Traversal time for tunneling

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Tunneling of carriers through a barrier is characterized not only by a transmission and reflection probability, but also by the time it takes a carrier to traverse the barrier. Recent work which discusses the traversal time is summarized, and its relevance is highlighted by discussing several tunneling phenomena.

Introduction and alternative viewpoint

Every quantum mechanics textbook discusses tunneling through a barrier and evaluates the transmission probability for incident particles. The actual time spent traversing the barrier has been a subject of discussion in the journals for over half a century, but with little clarity and unanimity. In Refs. [1–3], we have discussed our own approach and contrasted it with other work. We will not attempt to summarize the alternative answers again, but only stress the relationship of our result to discussions by Stevens [4] and by Jonson [5]. In this note, we first give an alternative approach to our results, also discussed in Ref. [6]. Subsequently, we cite some illustrative numerical examples.

Our original approach analyzed the transmission through the potential of interest, supplemented by a small oscillatory perturbation [1]. At low modulation frequencies, the transmission consists of the transmission as calculated for a time-independent potential but adjusted to the actual potential at each instant. As the modulation frequency is increased, serious departures from this behavior occur, and the corresponding oscillation period is taken as a measure of the time over which the tunneling particle interacts with the barrier.

Alternatively, we now consider a time-independent potential, and assume that we have a modulated incident wave

$$\psi = \exp[ikx - iEt/\hbar] + \exp[i(k + \Delta k)x - i(E + \Delta E)t/\hbar]$$
$$= 2\exp\left[i\left(k + \frac{1}{2}\Delta k\right)x - i\left(E + \frac{1}{2}\Delta E\right)t/\hbar\right]$$
$$\times \cos\left(\frac{1}{2}\Delta kx - \frac{1}{2}\Delta Et/\hbar\right) \tag{1}$$

consisting of two interfering plane waves. (If the region of incidence is not a region of constant potential, but one of slowly varying potential, we can still write a decomposition of the above form, based on the WKB approximation.) Now, if ΔE is small, the incident wave is modulated very slowly, and we can expect the emerging transmitted wave to follow the incident wave faithfully. Let $t(E)$ denote the complex ratio of transmitted wave to incident wave, at energy E. The transmitted wave is

$$\psi = t(E)\exp[ikx - iEt/\hbar]$$
$$+ t(E + \Delta E)\exp[i(k + \Delta k)x - i(E + \Delta E)t/\hbar]. \tag{2}$$

As long as $t(E)$ and $t(E + \Delta E)$ are close to each other this is simply ψ, as given in Equation (1), multiplied by $t(E)$. In that case, the terms in Equation (2) interfere constructively at the same time as the terms in Equation (1) and similarly the destructive interferences coincide. The immediate response in the outgoing wave to the variation in the incoming wave can be interpreted as a traversal time short compared to the period of oscillation. We have, in our earlier papers, criticized approaches which identify a peak in an outgoing wave with a peak in the incoming wave; physics has no law about a peak turning into a peak. In our present argument it is not just a peak, but the time variation of the total modulation waveform, which is invoked. As we increase ΔE and shorten the modulation period, $t(E)$ and $t(E + \Delta E)$ will increasingly differ. Once they differ appreciably, the transmitted wave will no longer reproduce the incident wave, and we can assume that a time delay, or dispersion in transit time, has become comparable to the modulation period.

In the case of transmission *over* a barrier the energy dependence of $t(E)$ comes primarily from the energy dependence of the phase. The reasoning just invoked, in the WKB approximation, leads to a transit time $\tau = \int dx/v$, where $v(x) = \hbar^{-1}dE/dk$. Within the WKB approximation, at energies above the barrier peak, $|t(E)| = |t(E + \Delta E)|$. Thus, in this range, it is *only* the phase that matters. If we go beyond the WKB approximation, then $|t(E)|$ will also, in general, as a result of reflections [7], be less than unity, and will have its own energy dependence. That is consistent, however, with the fact that in the presence of reflections we can expect an effect of these reflections on the traversal time, and can no longer expect it to be $\int dx/v$.

We stress that it is the energy sensitivity of $t(E)$ that matters. For tunneling under a barrier with a small transmission probability, it is the exponential decay that matters; questions of phase are secondary. A naive extrapolation, to the tunneling problem, of the phase considerations which apply to ordinary wave packets in the classically allowed range of motion, is at the root of some of the earlier approaches. For $V > E$ let us take

$$t = \exp\left(-\int |k|\,dx\right) = \exp(-\alpha), \tag{3}$$

with $\hbar^2k^2 = 2m(E - V)$. This expression applies within the WKB approximation. It is also a good approximation in the case of a rectangular barrier. In that case, however, the energy-dependent factors arising from matching at the potential discontinuities have been neglected. For barriers with small transmission probability, that will account for secondary corrections, independent of barrier length. We can then ask how large ΔE must be to yield $\Delta E\,|d\alpha/dE| \approx 1$. Let x_1 and x_2 be the turning points. Then

$$\begin{aligned}\frac{d\alpha}{dE} &= \frac{d}{dE}\int_{x_1}^{x_2} |k|\,dx\\ &= \frac{dx_2}{dE}\,|k(x_2)| - \frac{dx_1}{dE}\,|k(x_1)| + \int_{x_1}^{x_2} dx(d|k|/dE).\end{aligned} \tag{4}$$

In the WKB approximation $k(x_2) = k(x_1) = 0$. For a rectangular barrier $dx_2/dE = dx_1/dE = 0$. Thus, in either of these two cases

$$\Delta E \sim (|d\alpha/dE|)^{-1} = \int dx\,\frac{m}{\hbar^2|k|}, \tag{5}$$

where we have used $dE/d|k| = -\hbar^2|k|/m$. Now the traversal time is related to the modulation period, $\tau \sim \hbar/\Delta E$, yielding

$$\tau = \int dx\,\frac{m}{\hbar|k|} \tag{6}$$

in accordance with Refs. [1–6].

We have already mentioned the relationship to the work of Stevens and of Jonson, and have cited other early related results in Refs. [1–3]. Here we will, briefly, allude to more recent work. Pollak and Miller [8] state, "In quantal systems, the time is complex; the real part gives the actual duration of the collision, and the imaginary part is significant for systems with more than one degree of freedom." Their imaginary part is our traversal time. We quote this statement without any attempt to interpret it. Subsequently Pollak [9] pointed out that Equation (6) is the time which, in tunneling under a multi-dimensional saddle point, determines whether the transverse degrees of freedom can adjust adiabatically to the progress of the tunneling. Pollak and Miller [8, 9] are correct, of course, in stressing the relevance to systems with more than one degree of freedom. The traversal time is relevant only if there is some sensitivity to time in the problem, either through the adjustment to the tunneling process by other degrees of freedom, or through an externally impressed time variation as in Ref. [1].

A number of other recent and still unpublished contributions relate to our traversal time. Bruinsma and Bak [10] have analyzed macroscopic quantum tunneling from a viewpoint in which the reservoir oscillators coupled to the tunneling particle are treated in a way very close to that of Ref. [1]. Furthermore, the temperature at which they find a crossover from barrier penetration by tunneling to barrier crossing by thermal activation is given by $kT = \hbar/\tau$, where τ is our traversal time. Schmid [11] has approached macroscopic quantum tunneling through a many-dimensional WKB approximation. In this connection he has considered a tunneling particle damped by being tied to a linear chain of coupled masses (or violin string). He finds that during barrier penetration, by tunneling, the disturbance propagates along the string for a distance determined by the traversal time. Lane [12] has pointed to the relevance of the traversal time in fusion catalyzed by muons. During the tunneling event required in fusion, how much energy is picked up by the muon? This, in turn, relates to its escape and its subsequent availability for further catalysis. Guéret et al. [13] study the effect of a transverse magnetic field on the tunneling current through thick and low semiconductor barriers. The Lorentz force due to the applied field lengthens the tunneling path of the carriers through the barrier. This effect is proportional to the square of the traversal time.

Estimates of the traversal time

In this section we estimate the traversal time for some tunneling phenomena in solid state physics. Consider the *field-emission* of electrons out of a metal (Hartstein et al. [14], Hübner [15], Binnig et al. [16], and Jonson [5]). In that case we have a triangular barrier described by a potential $V = 0$ for $x < 0$ and $V = E_F + W - eFx$ for $x > 0$. E_F is the Fermi energy and W is the work function. Electrons at the Fermi energy have a probability [17]

$T \propto \exp[-(4/3)\sqrt{2m}W^{3/2}/\hbar eF]$

for tunneling out of the metal. Equation (6) yields a traversal time $\tau = (2mW)^{1/2}/eF$. With $W = 3.6$ eV and $F = 10^6$ V/cm, parameters taken from Hartstein et al. [14], we find $\tau \cong 7 \cdot 10^{-14}$ s. With decreasing field the traversal time increases in proportion to F^{-1}, but the tunneling probability decreases exponentially, effectively setting a lower limit to the observation of field-emission. Thus, it may be difficult to achieve longer traversal times. Hübner [15] applies an oscillating electric field, $F \cos \omega t$, with frequencies up to 18 GHz. He finds that the result can still be explained by replacing the static field, F, in the Fowler-Nordheim formula by the time-dependent field, a procedure valid for frequencies small compared to $1/\tau$. Hübner correctly concludes that the traversal time must be shorter than 2 ps in his experiment, in agreement with the rough estimate given above.

Another important tunneling mechanism is *Zener* tunneling. Zener [18] found that electrons tunnel through a small band gap ΔE with a probability

$T \propto \exp[-(ma/2\hbar^2)(\Delta E^2/eF)]$.

(See also Ref. [19].) Here a is the lattice constant and m is the free electron mass and F the field which tilts the band structure. Zener derived this result by calculating the wave vector in the lowest band gap, where $k = (\pi/a) + i\kappa$, and κ is given by

$$\kappa = \frac{\pi}{a}\frac{1}{E_b}[(\Delta E/2)^2 - (eFx)^2]^{1/2}. \tag{7}$$

$E_b = \hbar^2\pi^2/2ma^2$ is the energy a free electron would have at the zone boundary. Equation (7) is valid for nearly free electrons, which, near the gap, have an effective mass $m = (\Delta E/4E_b)m$. By analogy to the case of the ordinary barrier we can expect that the effective velocity of a Zener tunneling electron is given by $|v| = \hbar\kappa/m$, yielding a traversal time $\tau = \int dx |v|^{-1} = (ma/\hbar)(\Delta E/4eF)$. For $\Delta E = 0.75$ eV, $a = 5$ Å, and $F = 10^6$ V/cm, the tunneling probability turns out to be $T \cong 4.5 \cdot 10^{-5}$. Electrons take a time $(\hbar/eF)\int dk = 2\pi\hbar/eFa$ to traverse the Brillouin zone. In the absence of Zener tunneling the electrons execute Bloch oscillations with frequency $\nu_B = eFa/2\pi\hbar$. During each cycle they tunnel into the higher band with probability T. Thus the conduction band is depleted at a rate $r = \nu_B T \cong 6 \cdot 10^8$ Hz. The traversal time for these parameters is $\tau \cong 10^{-14}$ s. If we reduce the field by a factor of two, the traversal time is twice as long, but the Zener tunneling rate falls to $r \cong 1.3 \cdot 10^4$ Hz. (Note that our reasoning has invoked the Bloch oscillation frequency as a convenient device to characterize the frequency with which electrons approach the band edge. The particular conditions, however, needed for observation of actual Bloch oscillations, i.e., low scattering rates, are not required here.)

Josephson junction circuits are our third example. The junction has an energy $E = (C/2)V^2 + W(\theta)$, where C is the capacitance of the junction, $V = (\hbar/2e)d\theta/dt$ is the voltage, and θ is the phase-difference across the junction. The potential energy has the form of a tilted sinusoid given by

$W(\theta) = (\hbar I_m/2e)[(1 - \cos\theta) - (I/I_m)\theta]$,

where I_m is the maximum Josephson current and I the external current. The superconducting states $\theta_s = \arcsin(I/I_m)$ correspond to points of fixed phase at a local minimum of the tilted sinusoid. These are metastable states; the circuit can tunnel [20, 21] from this superconducting state to the voltage state, $V \neq 0$. In the metastable states the energy of the junction is $E = W(\theta_s) + \hbar\omega_0/2$. Here $W(\theta_s)$ is the energy at the bottom of a valley and $\hbar\omega_0/2$ is the zero-point energy,

$\omega_0 = \omega_p(1 - (I/I_m)^2)^{1/4}$,

where $\omega_p = (2eI_m/\hbar C)^{1/2}$ is the equilibrium plasma frequency. The traversal time, as determined by Equation (1), is given by

$\tau = \omega_p^{-1}\int d\theta/[2(2e/\hbar I_m)(W(\theta) - E)]^{1/2}$.

For the Josephson junction discussed in Ref. [22], with $I_m = 0.2$ mA, $I = 0.985\, I_m$, $C = 10^{-12}$ F, corresponding to a zero-point energy of $\hbar\omega_0/2 = 1.07 \cdot 10^{-4}$ eV and a barrier height of $1.4 \cdot 10^{-3}$ eV, we obtain $\omega_0 = 0.32 \cdot 10^{12}$ Hz. Numerical evaluation of the integral yields $\tau = 1.3 \cdot 10^{-11}$ Hz, almost three orders of magnitude larger than the typical traversal time in a field-emission or Zener tunneling experiment! The traversal time can, in turn, be used to calculate a reduction in tunneling rate, due to friction, as sketched in Ref. [1].

We would also like to point to the relationship between traversal time and the *single-electron tunneling* oscillations discussed by Averin and Likharev [23] and by Ben-Jacob and Gefen [24]. They point out that a tunneling event, in a structure with a small capacitance, can only occur if the electrostatic energy after tunneling is no larger than that before tunneling. If the tunneling structure is supplied by a current, i, through a resistor which delivers charge continuously, rather than in quantized amounts, then the voltage across the tunneling structure oscillates with frequency i/e. Observation of this effect will require a very small capacitance. Note, however, that only that part of the capacitance matters which can supply charge to the tunneling event within the tunneling time. Thus, if we use an STM configuration, charge transfer from parts of the structure where the electromagnetic wave propagation time exceeds the traversal time is irrelevant.

References

1. M. Büttiker and R. Landauer, "Traversal Time for Tunneling," *Phys. Rev. Lett.* **49**, 1739–1742 (1982).

2. M. Büttiker, "Larmor Precession and the Traversal Time for Tunneling," *Phys. Rev. B* **27,** 6178–6188 (1983).
3. M. Büttiker and R. Landauer, "Traversal Time for Tunneling," *Festkörperprobleme* **25,** P. Grosse, Ed., Vieweg, Braunschweig, Federal Republic of Germany, 1985, pp. 711–717.
4. K. W. H. Stevens, "A Note on Quantum Mechanical Tunnelling," *Eur. J. Phys.* **1,** 98–101 (1980); "A One-Dimensional Barrier and Time-Dependent Tunnelling," *J. Phys. C* **16,** 3649–3661 (1983).
5. M. Jonson, "The Dynamical Image Potential for Tunneling Electrons," *Solid State Commun.* **33,** 743–746 (1980).
6. M. Büttiker and R. Landauer, "Traversal Time for Tunneling," *Physica Scripta* **32,** 429–434 (1985).
7. R. Landauer, "Reflections in One-Dimensional Wave Mechanics," *Phys. Rev.* **82,** 80–83 (1951).
8. E. Pollak and W. H. Miller, "New Physical Interpretation for Time in Scattering Theory," *Phys. Rev. Lett.* **53,** 115–118 (1984).
9. E. Pollak, "The Significance of Imaginary Time in Quantal Reactive Scattering," *J. Chem. Phys.* **83,** 1111–1120 (1985).
10. R. Bruinsma and P. Bak, "Quantum Tunneling Dissipation, and Fluctuations," *Phys. Rev. Lett.* **56,** 420–423 (1986).
11. A. Schmid, "Quasiclassical Wave Function in Multidimensional Quantum Decay Problems," *Ann. Phys. (N.Y.)* **169** (1986, in press).
12. A. M. Lane, "The Physics of Muon-Catalyzed Fusion," lecture, IBM Thomas J. Watson Research Center, Yorktown Heights, NY, August 13, 1985.
13. P. Guéret, A. Baratoff, and E. Marclay, "Tunnelflugzeit aus Gleichstrommessung in Transversalem Magnetfeld," *Verhandlungen der Deutschen Physikalischen Gesellschaft*, Riehe 6, Band 21, 1446 (1986).
14. A. Hartstein, Z. A. Weinberg, and D. J. DiMaria, "Experimental Test of the Quantum-Mechanical Image-Force Theory," *Phys. Rev. B* **25,** 7174–7182 (1982).
15. H. Hübner, *Untersuchung der Feldemission bei Frequenzen Zwischen 12 und 18 GHz*, Ph.D. Thesis, Universität Karlsruhe, Federal Republic of Germany, 1982.
16. G. Binnig, N. Garcia, H. Rohrer, J. M. Soler, and F. Flores, "Electron-Metal-Surface Interaction Potential with Vacuum Tunneling: Observation of the Image Force," *Phys. Rev. B* **30,** 4816–4818 (1984).
17. R. H. Fowler and L. Nordheim, "Electron Emission in Intense Electric Fields," *Proc. Roy. Soc. (Lond.) A* **119,** 173–181 (1928).
18. C. Zener, "A Theory of the Electrical Breakdown of Solid Dielectrics," *Proc. Roy. Soc. (Lond.) A* **145,** 523–529 (1934).
19. C. B. Duke, *Tunneling in Solids*, Academic Press, Inc., New York, 1969, p. 44.
20. A. O. Caldeira and A. J. Leggett, "Quantum Tunneling in a Dissipative System," *Ann. Phys. (NY)* **149,** 374–456 (1983).
21. R. Voss and R. A. Webb, "Macroscopic Quantum Tunneling in 1-μm Nb Josephson Junctions," *Phys. Rev. Lett.* **47,** 265–268 (1981).
22. M. Büttiker, E. P. Harris, and R. Landauer, "Thermal Activation in Extremely Underdamped Josephson-Junction Circuits," *Phys. Rev. B* **28,** 1268–1275 (1983).
23. D. V. Averin and K. K. Likharev, "Possible Coherent Oscillations at Single-Electron Tunneling," *SQUID '85*, H. D. Hahlbohm and H. Lübbig, Eds., Walter de Gruyter, New York, 1985, pp. 197–202; "Coulomb Blockade of Single-Electron Tunneling, and Coherent Oscillations in Small Tunnel Junctions," *J. Low Temp. Phys.* **62,** 345–373 (1986).
24. E. Ben-Jacob and Y. Gefen, "New Quantum Oscillations in Current Driven Small Junctions," *Phys. Lett. A* **108,** 289–292 (1985).

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