers. In the case of barriers which have a small transmission probability we can, at least approximately, think in terms of a local set of states between adjacent barriers. As pointed out by Büttiker [27], the local density of states is very nonuniform and peaks near the resonances. This leads to a complex adjustment, resembling contact potentials, which permit alignment of Fermi levels between adjacent conductors, *even in the absence of transport*. We have not carried out this screening calculation; it is conceivable that it can leave us with uphill voltages. Remember that electrochemical potentials and voltages can move apart only over short distances in the regions where charges accumulate. Over most of the space between barriers they must be identical. Thus, an uphill voltage implies an uphill change in the electrochemical potentials and a negative power dissipation *associated* with the barrier in question. That would be surprising, but it cannot be ruled out without further calculation. In the case of correlated dissipation caused by adjacent barriers, it is only the net dissipation which clearly has to be positive. Thermoelectric effects which cool a junction between dissimilar materials are well known and, as already stated, our adjacent regions resemble dissimilar materials.[2] Uphill voltages have been seen experimentally [40], but not in measurements on a one-dimensional sample.

How does Figure 5 translate into two or three dimensions? We do not know; even though the answer may, in part, be implicit in some of the diagrammatic portions of the electromigration literature. Much of what has been said in connection with Figure 5 may carry over. At the simplest level, we might want to argue that carriers arriving at a given scattering site from different directions, i.e., from different preceding scatterer locations, have a very different history which makes them effectively incoherent, and that therefore the semiclassical arguments of [1] are directly applicable. The universal fluctuation literature, however, has taught us to be cautious; multiple scattering leads to subtle correlations. Incidentally, the universal fluctuation discussions also have a shortcoming which is not widely advertised: They assume point-defect scattering, whereas real mesoscopic samples are likely to have significant resistance contributions from grain boundaries, dislocations, surface ridges, etc. Even point defects cause long-range elastic distortions. These qualifications must be added to others [3, 5, 6, 41]. Universal fluctuations are only manifested between points separated by about one inelastic scattering length. The sample's resistance cannot be so small as to represent ballistic transmission, or so large as to represent localization. The conclusion: *Universal* is a somewhat exaggerated characterization. That, of course, is not intended to diminish the central insight involved, pointing to the importance of coherent multiple scattering effects.

The three-dimensional case, of course, just like the one-dimensional case, includes interference effects between the wave incident on an obstacle and the resulting scattered waves. This effect does not require coherence between scattering by successive obstacles. The effect is well known in electromigration theory [42], but the associated local fields are oscillatory and do not contribute to the overall voltage [32]. An additional effect, likely to be present, consists of higher-order multipole fields supplementing the residual resistivity dipole. For example, if either the incident velocity distribution or the dependence of scattering probability on angle is more complex than assumed in [1], these can be present. Once again, they do not contribute to the overall voltage drop.

[2] Thermoelectric cooling typically involves carriers which, at the junction, have to absorb energy to continue on their path at a higher energy in the new material. An n^+–n semiconductor junction is a typical example. These are inelastic effects, which we have ignored in our discussion. In fact, within our treatment, the dissipation *associated* with a barrier does not occur at the barrier. The heat is delivered to the reservoirs.

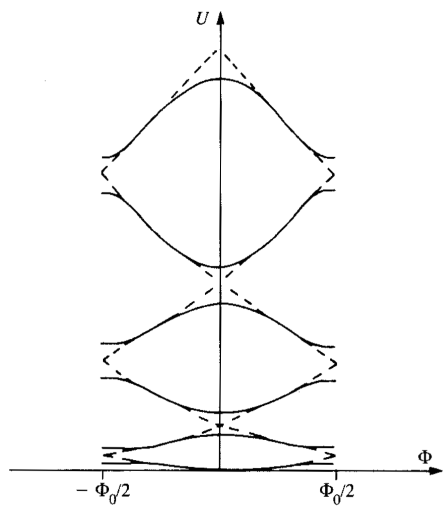

Figure 6

One-electron energies of the ring as a function of flux. The dashed lines are free electrons without elastic scattering. Φ_0 is the single electron flux quantum. If the horizontal axis measures k, instead of flux, the zone boundaries occur at $\pm\pi/L$, where L is the loop circumference.

5. Nonlinearity in closed loops

We have already stressed the role of nonlinearity in small samples. We supplement this, here, with a discussion of nonlinearity in closed loops *without leads*. First, a general comment about closed loops. Figure 3 shows the typical situation envisioned in most of this paper. The size of the resistance is determined by the scattering properties of the sample, but if this is entirely elastic scattering, then the dissipation occurs in the reservoirs. The reservoirs are "black bodies," which subject carriers to inelastic events and ensure loss of phase memory before the carriers coming from the sample can again return to the sample. What happens if we eliminate the reservoirs and tie the ends of the sample together to make a loop? Then we can only produce a current by a time-dependent flux through the sample. If the scattering in the sample is purely elastic, we have a Hamiltonian system. It can store energy; it cannot dissipate it. The system was analyzed in [43] and the analysis extended in [44–46].

Reference [43] invoked an equivalence between the electrons in the loop and electrons in a periodic potential, in which the variation in one period is that found in traversing the loop. **Figure 6,** adapted from [43], shows the energy of the electrons in the loop as a periodic function of the applied flux. The lattice momentum, k, in the equivalent periodic potential, is proportional to this flux. In the presence of a time-dependent flux, the electrons move through the band structure according to Bloch's theorem, $dk/dt = -eE/\hbar$. If we have a time-independent nonvanishing flux, we can expect a current from each occupied "band," $i \sim dU/dk \sim dU/d\Phi$. The simple band structure shown in Figure 6 can only be expected in the one-dimensional case. In the case of a field-effect-transistor structure, the one-dimensional behavior can be realized by choosing the transverse dimensions small enough so that only the lowest transverse state is occupied. In the metallic case, where we have a cross section with many atoms, and many transverse states are occupied, we can expect more complex three-dimensional "band" structures with many internal minima and maxima. In the one-dimensional case we can expect alternating signs for the contributions from successive bands, with some tendency for higher-lying bands to yield larger currents.

If the flux is increased from an initially vanishing value, and then kept fixed at a value which does not correspond to a zone boundary or zone center in Figure 6, we can expect a nonvanishing, or persistent, current. As explained in [46], even in the presence of some thermal relaxation, a reduced persistent current can be expected. The magnitude of this persistent current is hard to estimate, particularly in the many-dimensional case, where no detailed understanding of the equivalent of Figure 6 exists. The persistent current will, in any case, be less than that for a free-electron gas accelerated by application of a flux $\frac{1}{2}\Phi_0$, where Φ_0 is the Aharonov–Bohm flux quantum. Consider a one-dimensional loop with elastic scattering weak enough so that there is an appreciable probability for elastic transmission in one traversal around the loop. This means that the resistance of the loop, if opened up and measured as in Figure 3, would, according to Equation (2), yield a resistance $R_{\mathrm{el}} \sim \pi\hbar/e^2$. In that case, when the applied flux Φ is equal to $\frac{1}{4}\Phi_0$, we will be halfway between zone center and zone edge. The persistent current will be near its maximum value, and will be depressed only modestly below the free electron value for the same number of carriers and the same geometry.

There have been a number of suggestions[3] [47] that the gap and bandwidth in Figure 6 vary in a relatively random way as we go up in energy, from band to band. This is, of course, not true for a very simple scattering potential, e.g., a single scattering barrier with a smooth potential. At the other extreme, consider a large number of scattering centers within the loop, but weak enough to be treated as a perturbation on the free-electron case. In that case, the energy gaps depend on Fourier elements of the potential [48]

$$\int_0^L (e^{-2ikx}) \sum_i V(x-x_i)\, dx = \left(\sum_i e^{-2ikx_i}\right)\left[\int_0^L V(x)e^{-2ikx}\, dx\right]. \quad (3)$$

[3] Private communication: Ping Ao, Department of Physics, University of Illinois at Urbana–Champaign, 110 W. Green St., Urbana, IL 61801.

We see that this sum (resembling the spectral analysis of shot noise) has a smooth dependence on k through the final right-hand-side factor. The first r.h.s. factor can give a somewhat random variation between adjacent band gaps. The relative random variation between gaps, however, goes down as $1/\sqrt{N}$, where N is the number of scattering centers.

We are not, here, trying to answer the disorder question in detail, but it can be seen that even the one-dimensional case has a degree of complexity which makes it hard to evaluate the persistent current. The current is likely to be small. The method of measurement is likely to reflect losses into the ring, thereby reducing the persistent carrier magnitude. A large planar array of loops will give us loops which are unlikely to be exactly identical. Even the sign of the current will vary from loop to loop.

In view of these difficulties we propose a more indirect test for Figure 6. Let us apply a bias flux which puts us about halfway between zone center and zone edge, then apply an additional small sinusoidal oscillatory flux. At the zone center, before flux application, the energy varies symmetrically with flux,

$$U = a\phi^2 + b\phi^4. \quad (4)$$

With an oscillatory flux, $\delta\phi$, we find

$$\delta i \sim \frac{dU}{dk} \sim \frac{dU}{d\phi} = 2a\delta\phi + 4b(\delta\phi)^3. \quad (5)$$

Consider, alternatively, a bias flux $\phi_b \neq 0$, and an additional oscillatory flux $\delta\phi$. In that case

$$\delta i \sim \delta\left(\frac{dU}{dk}\right) \sim \delta\left(\frac{dU}{d\phi}\right) = U''(\phi_b)\delta\phi + \tfrac{1}{2}U'''(\phi_b)(\delta\phi)^2 + \cdots. \quad (6)$$

We can see that in Equation (5) the lowest harmonic is the third harmonic, whereas in Equation (6) there will be a second harmonic. This is a simple qualitative distinction, and, unlike the attempt to measure persistent currents, can utilize phase-locked detection techniques. A large array once again will give us contributions with random sign. Thus, the use of N loops in an array will only cause the detected signal to grow as $\sqrt{N}$. Note also that the *phase* of the second harmonic, in the reradiated field, can be used to distinguish between a nonlinear resistance and a nonlinear reactance. As we have stressed, the loop (at sufficiently low temperatures) acts as a reactance.

The distinction between Equations (5) and (6) utilizes only a symmetry argument and does not require the simple one-dimensional band structure of Figure 6. Our proposed experiment does not require a field-effect structure; it should work in the metallic case. The coil generating the oscillatory flux must, of course, generate no second harmonic. In the presence of a bias flux, dimensional changes in the coil will cause second-harmonic generation. If needed, the wave form driving the coil can be adjusted to compensate for this. For an alternative possible experimental approach to persistent current detection, see [45], invoking the measurement of small signal power absorption as a function of bias flux.

6. Conclusion

The ability to fabricate small samples has led to the study of a variety of phenomena exhibiting quantum interference between alternative carrier paths. This includes the Aharonov–Bohm effect and "universal" fluctuations. These studies have also brought attention to an approach to conductance which is based upon the overall scattering behavior of the sample. The literature in recent years has appreciated the distinction between an ensemble and a specific sample, as well as the perturbative role of additional measurement leads. The work in this field, however, continues to minimize questions about the explicit variation of currents and fields within the conductor, and the correlation of these variations with the positions of localized scatterers. Physics tends to be channeled by slogans and fashions, providing attention to an evolving variety of popular questions. Decades ago, these included the Kondo problem, deviations from Matthiessen's rule, and Fermi surface studies. Most recently, *universal fluctuations* has joined this list. This preoccupation with a limited range of topics draws attention from other commonsense questions, which one might expect to see discussed in every elementary text.

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