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Interferometric Wavelength Measurements through Post-Detection Signal Processing

Modern electronic signal processing techniques make it possible to use a conventional scanning Fabry-Perot interferometer for highly accurate measurement of (tunable) laser wavelengths. An experimental device is described in which the time intervals between resonances of an unknown wavelength are compared with those of a stabilized He-Ne reference laser by the use of highly accurate electronic timing devices. The accuracy is further enhanced through signal averaging over many scan periods. Accuracies approaching one part in 10^7 are expected to be readily achievable.

Introduction

The extraordinary resolving power routinely achieved with present tunable laser sources has rendered conventional methods for wavelength measurements inadequate. The resolution of all but the largest grating spectrographs has become insufficient and conventional interferometric methods for precision wavelength measurements are too time-consuming, tedious, and difficult for routine laboratory use. Several new approaches to precise wavelength measurements of tunable laser radiation have recently been explored [1]. These include arrays of fixed Fabry-Perot interferometers with optical multichannel readout, a Fizeau interferometer with a linear photodiode array interfaced to a minicomputer, and fringe counting traveling-arm Michelson-type interferometers. In this paper we describe another wavelength- or frequency-measuring device in which the burden of achieving a high degree of accuracy is placed on post-detection signal processing rather than on the optics.

Principles of operation

In its most rudimentary form, the device consists of a scanning Fabry-Perot interferometer (F-P) into which are coupled two laser beams: a highly stabilized He-Ne reference beam and the measured beam of unknown wavelength (see Fig. 1).

The intensity I_t of each beam transmitted through the interferometer is given by the Airy function [2]:

$$I_t = I_i/[1 + P \sin^2 (2\pi d/\lambda)]. \quad (1)$$

Here, I_i is the input intensity, P is a constant related to the finesse F through the relation $F = (\pi/2)P^{1/2}$, d is the spacing between the mirrors, and λ is the wavelength of light. Figure 2(a) shows the transmitted intensity I_t as a function of d.

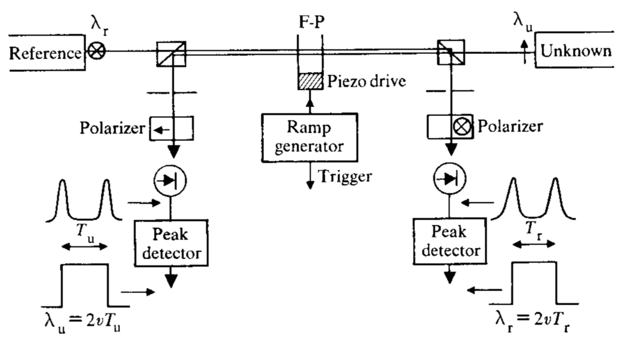

Figure 1 Schematic diagram of the experimental set-up.

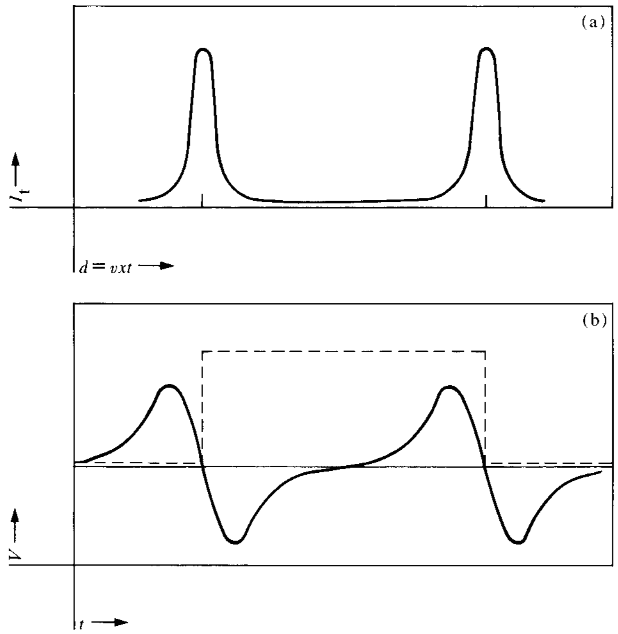

Figure 2 (a) Output signal from one of the two photodiodes. (b) The same signal after differentiation (—) and the time-interval gate (- - -).

Suppose that the distance between the mirrors is periodically scanned in a ramp-like manner so that it varies linearly with the time velocity of motion. Figure 2(a) then represents the output as a function of time. Since the two resonances are one-half wavelength apart, the time interval between the two peaks is equal to $T = \lambda/2v$, where v is the velocity of mirror motion.

If the wavelengths of the reference and unknown beams are denoted by λ_r and λ_u,

$$\lambda_u = \lambda_r(T_u/T_r). \tag{2}$$

The problem reduces to the accurate determination of the two time intervals T_u and T_r. In addition, as we shall see later, this accuracy can be enhanced beyond instrumental limitations by averaging the measurements over many periods. The accuracy of the measurement of the two time intervals depends both on the precision with which the four peaks are detected and the accuracy of the time measuring process itself.

From the variety of possible methods of peak detection, we have chosen one relying on signal differentiation and subsequent detection of the zero crossings [solid lines in Fig. 2(b)]. The actual time interval measurement in each channel is performed by means of a counter that is gated by a rectangular pulse [dashed lines in Fig. 2(b)] whose duration is determined by two successive zero crossings.

Basic sources of error are due to nonlinearities of the mirror motion and overall noise in the system. The former, although possibly large, is deterministic in nature and can therefore be minimized either by calibration or feedback control. Errors stemming from purely random noise in the system are much more difficult to remove. Clearly, the first step is to design a system so that noise is minimized within the acceptable limits of system complexity while the highest possible accuracy in time measurements is maintained. Any further increase in accuracy beyond that achieved by the "clever" design is possible only by post-detection signal processing, which takes advantage of the easily achieved periodicity of the signal.

Let us assume, for the sake of analysis, that the time intervals T_u (or T_r) for a large number of scans have been stored in a computer so that we can construct a quasi-periodic function:

$$y(t) = \sum_k f(t - kT_0 - \epsilon_k), \tag{3}$$

where $f(t)$ is the function representing individual pulses, such as the rectangular pulses in Fig. 3(a). The average interval between the pulses is given by T_0 and ϵ_k is the deviation of the kth interval from T_0 so that the average value of ϵ is zero.

Let the probability density function for ϵ be given by $p(\epsilon)$. We can then define the characteristic function

$$P(\nu) = \int_{-\infty}^{\infty} p(\epsilon) \exp(i2\pi\nu\epsilon) d\epsilon. \tag{4}$$

We also define the Fourier transform of Eq. (3) as

$$Y(\nu) = \int_{-\infty}^{\infty} y(t) \exp(-i2\pi\nu t) dt$$

$$= F(\nu) \sum_{k=-n}^{n} \exp(-i2\pi k\nu), \tag{5}$$

where

$$F(\nu) = \int_{-\infty}^{\infty} f(x) \exp(-i2\pi\nu x) dx \tag{6}$$

is the Fourier transform of the individual pulse $f(t)$. It can be shown [3] that in the limit of $n \to \infty$, the power or intensity spectrum $W(\nu)$ of such a signal is given by

$$W(\nu) = \frac{2}{T_0} |F(\nu)|^2 \times \left\{[1 - |P(\nu)|^2] + \frac{|P(\nu)|^2}{T_0} \sum_{n=0}^{\infty} \delta[\nu - (n/T_0)]\right\}. \tag{7}$$

A power spectrum of a typical signal is shown in Fig. 3(b) for the case of a Gaussian distributed ϵ. The signal is a mixed one, consisting of both a periodic component and a continuous background. This continuum, whose energy has been extracted from the periodic component, is the noise due to the interval jitter ϵ. Clearly, in the limit of no jitter [$\sigma^2 \rightarrow 0$ and $|P(\nu)| \rightarrow 1$], the power spectrum [Eq. (7)] reduces to a perfectly periodic signal consisting of discrete frequencies only.

This suggests that the accuracy of determination of T_0 can be improved by minimizing the continuous component in the spectrum. In practice, this can be accomplished by improving measurement accuracy of the time intervals T and by passing the signal through a "comb" filter. This filter, whose transfer function $|H(\nu)| = |[\sin(\pi\nu T_0 k)/\sin(\pi\nu T_0)]|$, can be realized either by cross-correlating the signal $y(t)$ with a string of periodic impulses [1] or by simply storing the measured time interval values T and averaging them in the computer. The latter is tantamount to taking the sum of a power series with time delays T, and mathematically leads to Eq. (7); *e.g.*, [3].

Generally, since the width of the peaks in the (square of the) "comb" transfer function goes down as the square root of k, the improvement in the accuracy is proportional to the square root of the number of averages. Thus, the final accuracy of the wavelength measurement process is proportional to the square root of the number of averages and to the precision with which the exact time interval between two Fabry-Perot resonances can be determined during a scan.

The expected time jitter ϵ in the position of a resonance can, on the other hand, be estimated. The Fabry-Perot transmission I_t/I_i, as a function of time t near a resonance t_m, is to a good approximation given by a Lorentzian. Let us assume that the detected signal near t_m is distorted by a superimposed sinusoidal noise component according to

$$s(t - t_m) \approx \left[\frac{1}{1 + (t - t_m)^2/\Delta T^2} + \frac{1}{r}\right] \times \sin[\omega_n(t - t_m) + \phi]. \quad (8)$$

Here,

$$\Delta T = T(\pi/4F) \quad (9)$$

is the width of the resonance pulse, r is the signal-to-noise ratio, ω_n is a frequency characteristic for the noise, and ϕ_m is some arbitrary phase angle. Setting the time derivative of the signal [Eq. (8)] equal to zero, we find a maximum time jitter of

$$|t - t_m|_{\max} = \frac{\Delta T^2}{2}\left(\frac{\omega_n}{4}\right). \quad (10)$$

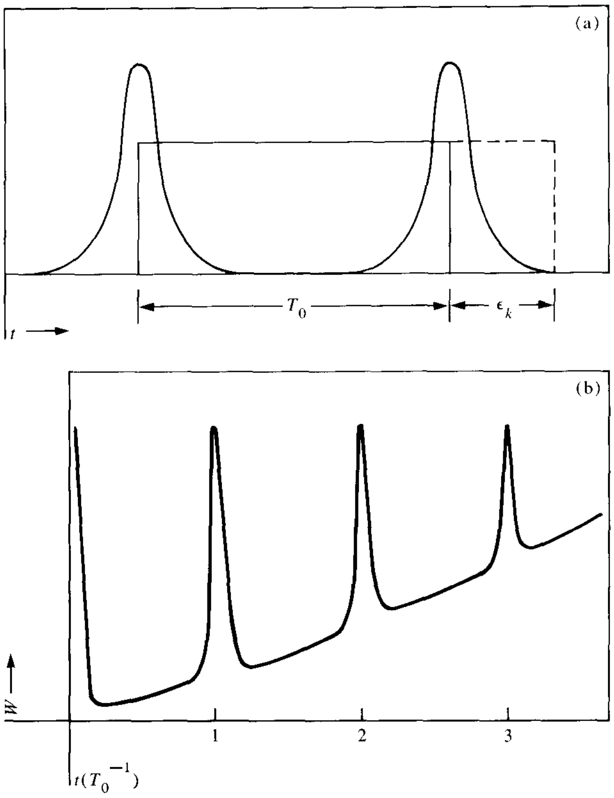

Figure 3 (a) Definition of the time jitter ϵ_k and (b) the power spectrum $W(\nu)$ of a Gaussian distributed ϵ and the root mean square deviation σ = 1/200 000.

Since the signal processing electronics have to transmit frequencies up to at least ΔT^{-1}, the estimated "worst case" jitter is approximately

$$|t - t_m|^2 \approx \pi\Delta Tr = (\pi^2/4)(T_0/Fr), \quad (11)$$

or its relative value

$$\epsilon_m = \frac{|t - t_m|}{T_0} = \frac{\pi^2}{4Fr}, \quad (12)$$

which implies that the effective finesse has been improved by a factor corresponding to the signal-to-noise ratio r of the system. Since this jitter can be further decreased by averaging, the total uncertainty δ_t in the measurement of the wavelength is on the order of

$$\delta_t = \left(\frac{\pi^2}{4}\right)\frac{1}{Frk^{1/2}}, \quad (13)$$

where k is again the number of averages.

Since for time intervals T_0 on the order of 1 ms an r of 10^3 could be readily achieved, an accuracy of one part in 10^7, for a finesse of F = 200, can be obtained by averaging over roughly a ten-second interval. Better estimates of the achievable measurement accuracy can be obtained by deriving the actual $p(\epsilon)$ in a given system rather than only estimating ϵ_m. In that case, the power spectrum [such as that shown in Fig. 2(b)] may provide insight into the uncertainty and/or the accuracy of the wavelength measurement. This curve was derived for a Gaussian $p(\epsilon)$ and $\sigma = 1/Fr$ with $F = 200$ and $r = 10^3$ in Fig. 2(b).

Experimental details

The arrangement of the wavelength meter was shown schematically in Fig. 1. The two beams λ_r and λ_u traverse the interferometer in opposite directions; the two laser sources are in line. After having passed the interferometer, the beams are reflected to their respective photodetectors. Because of internal reflection in the beam splitter and the light reflected back from the interferometer, light from the reference beam can reach the photodetector for the unknown and *vice versa*. To prevent this, iris diaphragms have been placed in front of the photodetectors in order to block the off-axis light. In addition, the polarization directions of the two lasers are chosen perpendicular, and polarizers are placed in front of the photodetectors to stop unwanted beams.

The interferometer is a commercially available scanning interferometer driven by a ramp generator. The signals from the photodetectors are differentiated, the zero crossing detected, and the resulting gating signals (see Fig. 2) fed into the time interval counters in order to measure T_r and T_u.

The whole measurement is controlled by a microprocessor that performs several tasks. During one scan of the interferometer, more than two intensity peaks occur in one channel. Together with the gating circuits, the microprocessor selects the first two successive pulses during each scan and directs them into the counters. After the time measurements, the results are transferred from the counters into the microprocessor and then into a minicomputer. Here, the ratio T_r/T_u is taken and multiplied by λ_r (in the memory of the minicomputer). The average over several results can be calculated. Finally, the calculated λ_u is transmitted to the microprocessor and shown on an LED display [4].

The reference laser is a stabilized single-mode He-Ne laser. Since we have not attempted to determine the actual stability of this laser, our results are only as accurate as this reference. So far, we have been able to achieve such a *relative* accuracy on the order of one part in 10^5 during sample measurement, and several parts in 10^6 by averaging over several seconds. Although somewhat less than expected, we have traced the problem to previous noise sources in the microprocessor and the counters, and expect to resolve this problem in the near future.

Conclusions

We have described a wavelength meter based on a multiple path interferometer of the Fabry-Perot type. The advantage of this approach is that the device is rather compact and the total mirror motion small. The measurement accuracy, which in a Michelson-type wave meter is associated with the long pathlength traversed by the moving mirror, has been at least partly recaptured through the long "lifetime of the photon" in the multiple-path interferometer, or in other words, through its higher finesse. Further enhancement in measurement accuracy has been achieved by precision time measurements associated with high signal-to-noise ratios of modern electronics and by averaging the results of repetitive measurements.

Although the inaccuracies associated with the nonlinearity of the mirror motion are considerable, we believe they will not be too troublesome since they can be substantially reduced by proper calibration of the system.

Acknowledgment

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References and note

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4. For additional information concerning the electronics, contact J. D. Hope, IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, NY 10598.

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