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An Improved Regional Correlation Algorithm for Signature Verification Which Permits Small Speed Changes Between Handwriting Segments

If two nearly coincident accelerometers on a pen axis measure orthogonal acceleration components perpendicular to this axis, then the regional correlation algorithm *for signature verification divides these data into plausible segments, it compares each segment with a corresponding reference, and it combines the results into a global similarity index. The presently used intersegment distance permits certain natural data transformations: (1) translating a segment by small integer multiples of the sampling interval; (2) moving the pen with uniformly larger amplitude throughout a segment; and (3) rotating the pen about its axis between any two segments. We propose a new intersegment distance which permits further natural transformations: (4) translating a segment by a fraction of the sampling interval; and (5) writing at slightly different uniform speed within each segment. The new distance, like the old one, is the minimum of a certain function. We describe an algorithm which computes this minimum.*

1. Introduction

Recent attempts at signature verification have measured various concomitants of human handwriting [1–25]. Specifically, a group at the IBM Thomas J. Watson Research Center has proposed measurement of the related accelerations [8, 14]. They have developed a pen with imbedded accelerometers and have achieved high selectivity with the resulting data [12, 13, 18]. This author has provided a mathematical structure for such measurements and has deduced optimal layouts of imbedded accelerometers [15–17]. His work supports the idea that the acceleration of the pen point is the essential carrier of signature information; it offers an argument [16, Section 5] that full recovery of the point motion demands six accelerometers inside the pen. However, diagnostic information from fewer instruments has furnished good results through astute data analysis, and one statistical technique with remarkable success has been the regional correlation algorithm of the cited accelerometer research [12, 13, 18]. This paper suggests a device which may sharpen that technique.

Any recognition algorithm saves data from prior signatures and takes corresponding data from each new signature. It digests these into numerical indices and compares them with some threshold values. But proper matching of such data requires an allowance for certain normal variations. Thus, procedures for *nonlinear time-stretching* allow nonconstant distortions of the time axis, to superimpose signatures with irregular writing speeds [1, 11, 24]. However, techniques with such flexibility demand computations of some length. Hence the regional correlation algorithm takes acceleration values at equal time intervals, tries moderate shifts of natural data segments which align their sampling times with reference sampling instants, and finds the best "correlation" among these various alternatives. However, this algorithm admits no variations in writing *speed*, and its time-shifts include no fractions of the sampling interval. The proposed improvement incorporates time-shifts by arbitrary small amounts—and it allows a different, constant speed in each handwriting segment.

The author's previous model [16, Section 9] assumes two accelerometers, nearly coincident on the pen axis, measuring orthogonal components perpendicular to this axis. Such layouts record different numerical values if the same person, between two signatures, merely rotates the pen about its axis; but the regional correlation algorithm allows such rotations [12, 13, 18]. Also, it lets each handwriting segment have a different, constant amplitude. The proposed improvement retains these features, and it involves similar computations. Previous workers [3, 6, 7, 16], among them the author, have considered various analogs of covariance analysis to weight important features of signature data. This new technique simply obtains the best fit after permitting the natural transformations.

The algorithm uses the same global calculation, but it redefines the intersegment distance. Indeed, measurements from a segment and its reference form vectors z and z_0 in the complex inner-product space C^n, and the inner product defines a "correlation coefficient" γ_0 such that $1 - |\gamma_0|^2$ is the present segment distance. We interpret such vectors in a $2n$-dimensional *real* inner-product space, then introduce auxiliary reference vectors for *small* time-shifts or speed variations, and replace the old distance formula by a more general minimization problem. Here the plausible generalization is not an unrestricted minimum, so the domain boundary may contain the minimizing point. However, the calculation on the boundary is a lower-dimensional analog of the original problem; therefore, inductive use of our basic Lemma gives the global minimum on the restricted domain.

Section 2 introduces our concepts and calculates the unrestricted minimum. Section 3 reviews the regional correlation algorithm and motivates our proposed change. Section 4 describes the new algorithm and proves its validity. All computational details are standard linear algebra [26].

2. Preliminaries

If ξ_j, η_j are real numbers and $\zeta_j = \xi_j + i\eta_j$, then the set C^n of complex n-tuples $z = (\zeta_1, \cdots, \zeta_n)$ forms an additive group under the usual componentwise addition. Indeed, this group becomes an n-dimensional complex inner product space if we introduce complex scalar multiplication and the inner product

$$(z_1, z_2)_C = \sum_{j=1}^{n} \bar{\zeta}_{1,j}\zeta_{2,j}, \bar{\zeta}_j = \xi_j - i\eta_j . \tag{1}$$

However, this group becomes a $2n$-dimensional *real* inner product space if we employ only real scalar multiplication and the inner product

$$(z_1, z_2)_R = \sum_{j=1}^{n} [\xi_{1,j}\xi_{2,j} + \eta_{1,j}\eta_{2,j}]. \tag{2}$$

These products obey the simple relation

$$(z_1, z_2)_R = \text{Re}\,[(z_1, z_2)_C], \tag{3}$$

whence both define the same norm

$$\| z \|^2 = (z, z)_R = (z, z)_C . \tag{4}$$

Thus the normalization of a nonzero vector z will yield the same $z/\| z \|$ in both spaces, but the orthonormalization of a sequence $z_1, z_2, \cdots$ may give different results after the first vector. Moreover, any orthonormal basis for the real inner product $(\cdot, \cdot)_R$ defines an isomorphism-isometry onto real Euclidean $2n$-space, even when the basis vectors have complex components. Since the norm (4) defines the topology, clearly, both spaces have the same topology. This notation clarifies the model in Section 3; the next lemma simplifies the proof in Section 4.

Given a finite sequence $z_0, z_1, \cdots, z_m$ in C^n which is linearly independent over the real numbers, form the corresponding orthonormal sequence $w_0, w_1, \cdots, w_m$ in C^n which is determined by the inner product $(\cdot, \cdot)_R$. Then each vector v in the domain

$$D(z_0, \cdots, z_m) = \left\{ e^{i\phi} \cdot \sum_{j=0}^{m} \rho_j z_j : \phi, \rho_0, \cdots, \rho_m \text{ real numbers} \right\} \tag{5}$$

has a representation

$$e^{i\phi} \cdot \sum_{j=0}^{m} \sigma_j w_j$$

for some real numbers $\sigma_0, \cdots, \sigma_m$. If $\phi^* = \rho + r\pi$ and $\sigma_j^* = (-1)^r \sigma_j$, where r is any integer, then $\phi^*, \sigma_0^*, \cdots, \sigma_m^*$ fix the same vector. Also, any vector w defines two others

$$a = \sum_{j=0}^{m} \alpha_j w_j , \qquad b = \sum_{j=0}^{m} \beta_j w_j , \tag{6}$$

where the real numbers, α_j, β_j satisfy

$$\alpha_j + i\beta_j = \gamma_j = (w_j, w)_C, \qquad j = 0, 1, \cdots, m. \tag{7}$$

Let the function $f(v) = \| v - w \|^2$, restricted to the domain $D(z_0, \cdots, z_m)$, have local minima precisely on a set $M(w, z_0, \cdots, z_m)$. Note the following analog of Bessel's inequality, extending a remark by R. K. Brayton [1]. (The corollary is well known, but its statement is convenient.)

Lemma

Any vector in $M(w, z_0, \cdots, z_m)$ has the form

$$v = \sum_{j=0}^{m} w_j \cdot \text{Re}\,[\gamma_j e^{-i\phi}] = a \cdot \cos\phi + b \cdot \sin\phi \tag{8}$$

for some real number ϕ. If

$$\sum_{j=0}^{m} \gamma_j^2 = 0,$$

then $\| a \| = \| b \|$ and $(a, b)_R = 0$, while ϕ is an arbitrary real number; so $M(w, z_0, \cdots, z_m)$ is a circle, and its points yield the same minimum value. If

$$\sum_{j=0}^{m} \gamma_j^2 \neq 0,$$

then

$$e^{2i\phi} = \sum_{j=0}^{m} \gamma_j^2 \bigg/ \left| \sum_{j=0}^{m} \gamma_j^2 \right|, \tag{9}$$

and any solutions ϕ_1, ϕ_2 have difference $r\pi$, where r is an integer; so $M(w, z_0, \cdots, z_m)$ is a single vector, and this point is the only local minimum. Hence all local minima are global minima, and the minimum value is

$$\mu(w, z_0, \cdots, z_m) = \| w \|^2 - \frac{1}{2}\left\{ \left| \sum_{j=0}^{m} \gamma_j^2 \right| + \sum_{j=0}^{m} |\gamma_j|^2 \right\}. \tag{10}$$

Proof Expand $\| v - w \|^2$ via the w_j, using the orthonormality of these vectors:

$$\begin{aligned}
&\left\| w - e^{i\phi} \cdot \sum_{j=0}^{m} \sigma_j w_j \right\|^2 \\
&\quad = \left\| e^{-i\phi} w - \sum_{j=0}^{m} \sigma_j w_j \right\|^2 \\
&\quad = \|w\|^2 + \sum_{j=0}^{m} \sigma_j^2 - 2 \cdot \mathrm{Re}\left[\sum_{j=0}^{m} \sigma_j (e^{-i\phi} w, w_j)_C \right] \\
&\quad = \|w\|^2 + \sum_j \{\sigma_j - \mathrm{Re}\,[(w_j, e^{-i\phi} w)_C]\}^2 \\
&\qquad - \frac{1}{4} \sum_j [(e^{-i\phi} w, w_j)_C + (w_j, e^{-i\phi} w)_C]^2 \\
&\quad = \|w\|^2 + \sum_{j=0}^{m} \{\sigma_j - \mathrm{Re}\,(\gamma_j e^{-i\phi}]\}^2 \\
&\qquad - \frac{1}{2} \sum_{j=0}^{m} |\gamma_j|^2 - \frac{1}{2} \mathrm{Re}\left[e^{-2i\phi} \cdot \sum_{j=0}^{m} \gamma_j^2 \right].
\end{aligned} \tag{11}$$

Clearly

$$\sum_{j=0}^{m} \gamma_j^2 = \| a \|^2 - \| b \|^2 + i(a, b)_R;$$

hence the last form yields the stated results.

- *Corollary*

If $m = 0$ and $\| w \| = 1$, then $\gamma_0 = (w_0, w)_C = (z_0, w)_C / \| z_0 \|$ and

$$\mu(w, z_0) = 1 - |\gamma_0|^2. \tag{12}$$

3. Distance

Suppose two accelerometers fixed inside a pen, having the same location (nearly) on the pen axis and measuring orthogonal acceleration components perpendicular to this axis. If $\xi(\tau)$, $\eta(\tau)$ are the measured components, where τ denotes time, then

$$\zeta(\tau) = \xi(\tau) + i\eta(\tau) \tag{13}$$

holds the same information. Let equally spaced times τ_1, τ_2, $\cdots$ during a signature yield measured values $(\zeta(\tau_1), \zeta(\tau_2), \cdots)$. Let some processing of earlier data yield a reference signal $(\zeta_0(\tau_1), \zeta_0(\tau_2), \cdots)$. The regional correlation algorithm [12, 13, 18] divides the new signal into natural segments $(\zeta(\tau_{j+1}), \zeta(\tau_{j+2}), \cdots, \zeta(\tau_{j+n}))$ and evaluates their "similarity distance" from translated segments $(\zeta_0(\tau_{k+1}), \zeta_0(\tau_{k+2}), \cdots \zeta_0(\tau_{k+n}))$. Then it chooses the closest fit among possible translations, and it combines these segment distances into one global index. If new and reference segments have different lengths, then adjoined zeros first equalize them.

Our proposed improvement keeps the same global calculation; it simply changes the segment distance formula. Hence, relabeling observation times, we treat only one segment

$$z = (\zeta(\tau_1), \zeta(\tau_2), \cdots, \zeta(\tau_n)), \tag{14}$$

and, choosing a particular translation, we compare an equal-length reference

$$z_0 = (\zeta_0(\tau_1), \zeta_0(\tau_2), \cdots, \zeta_0(\tau_n)). \tag{15}$$

Now, the complex inner product space C^n contains z and z_0. But the presently used [12, 13, 18] intersegment distance is $1 - |\gamma_0|^2$, where the "correlation coefficient" γ_0 is

$$(z_0, z)_C / \| z_0 \| \cdot \| z \|;$$

so the Schwarz inequality implies nonnegative $1 - |\gamma_0|^2$ and proportional z, z_0 have vanishing segment distance.

Indeed, multiplying either z or z_0 by a positive number ρ or a phase factor $e^{i\phi}$ yields precisely the new or reference data for uniformly scaled measurements or an axially rotated pen, while multiplying either z or z_0 by any nonzero complex number yields, obviously, the same numerical values of $|\gamma_0|$ and $1 - |\gamma_0|^2$. This property for individual segments shows the claimed invariance of the global index. Also, normalizing z does not change $1 - |\gamma_0|^2$. But if $w = z / \| z \|$, then

$$\begin{aligned}
1 - |\gamma_0|^2 &= \mu(w, z_0) \\
&= \min \{ \| w - \rho_0 e^{i\phi} z_0 \|^2 : \phi, \rho_0 \text{ real numbers} \}
\end{aligned} \tag{16}$$

by our Corollary; so if the reference admits arbitrary scaling and rotation, then the present segment distance is just the closest fit to this w. Hence an improved distance would be the corresponding minimum when the reference segment undergoes further natural transformations: (1) time translations by a fraction $\Delta\tau$ of a sampling interval; (2) uniform change in writing speed during a segment.

The time-dependent reference $\zeta_0(\tau)$, after such transformations, takes the approximate form

$$\begin{aligned}
&\zeta_0((1 + \delta)(\tau - \tau_{\mathrm{av}}) + \tau_{\mathrm{av}} + \Delta\tau) \\
&\quad \simeq \zeta_0(\tau) + \Delta\tau \cdot \zeta_0'(\tau) + \delta \cdot (\tau - \tau_{\mathrm{av}}) \zeta_0'(\tau)
\end{aligned} \tag{17}$$

by Taylor's theorem. Here $\Delta\tau$ is a short time and δ is a small real number, while $\tau_{av} = (\tau_1 + \tau_n)/2$ and $\zeta_0'(\tau) = d\zeta_0/d\tau$. If we can smooth $\zeta_0(\tau)$ well enough so that we can find $\zeta_0'(\tau)$ numerically, then we can define the three vectors

$$z_0 = (\zeta_0(\tau_1), \zeta_0(\tau_2), \cdots, \zeta_0(\tau_n)),$$

$$z_1 = (\zeta_0'(\tau_1), \zeta_0'(\tau_2), \cdots, \zeta_0'(\tau_n)),$$

$$z_2 = ((\tau_1 - \tau_{av})\zeta_0'(\tau_1), \cdots, (\tau_n - \tau_{av})\zeta_0'(\tau_n)), \tag{18}$$

and we can introduce the domain

$$D_0(z_0, z_1, z_2) = \{\rho e^{i\phi}(z_0 + \lambda_1 z_1 + \lambda_2 z_2) : \phi, \rho \text{ real numbers}, -\kappa_j \leq \lambda_j \leq \kappa_j\}. \tag{19}$$

Here the domain admits negative scale factors ρ because these have physical interpretations: namely, the domain admits all angles ϕ, and $-\rho e^{i\phi} = \rho e^{i(\phi+\pi)}$. Moreover, the positive constants κ_1 and κ_2 in (19) bound respectively the small parameters $\Delta\tau$ and δ in (17), and thus limit the transformations (1) and (2) in this approximate form.

But the old segment distance is the minimum (16), so our proposed replacement is the quantity

$$\nu(w, z_0, z_1, z_2) = \min\{\| v - w \|^2 : v \in D_0(z_0, z_1, z_2)\}. \tag{20}$$

We now present an algorithm which easily computes this minimum.

4. Algorithm

The broader context of Section 2 permits a simpler discussion of the algorithm. Hence, take any finite sequence $z_0, \cdots, z_m$ linearly independent over the real numbers. Again form the orthonormal sequence $w_0, \cdots, w_m$ determined by the inner product $(\cdot, \cdot)_R$. Choosing positive constants $\kappa_1, \cdots, \kappa_m$, introduce the domain

$$D_0(z_0, \cdots, z_m) = \left\{\rho e^{i\phi}\left(z_0 + \sum_{j=1}^{m} \lambda_j z_j\right) : \phi, \rho \text{ real numbers}, -\kappa_j \leq \lambda_j \leq \kappa_j\right\}. \tag{21}$$

Now, given nonzero vector z, take $w = z/\| z \|$ and restate problem (20): find

$$\nu(w, z_0, \cdots, z_m) = \min\{\| v - w \|^2 : v \in D_0(z_0, \cdots, z_m)\}. \tag{22}$$

Clearly, the nonvoid compact subdomain $B \cap D_0(z_0, \cdots, z_m)$ contains any optimal point v, where

$$B = \{v \in C^n : \| v - w \| \leq \| z_0 - w \|\}. \tag{23}$$

Thus the continuity of $\| v - w \|^2$ implies the existence of this minimum. Also, by definition (5), the previous domain $D(z_0, \cdots, z_m)$ includes the new set $D_0(z_0, \cdots, z_m)$; and, for a single vector z_0,

$$D(z_0) = D_0(z_0). \tag{24}$$

Given any disjoint subsets S^+ and S^- of $\{1, \cdots, m\}$, alter the definition (21) of $D_0(z_0, \cdots, z_m)$ so that $\lambda_j = \pm\kappa_j$ when $j \in S^\pm$. The resulting subsets for any S^+, S^- are the *faces* of $D_0(z_0, \cdots, z_m)$. If $S^+ = S^- = \phi$, then the defined set is the *improper* face $D_0(z_0, \cdots, z_m)$. If $S^+ \cup S^-$ is a singleton $\{k\}$, then the defined set is a *maximal* face. Indeed, $\lambda_k = \pm\kappa_k$; so the set is

$$D_0(z_0 \pm \kappa_k z_k, z_1, \cdots, z_{k-1}, z_{k+1}, \cdots, z_m). \tag{25}$$

Inductively, each *proper* face has form $D_0(z_0^*, \cdots, z_r^*)$, where the integer $r <$ the given m. Moreover, the norm topology for C^n defines a relative topology on $D(z_0, \cdots, z_m)$ such that the union of the maximal faces includes the boundary points of $D_0(z_0, \cdots, z_m)$.

- *Inductive algorithm*

Use the results of our Lemma to find the set $M(w, z_0, \cdots, z_m)$ in the previous domain $D(z_0, \cdots, z_m)$. If $M(w, z_0, \cdots, z_m) \cap D_0(z_0, \cdots, z_m)$ is nonvoid, then its elements yield the minimum (22). If this intersection is void, then take all maximal faces of $D_0(z_0, \cdots, z_m)$, each a set having form (25), and use this algorithm on every such face to find *its* minimizing set for $\| v - w \|^2$; finally, choose the computed vector or set with the smallest $\| v - w \|^2$.

- *Theorem*

This algorithm finds the minimum (22), and it takes finitely many steps.

Proof If $m = 0$, then (24) implies the result. If the result holds for integers up to $m - 1$, then it holds for all maximal faces of $D_0(z_0, \cdots, z_m)$. If no point of $D_0(z_0, \cdots, z_m)$ yields a global minimum over $D(z_0, \cdots, z_m)$, then the Lemma implies that no interior point of $D_0(z_0, \cdots, z_m)$ yields a local minimum of $\| v - w \|^2$; thus, some maximal face contains the desired minimum.

Clearly, the domain $D_0(z_0, \cdots, z_m)$ has 3^m faces, and probably large m need faster algorithms. However, our motivation is (20), and there $m = 2$. Moreover, our discussion has concealed one small problem, namely, that the algorithm must determine the set $M(w, z_0, \cdots, z_m) \cap D_0(z_0, \cdots, z_m)$. Hence, using the notation of Section 2, we treat the two cases of the Lemma. Our recent disclosure [26] contains more computational details.

1. If

$$\sum_{j=0}^{m} \gamma_j^2 = 0,$$

then $M(w, z_0, \cdots, z_m)$ is the circle $\{a \cdot \cos\phi + b \cdot \sin\phi : \phi \text{ real}\}$, where $\| a \| = \| b \|$ and $(a, b)_R = 0$. If

$$a = \sum_{j=0}^{m} \rho_{a,j} z_j \text{ and } b = \sum_{j=0}^{m} \rho_{b,j} z_j,$$

then the intersection is void when $\rho_{a,0} = \rho_{b,0} = 0$. Otherwise, let σ be any real number, and define

$$\begin{aligned} z(\sigma) &= [(\rho_{a,0})^2 + (\rho_{b,0})^2]^{-1} \\ &\quad \times [(\rho_{a,0} - \sigma\rho_{b,0})a + (\rho_{b,0} + \sigma\rho_{a,0})b] \\ &= z_0 + [(\rho_{a,0})^2 + (\rho_{b,0})^2]^{-1} \\ &\quad \times \sum_{j=1}^{m} [\rho_{a,0}\rho_{a,j} + \rho_{b,0}\rho_{b,j} + \sigma(\rho_{a,0}\rho_{b,j} - \rho_{b,0}\rho_{a,j})]z_j. \end{aligned} \tag{26}$$

Then the intersection contains $z(\sigma)/\|z(\sigma)\|$ precisely when $D_0(z_0, \cdots, z_m)$ contains $z(\sigma)$. Thus the admissible values σ satisfy

$$|\rho_{a,0}\rho_{a,j} + \rho_{b,0}\rho_{b,j} + \sigma(\rho_{a,0}\rho_{b,j} - \rho_{b,0}\rho_{a,j})| \le \kappa_j[(\rho_{a,0})^2 + (\rho_{b,0})^2], \tag{27}$$

where $j = 1, \cdots, m$, and these inequalities delimit a (possibly void) σ-interval.

2. If

$$\sum_{j=0}^{m} \gamma_j^2 \neq 0,$$

then $M(w, z_0, \cdots, z_m)$ contains a single vector

$$v = e^{i\phi} \cdot \sum_{j=0}^{m} \sigma_j w_j = e^{i\phi} \cdot \sum_{j=0}^{m} \rho_j z_j. \tag{28}$$

But the intersection is void when $\rho_0 = 0$; otherwise the intersection is $\{v\}$ when all $|\rho_j/\rho_0| \le \kappa_j$.

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Received July 22, 1982; revised September 16, 1982

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