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Pattern Optimization for UPC Supermarket Scanner

Different scanning patterns χ are analyzed in order to determine the degree of redundancy. To this purpose, we evaluate the number of resolution points N_χ which are generated by a sweeping laser beam, while the merchandise is moved across the scanning window. The goal is to find the pattern which minimizes N_χ for an acceptable detection rate.

Introduction

The majority of grocery products in the USA now have the Universal Product Code (UPC) symbol, which allows the automatic recognition of the product at the supermarket checkout counter. This symbol consists of several dark and light lines representing a binary code for an eleven-digit number, ten of which identify the particular product. Five digits are assigned to the manufacturer and five digits to the product in the most typical configuration.

The symbol is read optically when the product moves over a glass window. To this purpose a focused laser beam of weak intensity is swept across the glass window in a specially designed scanning pattern. The reflected light is measured and analyzed successively by a computer. If the UPC symbol is correctly identified, the product's name, price, and other important data are displayed and printed out. The advantages are obvious: Besides a faster product throughput and a more reliable use of the product's proper price, the automatic reading system allows the user to determine the sales rate of each product and to more tightly control the product inventory.

The deflection of the laser light is normally accomplished by a setup of oscillating and rotating mirrors. Independent of the method used, the problem has arisen of determining the most suitable scanning pattern, *i.e.*, the pattern that allows readout of the UPC symbol with minimum effort for a predetermined error rate. So far, the selection of various patterns seems to have been governed by less than optimal means on one hand and ease of technical realization on the other. In this paper, we attempt to quantify the problem and describe a possible method of evaluating the overall usefulness of such patterns.

Label configuration

- *Label parameters*

In what follows, we consider a "label" to be only one-half of the printed UPC symbol. This is legitimate because each half of the UPC symbol can be independently decoded. However, both halves carry information (left half about the manufacturer, right half about the product) and both must be identified. Scanning is accomplished while the item moves across a rectangular scanning window W of area AB. Typically, A = 12.5 cm and B = 10 cm.

We characterize labels by both fixed and varying parameters. Fixed parameters should be the label dimensions a, b and the velocity u_y with which the item is moved in the y direction [Fig. 1(a)]. In practice, these parameters are allowed to vary within the following ranges: 1.27 cm $\leq a \leq$ 3.2 cm; 2.03 cm $\leq b \leq$ 5.1 cm and $u_y \leq$ 2.5 m/s [1]. It will simplify our analysis if for the moment a, b and u_y are assumed to be constant. By setting a = 3.2 cm, b = 5.1 cm, and u_y = 2.5 m/s, we concentrate on the case of large labels and fast-moving products. The case of smaller and slower-moving labels is a simpler subset of this more general case.

Successful scanning must be performed during the window crossing time of the label

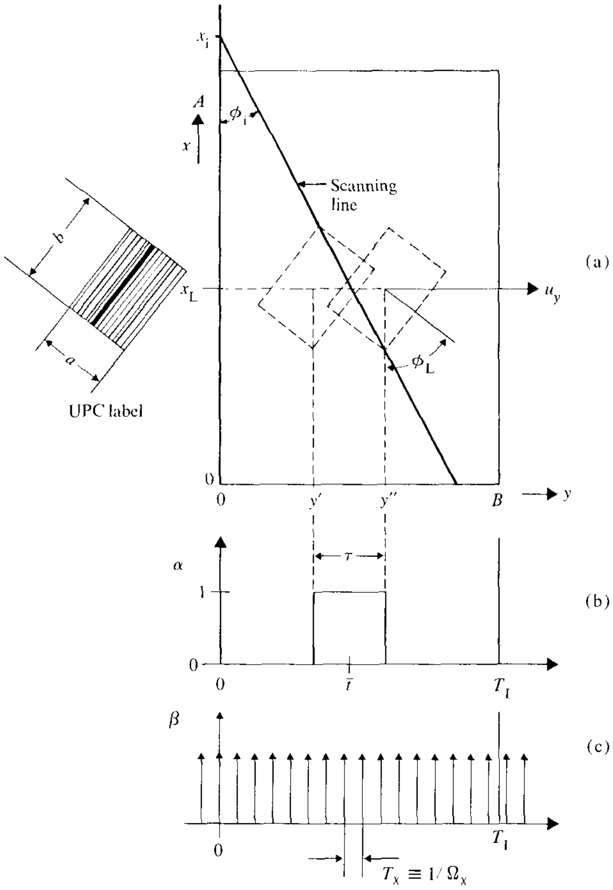

Figure 1 (a) Scanning window and UPC symbol: UPC symbol (x_L, ϕ_L) moves from left to right; laser beam, which scans along line (x_i, ϕ_i), encounters all bar code lines between y' and y''. (b) Label's availability function α is nonzero only between y' and y''. (c) Scanning pulses: Beam is swept along line every T_χ seconds. In this example four correct readings are performed during window crossing time T_I.

$$T_I = B/u_y, \tag{1}$$

which is at least 40 ms.

- *Label configuration space*

We consider the position and the orientation of the label at the entrance to the scanning area as varying parameters. Since merchandise is moved in most cases in the y direction, it is sufficient to describe the label position by x_L, the x coordinate of the label's center of mass. The orientation can be described by the angle ϕ_L, the angle of the normal to the bar code lines and the x axis. Thus, a label is completely described by vector $\vec{r}_L \equiv (x_L, \phi_L)$, which is a constant of motion during T_I.

All detectable label vectors lie within a two-dimensional configuration space, in which all our calculations are performed:

$$C_L \equiv \left\{ \vec{r}_L = (x_L, \phi_L), \text{ where } \begin{array}{c} \frac{a}{2} \le x_L \le A - \frac{a}{2} \\ \\ -\frac{\pi}{2} \le \phi_L \le +\frac{\pi}{2} \end{array} \right\}. \tag{2}$$

It can be shown that the range of x_L depends in a complicated way on ϕ_L and on the scan pattern used; however, neglecting this dependence as in Eq. (2) does not seriously affect the validity of our analysis.

Scanning pattern

Our goal is to detect and decode a moving label in an optimal manner. In this regard we consider scanning optimal if the total decision effort can be kept as small as possible. In order to derive a quantitative measure for the decision effort, we consider the number of resolved spots needed to read a label. If the laser beam has spot size $\delta\ell$, then at least $N_L = a/\delta\ell$ spots are necessary. Here a is a label dimension normal to the bar code lines. Thus, if L_χ is the path length of the entire scanning pattern and Ω_χ the pattern repetition frequency, the number of resolvable spots addressed by the beam during time T_I is $N_\chi \approx (L_\chi \Omega_\chi T_I)/\delta\ell$. Dividing N_χ by N_L leads to the dimensionless quantity

$$R_\chi \equiv N_\chi/N_L = L_\chi \Omega_\chi T_I/a, \tag{3}$$

which is the length of the scanning path of the beam during T_I, expressed in units of label size a. The quantity R_χ indicates how much scanning effort is needed to find an arbitrary label vector $\vec{r}_L$ within its configuration space. The optimization goal is to detect each label $\vec{r}_L$ within a given error $e_0(\vec{r}_L)$ with the constraint of minimum decision effect R_χ. We express e_0 by a minimum number of correct label readings n_0, which should increase as the preset e_0 decreases.

In summary, we try to evaluate that pattern χ for which R_χ is as small as possible so that for all labels the number of correct readings n is not less than a preset value n_0:

$$n(\vec{r}_L, \chi) \ge n_0(\vec{r}_L) \qquad \forall\, \vec{r}_L \in C_L. \tag{4}$$

Minimization of R_χ results in a pattern of either short scan length L_χ and/or low repetition rate Ω_χ. Both conditions are desirable when constructing the light deflector because they lead to a more compact, more slowly rotating and hence less costly device. Since in Eq. (3) a and T_I are given by the specifications, and since L_χ is easily determined for each pattern, the optimization problem reduces to the evaluation of the pattern repetition frequency Ω_χ.

Pattern repetition frequency

We consider first the interaction of a moving label with a single straight-scanning line χ_i [Fig. 1(a)]. The scanning line is described by the coordinate x_i of the intercept at the window entrance boundary and by ϕ_i, the angle that the line forms with the x axis.

A necessary condition for correct label scanning is that the scanning line encounter the moving label in such a way that all bar code lines are crossed. As can be seen from Fig. 1(a), this is only possible between label positions y' and y''. We refer to the time which the label needs to move from y' to y'' as the label availability time $\tau = (y' - y'')/u_y$. It is shown in the Appendix that τ can be written as

$$\tau\left(\frac{b}{u_y}\right) f, \tag{5}$$

where f depends on angles ϕ_L and ϕ_i and on x_L if scanning occurs near window boundaries. We express this partial detectability by the function

$$\alpha(\vec{r}_L, \chi_i, t) = \text{rect}\left\{\frac{t - \bar{t}(\vec{r}_L, \chi_i)}{\tau(\vec{r}_L, \chi_i)}\right\} = \begin{cases} 1 & |t - \bar{t}| \leq \dfrac{\tau}{2}, \\ 2 & \text{else.} \end{cases} \tag{6}$$

This is a rectangular time function of duration τ, centered at $\bar{t} \equiv \{|x_L - x_i|/u_y\} \tan \phi_i$ [Fig. 1(b)].

Since the laser beam is swept along the scanning line every $T_\chi = 1/\Omega_\chi$ seconds with much higher speed than that of the label itself, we may approximate the time behavior of the scanner by

$$\beta(\chi_i, t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_\chi), \tag{7}$$

an infinite series of δ functions. Every δ function represents the moment during which the scanning beam crosses the window [Fig. 1(c)]. The number of correct readings is then the time integral of the product of both functions α and β,

$$n(\vec{r}_L, \chi_i) = \int_{-\infty}^{+\infty} \alpha(\vec{r}_L, \chi_i, t')\beta(\chi_i, t')dt'. \tag{8}$$

In the special case of Figs. 1(a–c), four successful readings are performed ($n = 4$).

Equation (8) only holds if both functions are synchronized, *i.e.*, if the light-deflecting device is started at the moment when the product enters the scanning area. In most applications, because the scanning pattern is independently generated in time, the label may enter the scanning area with varying delays t with respect to the δ functions. Consequently, we must replace the simple scalar product in Eq. (8) with the convolution integral

$$n(\vec{r}_1, \chi_i, t) \equiv \alpha(^*)\beta = \int_{-\infty}^{+\infty} \alpha(\vec{r}_L, \chi_i, t - t')\beta(\chi_i, t')dt'. \tag{9}$$

Since we are only interested in the average number of scans, we take the expectation value of $n(t)$ over one period T_χ,

$$\bar{n} = \frac{1}{T_\chi}\int_0^{T_\chi} n(t)dt. \tag{10}$$

Inserting Eqs. (6) and (7) into Eq. (9) and applying Eq. (10) leads to

$$\bar{n}(\vec{r}_L, \chi_i) = \Omega_\chi \tau(\vec{r}_L, \chi_i), \tag{11}$$

which demonstrates that the average number of scans is proportional to the pattern frequency and the label availability time. Equation (11) is now easily generalized to the case of a pattern of M different scan lines:

$$\bar{n}(\vec{r}_L, \chi) = \Omega_\chi \tau(\vec{r}_L, \chi), \tag{12}$$

where $\tau(\chi)$ is the sum of all M availability time values $\tau(\chi_i)$. It follows from Eq. (12), together with the optimization constraint of Eq. (4), that $\Omega_\chi \geq n_0/\tau$. Since Ω_χ also should be as small as possible, the optimal choice for Ω_χ is

$$\Omega_\chi = \max_{\vec{r}_L \in C_L} \{n_0/\tau\}. \tag{13}$$

At this point it is sensible to define the Tchebychef decision coefficient

$$\gamma_T \equiv \max_{\vec{r}_L \in C_L} \{n_0/f\}. \tag{14}$$

Using γ_T and Eq. (5) allows us to write Ω_χ in its final form as

$$\Omega_\chi = \gamma_T u_y/b. \tag{15}$$

Equation (15) clearly indicates that the scanning frequency must be higher for both smaller and faster-moving labels and for a greater number of required scans n_0. Finally, by inserting Eq. (15) into Eq. (3) and using Eq. (1), we obtain the optimization quantity

$$R_\chi = \gamma_T(L_\chi B)/(ab), \tag{16}$$

which depends on both γ_T and the ratio of the "active" scanning area $L_\chi B$ to the label area ab.

Numerical results

We calculated Ω_χ and R_χ according to Eqs. (14) and (15) for the nine different pattern configurations shown in Fig. 2. These patterns were chosen either because of their simplicity or their similarity to patterns in use. The calculations were performed assuming $n_0 = 5$ for all label vectors. The results for the large label sizes are listed in the left column of Table 1. We see that Pattern No. 1 yields the lowest R_χ value.

Figure 3(a) shows function τ, which is defined on the two-dimensional label space C_L. The figure indicates that

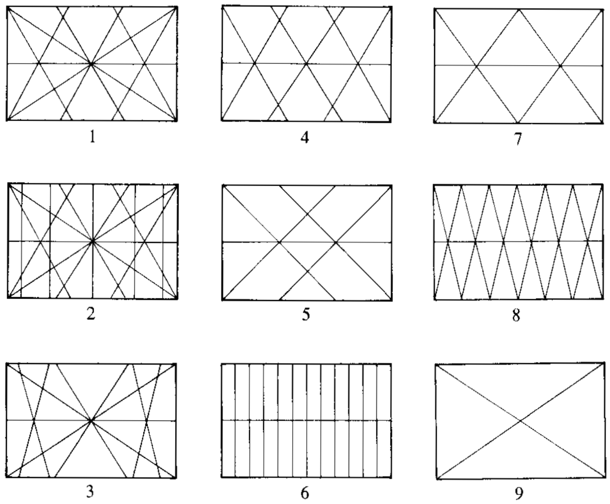

Figure 2 Scanning pattern configurations.

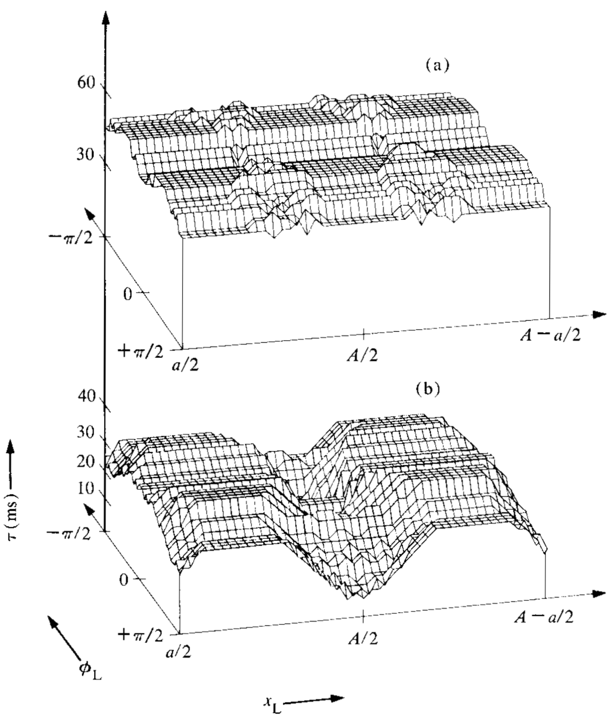

Figure 3 Label's availability time τ as a function of label position x_L and orientation ϕ_L. (a) Pattern No. 1: Shows best results; τ is rather homogeneous and each possible label situation (x_L, ϕ_L) has the same probability. (b) Pattern No. 7: τ is strongly structured and the deep-lying minimum affects higher scanning rates, which are necessary to obtain the same error rate as in (a).

variation of τ was fairly small and that the minimum lies rather high. Consequently R_x, which depends on the minimum value for constant n_0, is desirably low. Figure 3(b) shows the τ distribution for a less suitable pattern (No. 7). Here, τ exhibits much more structure and the minimum lies rather low, which results in undesirably higher Ω_x and R_x values.

Our results also indicate how to improve Pattern No. 4, which is very similar to a pattern currently used by scanners. If we add two diagonal scan lines to Pattern No. 4, we obtain Pattern No. 1, the best of our collection. As can be seen from Table 1, an improvement in scan efficiency of approximately 50% might be obtained using the assumptions stated earlier.

Integral decision measure

Our calculations so far have been based on the Tchebychef constraint of Eq. (4), which requires that the number of successful scans should not be smaller than a predetermined number. However, a certain amount of underscanning may be tolerable in practical situations if *in toto* the number of correct scannings predominates. One should then replace Eq. (4) with a Gaussian decision measure, *e.g.*, by calculating the mean quadratic deviation of n from n_0,

$$S = \int_{C_L} (n - n_0)^2 d\bar{r}_L, \tag{17}$$

and by considering a pattern acceptable if S does not exceed a preset fidelity value S_0. This can be decided after minimization of S, which is performed in the usual way by evaluating the Ω_x value that satisfies $dS/d\Omega_x = 0$.

Before applying this procedure to our patterns, we substitute for n and n_0 their logarithms. This results in risky underscanning ($n < n_0$) being more weighted in S than useless scanning ($n > n_0$):

$$S = \int_{C_L} (\log n - \log n_0)^2 d\bar{r}_L. \tag{18}$$

Differentiating S with respect to Ω_x and setting the derivative equal to zero leads to the optimal pattern frequency

$$\Omega_x = \exp\left\{\left[\int_{C_L} \log (n_0/\tau) d\bar{r}_L\right] \Big/ \int_{C_L} d\bar{r}_L\right\}. \tag{19}$$

Results of calculations of Ω_x and R_x, for some patterns listed in the right column of Table 1, indicate two things. First, using the weaker Gaussian measure generally results in much lower frequencies than using the more exclusive Tchebychef constraint. Second, differences in the patterns are now much smaller, but Pattern No. 1 still shows the lowest R_x value.

Summary and conclusion

Our pattern analysis was performed using two rather extreme decision criteria. It was found in both cases that patterns which cover the label space more homogeneously yield desirable smaller redundancy factors R_x.

The main reason for this result lies in the assumption that all label vectors should be detected with the *same* number of correct readings n_0. This assumption is only sensible if all label vectors occur statistically with the same probability $p(\vec{r}_L)$.

Analyzing practical situations, which to our knowledge so far has not been done, may show that certain label vectors appear more frequently: For example, horizontal ($\phi_L = 0$) and vertical ($\phi_L = \pm\pi/2$) label orientations in the center of scanning area ($X_L = A/2$) are usually more frequent if the majority of products are packed in rectangular boxes. These label situations should then be scanned with a higher detection rate in order to increase the overall scan efficiency.

In summary, the efficiency of a scanner can be increased by using all kinds of *a priori* information to adapt $n_0(\vec{r}_L)$ to the probability $p(\vec{r}_L)$ of the label's occurrence. One possibility is to make $n_0 = \hat{n} + \log p$, where $\hat{n}$ is a chosen appropriate bias value. In the example just mentioned, using an adaptive error rate would consequently favor patterns like No. 6.

Acknowledgment
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Appendix

- *Availability time*

It can be found from geometrical considerations that for an infinitely extended scanning line χ_i, the label's availability time can be expressed by

$$\tau_\infty(\vec{r}_L, \chi_i) = \tau_0 f(\vec{r}_L, \chi_i), \tag{A1}$$

with

$$f = \{1 - \kappa \tan|\phi_L - \phi_i|\}\{\cos\phi_L + \sin\phi_L \tan\phi_i\} \times \left\{\mathrm{rect}\,\frac{\phi_L - \phi_i}{2\hat{\phi}}\right\}. \tag{A2}$$

Here, $\tau_0 = b/u_y$, $\kappa = a/b = 0.63$, and $\hat{\phi} = \tan^{-1}(b/a) = 58°$. The rectangular function indicates that correct scanning is only possible if the scanning line encounters the label at both large side lines b; *i.e.*, only label orientations ϕ_L with $|\phi_L - \phi_i| \le \hat{\phi}$ are readable by scanning line χ_i. Maximum availability time is obtained for $\phi_L = \phi_i$ if the bar pattern is met normally by the scanning line.

Furthermore, Eq. (A2) states that generally steeper scanning lines provide larger τ values. The divergence of τ at $\phi_i = \pm\pi/2$ is, however, not realistic since scanning lines of only finite length are used practically. The effect of window boundaries on τ must therefore be taken into account.

Table 1 Comparison of Ω_x and R_x values obtained using Tchebychef constraint [see Eqs. (4), (13), (15), (16)] *versus* Gaussian decision measure [see Eq. (19)].

Pattern number	L_x (cm)	*Tchebychef* Ω_x (Hz)	*Tchebychef* R_x^a	*Gaussian* Ω_x (Hz)	*Gaussian* R_x
1	122	153	237	105	158
2	173	140	307	84	179
3	117	222	329	114	164
4	87	325	359	160	172
5	74	649	610	179	163
6	127	448	722	b	b
7	66	1005	842	199	162
8	142	493	890	137	240
9	36	b	b	b	b

[a]Low numbers indicate better efficiencies.
[b]Insufficient results.

- *Boundary interaction*

If scanning occurs near window boundaries (very often the case for larger labels), the label's availability time is generally shortened:

$$\tau(\vec{r}_L, \chi_i) = \tau_\infty(\vec{r}_L, \chi_i) \cdot G(\vec{r}_L, \chi_i), \tag{A3}$$

where

$$|G(\vec{r}_L, \chi_i)| \le 1 \qquad \forall\, \vec{r}_L \in C_L.$$

The function G is linear in x_L:

$$G = \begin{cases} 0 & x_L \le x_s, \\ \dfrac{x_L - x_s}{x_r - x_s} & x_s \le x_L \le x_r, \\ 1 & x_r \le x_L \le x_r', \\ \dfrac{x_L - x_s'}{x_r' - x_s'} & x_r' \le x_L \le x_s', \\ 0 & x_s' \le x_L. \end{cases} \tag{A4}$$

If $x_r \le x_r'$ and for $x_r > x_r'$,

$$G = \begin{cases} 0 & x_L \le x_s, \\ \dfrac{x_L - x_s}{x_r - x_s} & x_s \le x_L \le x_r', \\ \dfrac{x_r' - x_s}{x_r - x_s} & x_r' \le x_L \le x_r, \\ \dfrac{x_L - x_s'}{x_r' - x_s'} & x_r \le x_L \le x_s'. \end{cases} \tag{A5}$$

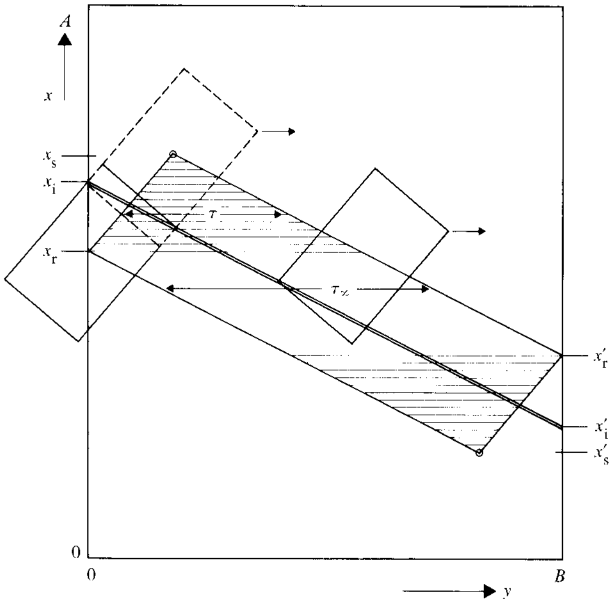

Figure 4 For a special scanning line and for fixed label orientation ϕ_L, the label's position x_L is varied. It can be seen that τ is constant only between x_r and x'_r, while beyond these limits τ drops linearly to zero.

In Eqs. (A4) and (A5), x_s and x'_s are the x coordinates for the label's extreme left- and right-hand positions, respectively, for which detection is still possible;

$$\tau(x_L) \to 0 \qquad \text{for } x_L \to x_s \text{ or } x'_s. \tag{A6}$$

The values x_r and x'_r describe those left- and right-hand positions at which boundary effects disappear;

$$\tau(x_L) \to \tau_\infty \qquad \text{for } x_L \to x_r \text{ or } x'_r. \tag{A7}$$

Figure 4 illustrates the definition of these extreme label positions and shows the linear shortening of the availability time τ. As can be seen from the figure, the following relations hold:

$$\Delta_s \equiv x_s - x_i = x'_i - x'_s,$$
$$\Delta_r \equiv x_r - x_i = x'_i - x'_r. \tag{A8}$$

Both values (Δ_s, Δ_r) are themselves functions of the label parameters (x_L, ϕ_L) and the scanning line parameters (x_i, ϕ_i). We find that

$$\Delta_s = [g_0 - g_1] \text{ rect (I)}$$
$$+ g_1 \text{ rect (II)} + [g_0 - g_2] \text{ rect (III)} \tag{A9}$$

and

$$\Delta_r = g_1 \text{ rect (I)}$$
$$+ [g_0 - g_1] \text{ rect (II)} + g_2 \text{ rect (III)}, \tag{A10}$$

where the following abbreviations are used:

$$g_0 = a \cos \phi_i / \cos (\phi_L - \phi_i),$$
$$g_1 = b \cos (\phi_L + \hat{\phi})/(2 \sin \hat{\phi}),$$
$$g_2 = b \cos (\phi_L - \hat{\phi})/(2 \sin \hat{\phi}),$$
$$\text{rect (I)} = \begin{cases} 1 & \phi_L \le 0, \\ 0 & \text{else;} \end{cases}$$
$$\text{rect (II)} = \begin{cases} 1 & 0 \le \phi_L \le \phi_i, \\ 0 & \text{else;} \end{cases}$$
$$\text{rect (III)} = \begin{cases} 1 & \phi_i \le \phi_L \le \phi_i + \hat{\phi}, \\ 0 & \text{else.} \end{cases} \tag{A11}$$

Reference

1. D. Savir and G. J. Laurer, *IBM Syst. J.* **14,** 16 (1975).

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