

Polarization Problems of Parallel Plate Lasers

Parallel plate laser technology requires that the plasma envelope and particularly the Brewster windows be "hard sealed." The first part of this study reports the effects of misalignments of the Brewster windows on laser performance. This turned out not to be a serious problem. In the second part the increase in the birefringence of the Brewster windows resulting from the hard seal technique is examined. We found that this problem, which is serious, can be eliminated by careful control of the temperature during sealing and the annealing process.

Introduction

Parallel plate laser technology requires that the plasma envelope be entirely hard sealed. Affixing the resonator mirrors directly onto the structure by this technique, while maintaining the proper geometry, was problematic. It was also not known whether the reflective properties of the mirror could remain unaltered during sealing. To avoid these difficulties, Brewster windows were included in the design. This also had the advantage that the polarization of the output was fixed. However, polarization related problems, not normally encountered in laser technology, resulted, which manifested themselves as an increase in the losses in the optical resonator. These additional losses were due to the reflections, occurring at the Brewster window surfaces, of the energy stored in the resonator (see Fig. 1). Losses are always critical in a low gain laser and must be kept to a minimum; otherwise, power output is adversely affected.

Keeping the efficiency of parallel plate lasers comparable with that of conventional helium neon lasers by identifying and eliminating the above losses and their origins was the aim of the work reported here.

The positioning or alignment of the Brewster windows in the resonator was investigated first, since angular departures from their prescribed positions was an obvious explanation for the previously mentioned losses. The hard seal technique entails the use of seal glass between the components that must be bonded. The presence of seal glass between a Brewster window and the laser structure

prevents the window from being as accurately positioned as it would be without seal glass. The small annular seal glass volume between the Brewster window and the laser structure may be compared to a gasket. Ensuring that this gasket has a reasonably uniform thickness is difficult. Conventional lasers generally do not use seal glass but rather epoxy cement as the bonding agent. This makes it much easier to obtain intimate contact between the Brewster window and the laser structure. In any case the alignment problem did not turn out to be critical. The angular departures can be relatively large (i.e., $\pm 1^\circ$) without having harmful effects; they are easy to detect; and they can be reduced to tolerable values by suitable modifications of the sealing process.

After implementing these modifications, it was discovered, however, that the losses at the Brewster windows persisted. After examination, this second problem was found to be related to the stress birefringence which the hard seal technique, when improperly carried out, induced in the Brewster windows. The optical glass used for the Brewster windows was chosen on the basis of its characteristics. In particular, its thermal expansion coefficient must closely match that of the seal glass [1]. However, this glass exhibited a slight inherent birefringence [2]. This birefringence, while not harmful in itself, was increased by the sealing operation and reached levels such that it seriously impaired laser action. Conventional lasers do not use optical glass but rather fused silica as Brewster window material. The residual birefringence of

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fused silica is negligible, and the epoxy bonding process does not appear to induce any stress birefringence. However, the extremely low thermal expansion coefficient of fused silica precludes its use for the Brewster windows of parallel plate lasers. The birefringence problem was more serious and not as easily disposed of as the alignment problem. While birefringence in the Brewster windows of a laser resonator is generally very apparent (due to the reflections mentioned previously), it can, however, at times be masked by self-compensation, as will become clear later. The induced birefringence was eliminated in our work through careful control of the temperature during sealing, with particular emphasis on annealing.

The relationships among Brewster window misalignments, birefringence, polarization, and losses in optical resonators are now examined.

Theory

The resonance of an electromagnetic mode in an optical resonator manifests itself by the repetition of the polarization state at a particular point inside the resonator after any number of round trips [3, 4]. [A polarization state is defined as a vector whose components may be real (linear polarization) or complex (elliptical polarization) and which must be in a constant ratio.] The search for these states or eigenpolarizations consists in deriving, using the appropriate formalism, the eigenvalues λ_1 and λ_2 and the eigenvectors V_1 and V_2 of the resonator under consideration. The magnitude of an eigenvalue λ is a measure of the amplitude decrease of the associated eigenpolarization after one round trip. The corresponding relative decrease in intensity and the polarization loss coefficient are given by $\lambda\lambda^*$ (where λ^* is the complex conjugate of λ) and $L = 1 - \lambda\lambda^*$, respectively. Since the resonators that are examined here are strongly anisotropic, the eigenvalues λ_1 and λ_2 are very different. Only the eigenpolarization associated with the eigenvalue of largest modulus, say λ_1 , is considered. The modes which correspond to the other eigenpolarization are not present due to their large losses, which cannot be compensated by the gain of the lasing medium present in the resonator.

A given Brewster window may be characterized by its angular relationships to the resonator optical axis and to the other Brewster window. This purely geometrical description is sufficient if the Brewster window in question is assumed to be only a *partial linear polarizer*. The first part of the present study is devoted to the effects which angular departures from these ideal configurations have (under the assumption that the windows are partial linear polarizers) on the eigenpolarization of interest and on the corresponding intensity loss coefficient. Angular tolerances on the position of Brewster windows in laser reso-

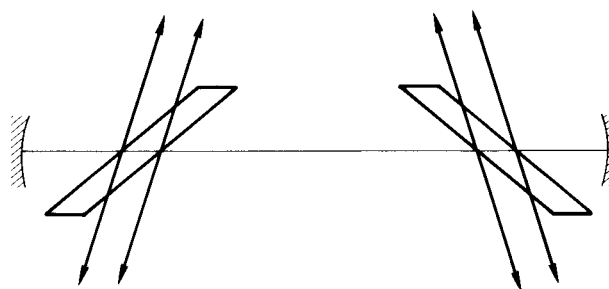


Figure 1 Losses originating as reflections at the Brewster window surfaces.

nator structures are subsequently derived. The second part deals with the fact that in order to account fully for the polarization phenomena observed with parallel plate lasers one must also assume that the Brewster windows were not only partial polarizers but *linear retarders* as well. This new characteristic ascribed to the Brewster windows is merely an expression of the fact that they may also be birefringent.

The Jones matrix polarization calculus [5] is ideally suited for this type of investigation. In a first step, the coefficients of the amplitude transmission matrix which correspond to a round trip inside the resonator are derived. The eigenvalues of this matrix are then determined. The resonator amplitude transmission matrix results from the product of a certain number of elementary matrices, which are now defined.

Let $oxyz$ be an orthogonal set of reference axes. Light propagates in a direction parallel to ox .

The Jones matrix of a partial polarizer whose axes are parallel with oy and oz is given by

$$PP = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}, \quad \text{where } p, q \leq 1. \quad (1)$$

If the axes of the partial polarizer are no longer aligned with oy and oz but tilted from them at an angle β (i.e., the azimuth of the polarizer), this matrix takes the more general form

$$PPG = R(-\beta) \cdot PP \cdot R(\beta), \quad (2)$$

where $R(\beta)$ and $R(-\beta)$ are the corresponding direct and inverse rotation matrices

$$R(\beta) = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}, \quad (3)$$

$$R(-\beta) = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix}. \quad (4)$$

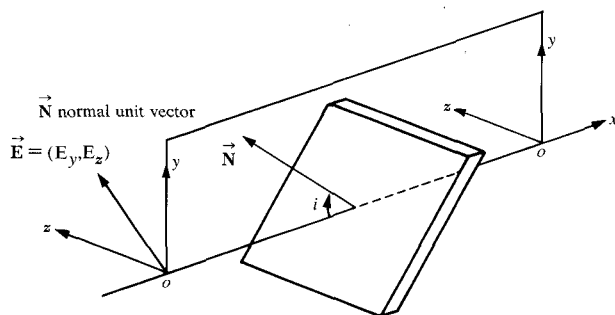


Figure 2 Reference axes of a Brewster window. The normal unit vector $\vec{N}$ is located in the plane of incidence yox .

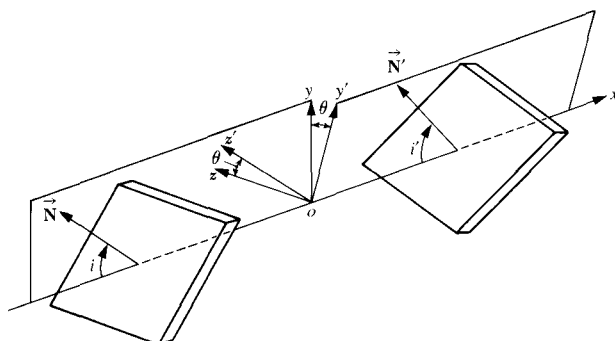


Figure 3 Misaligned Brewster windows. The skewness angle θ is that formed by the incidence planes yox and $y'ox$.

The Jones matrix of a linear retarder whose axes are parallel to oy and oz is given by

$$LR = \begin{pmatrix} \exp i\varphi/2 & 0 \\ 0 & \exp -i\varphi/2 \end{pmatrix}, \quad (5)$$

where φ is the retardation angle. If the azimuth of the retarder with respect to oy and oz is α , the general form of the previous matrix is given by

$$LRG = R(-\alpha) \cdot LR \cdot R(\alpha). \quad (6)$$

The matrices M and M' , which characterize the first and second Brewster windows, respectively, are expressed as a product of elementary matrices of the above types. This product states mathematically that a given Brewster window is considered to consist of a partial polarizer (the first surface), one retarder (the bulk of the Brewster window itself), and another partial polarizer (the second surface). The reference axes used for M are defined as follows: One axis (oz) is perpendicular to the propagation axis (ox) and the plane of incidence (see Fig. 2); the other axis (oy) is perpendicular to the propagation axis and parallel to the plane of incidence. The surfaces of each Brewster window are, of course, assumed to be par-

allel; a departure of a few minutes from this condition, as might be encountered in practice, is of no consequence.

Since one has no *a priori* information as to the direction of the principal stress axes in the glass, the retarder matrix must be used in its general form, as in (6). Uniform anisotropy in the glass volume occupied by the laser beam has been assumed. Since the reference axes oy' and oz' used for M' are not necessarily coincident with the axes oy and oz , but may be tilted from them at an angle θ , the appropriate rotation matrices must be inserted while going from one Brewster window to the next (see Figs. 2 and 3). The angle θ characterizes the skewness of the system.

In all the following derivations, it has further been assumed that the end mirrors have no influence on the polarization.

The matrices M and M' are given by

$$M = M_2 \cdot R(-\alpha) \cdot F(\varphi) \cdot R(\alpha) \cdot M_1, \quad (7)$$

$$M' = M'_2 \cdot R(-\alpha') \cdot F(\varphi') \cdot R(\alpha') \cdot M'_1, \quad (8)$$

where

1. The partial polarizer type matrices M_1 , M_2 and M'_1 , M'_2 refer to the surfaces of the first and second Brewster windows, respectively, and
2. The quantities φ , α and φ' , α' refer to the birefringence-induced retardation and its azimuth in the first and second Brewster windows, respectively. The products $R(-\alpha) \cdot F(\varphi) \cdot R(\alpha)$ and $R(-\alpha') \cdot F(\varphi') \cdot R(\alpha')$ are the linear retarder type matrices associated with each Brewster window.

Many possible matrices correspond to a round trip in the resonator. Complete rigor requires that the point of departure be specified. All round trip matrices may be derived from each other by a cyclic permutation of the individual matrices entering into their definitions. All round trip matrices have the same eigenvalues but *not* the same eigenvectors. In other words, the state of polarization is not uniform in the resonator, a conclusion already stated in [3]. The point of departure that was used in deriving the following round trip matrix $\mathcal{M}$ lies somewhere in the space included between the first mirror and the first Brewster window:

$$\mathcal{M} = M \cdot R(-\theta) \cdot M'^2 \cdot R(\theta) \cdot M. \quad (9)$$

The general case where the Brewster windows are assumed to be *misaligned* partial polarizers and linear retarders is intractable analytically taking into account the large number (seven) of variables involved. The general

case has been treated numerically in a study [6] whose conclusions justify the breaking down of the general case into two sub-cases:

- The Brewster windows are assumed to be *misaligned* partial polarizers.
- The Brewster windows are assumed to be partial polarizers *and* linear retarders.

In either case an analytic solution is possible.

Brewster windows as misaligned partial polarizers

Ideally, the two Brewster windows should be located in the laser resonator such that:

- The angle of incidence of the axis of the laser beam with each Brewster window is "Brewsterian";
- The corresponding planes of incidence are coincident.

Two possible configurations fulfill these requirements, as shown in Fig. 4.

In the following text, the quantities i and r , i' and r' refer to the angles of incidence and refraction of a ray parallel to the laser beam for the first and second Brewster windows, respectively (Fig. 5). It is also assumed that whatever departures from the ideal configurations described previously that may be present in the resonator undergoing examination are small. Therefore, if i_B represents the Brewsterian angle (defined by $i_B = \tan^{-1} n$), i and i' may be written

$$i = i_B + \delta i \text{ and } i' = i_B + \delta i', \quad (10)$$

where δi and $\delta i'$ are small. Naturally the angle θ defined previously (see Fig. 3) is also small.

$$\mathbf{M} = \mathbf{M}_2 \cdot \mathbf{M}_1, \quad (11)$$

$$\mathbf{M}' = \mathbf{M}'_2 \cdot \mathbf{M}'_1. \quad (12)$$

The matrices $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}'_1$, and $\mathbf{M}'_2$ in (11) and (12) may be derived from the Fresnel transmission coefficients [7] (the angles i and r are shown in Fig. 5):

$$\mathbf{M}_1 = \begin{pmatrix} p_1 & 0 \\ 0 & q_1 \end{pmatrix}, \quad \text{with } p_1 = \frac{2 \cos i}{(n \cos i + \cos r)}, \quad q_1 = \frac{2 \cos i}{(\cos i + n \cos r)}, \quad (13)$$

$$\mathbf{M}_2 = \begin{pmatrix} p_2 & 0 \\ 0 & q_2 \end{pmatrix}, \quad \text{with } p_2 = \frac{2n \cos r}{(\cos r + n \cos i)}, \quad q_2 = \frac{2n \cos r}{(n \cos r + \cos i)}. \quad (14)$$

The matrices $\mathbf{M}'_1$ and $\mathbf{M}'_2$ are given by similar expressions with the quantities i' and r' instead of i and r .

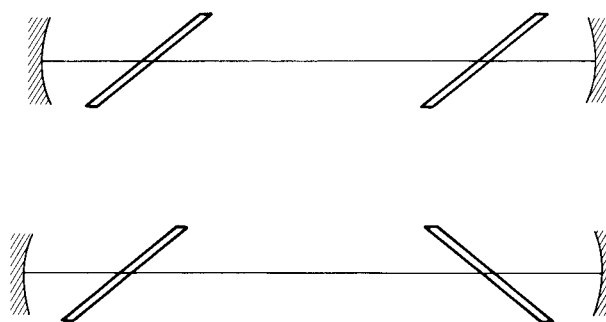


Figure 4 Two possible configurations for Brewster windows inside a resonator.

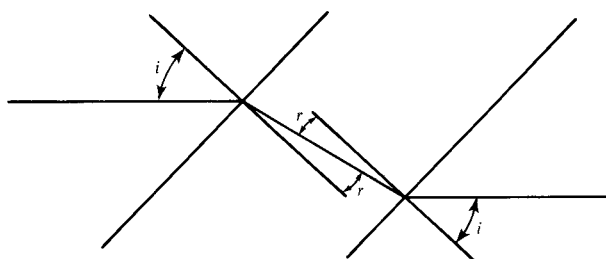


Figure 5 Definitions of the angles i and r .

The matrices $\mathbf{M}$ and $\mathbf{M}'$ may now be written as

$$\mathbf{M} = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}, \quad \text{with } p = p_1 p_2, \quad q = q_1 q_2, \quad (15)$$

$$\mathbf{M}' = \begin{pmatrix} p' & 0 \\ 0 & q' \end{pmatrix}, \quad \text{with } p' = p'_1 p'_2, \quad q' = q'_1 q'_2. \quad (16)$$

The matrix $\mathcal{M}$ is given by (9).

The eigenvalues λ_1 and λ_2 are the roots of the characteristic polynomial of the matrix $\mathcal{M}$:

$$\lambda^2 - \lambda(q^2 q'^2 \cos^2 \theta + p^2 q'^2 \sin^2 \theta + p'_2 q^2 \sin^2 \theta + p^2 p'^2 \cos^2 \theta) + p^2 q^2 p'^2 q'^2 = 0. \quad (17)$$

The eigenvalues defined by (17) are real. The discriminant of (17) falls between $(q^2 q'^2 - p^2 p'^2)^2$ for $\theta = 0$ and $(p^2 q'^2 - q^2 p'^2)^2$ for $\theta = \pi/2$. The eigenvalues are smaller than unity; one can determine that they fall between $p^2 p'^2$ and $q^2 q'^2$ for the first one and $p^2 q'^2$ and $p'^2 q^2$ for the other. Since this study is confined to configurations where θ is small, the quantity $\lambda_1 - \lambda_2$ is approximately equal to $(p^2 p'^2 - q^2 q'^2)$, λ_1 being the larger eigenvalue. The eigenvalues λ_1 and λ_2 are indeed quite distinct.

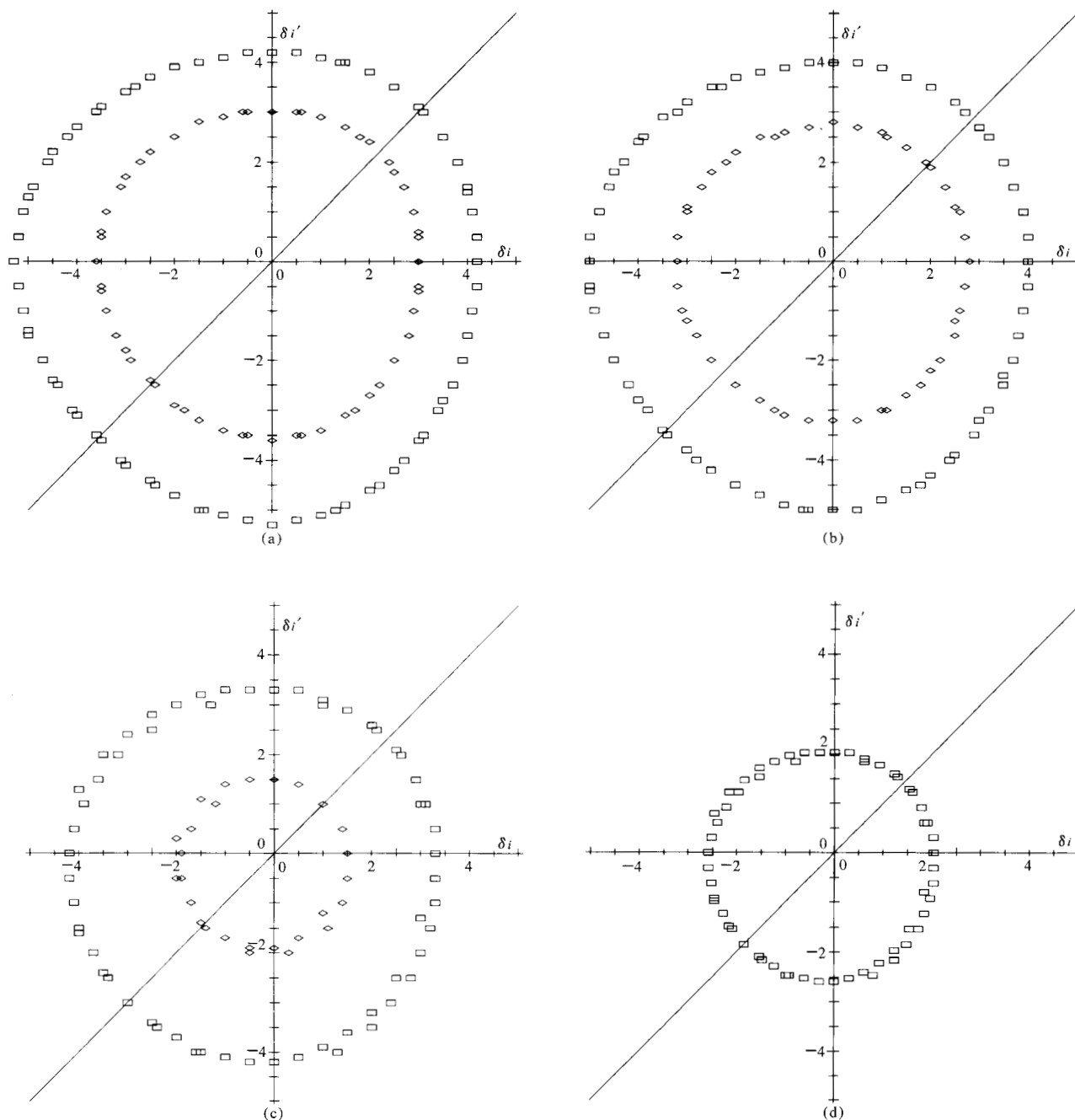


Figure 6 Sections of the surface S for the skewness angle $\theta = 0, 3, 6$, and 9 degrees at (a), (b), (c), and (d), respectively. The inner curve corresponds to the value $T = 0.005$ of the polarization loss coefficient T . The outer curve corresponds to $T = 0.01$. Only one curve is shown at (d) since the curve that corresponds to $T = 0.005$ does not exist for $\theta = 9$ degrees. Note the 45° symmetry axis.

Actual computations for the misaligned partial polarizer case

The coefficients of Eq. (17), the eigenvalue of interest λ , and the polarization loss coefficient L are all functions of the three variables δi , $\delta i'$, and θ . Careful examination of Fig. 3 indicates that δi and $\delta i'$ may be interchanged and that the sign of θ need not be considered. Accordingly,

$$\lambda_1(\theta, \delta i', \delta i) = \lambda_1(\theta, \delta i, \delta i')$$

and

$$L(\theta, \delta i', \delta i) = L(\theta, \delta i, \delta i'); \quad (18)$$

$$\lambda_1(-\theta, \delta i, \delta i') = \lambda_1(\theta, \delta i, \delta i')$$

and

$$L(-\theta, \delta i, \delta i') = L(\theta, \delta i', \delta i). \quad (19)$$

Angular tolerances on the location of the Brewster windows

Taking into account the combined effects of all the known loss factors inside the resonator (e.g., diffraction loss, transmission loss, scattering loss, misalignments, Brewster window loss, etc.), the permissible value for each factor must be small, so as not to reduce the laser power output too drastically. Let T be the permissible limit on the loss coefficient L , so that

$$L(\theta, \delta i, \delta i') \leq T. \quad (20)$$

Let us consider a surface S whose equation is given by

$$L(\theta, \delta i, \delta i') - T = 0. \quad (21)$$

The value taken by $L(\theta, \delta i, \delta i')$ when the point of intersection of coordinates $(\theta, \delta i, \delta i')$ is located inside, or on, S fulfills the inequality (20). Therefore, considering that θ , δi , and $\delta i'$ are departures from an ideal situation, the volume of S defines the bounds (or tolerances) on these quantities.

The sections of S for the skewness angle $\theta = 0, 3, 6$, and 9 degrees have been represented in Figs. 6(a)–(d). Relationships (18) and (19) imply symmetry with respect to the planes $\delta i = \delta i'$ and $\theta = 0$. The permissible values of T for the polarization loss coefficient L have been set to $T = 0.01$ and $T = 0.005$. The surface S appears to be a very elongated surface of a somewhat ellipsoidal shape.

For example, if it has been found from measurements that $\delta i = 2^\circ$, $\delta i' = 1^\circ$, and $\theta = 3^\circ$, examination of Fig. 6(b) shows that the corresponding point lies inside the curve $T = 0.005$. The misalignment losses cannot therefore exceed 0.5%. If similarly one finds $\delta i = 3^\circ$, $\delta i' = 3^\circ$, and $\theta = 0$, examination of Fig. 6(a) indicates that the corresponding point lies between the curves $T = 0.005$ and $T = 0.01$, respectively. The misalignment losses are therefore between 0.5% and 1%.

One can infer from the above that if all three angles δi , $\delta i'$, and θ are smaller than one degree the resulting misalignment losses will be negligible. The angular tolerances on the location of the Brewster windows are not particularly tight. Considering that the Brewsterian incidence corresponds to an extremum for the reflection coefficient of the vibrations parallel to the plane of incidence, this is not surprising. However, since the Brewster windows are affixed to the parallel plate structure by means of a seal glass, if the thickness of the seal is not absolutely uniform, the Brewster windows will be displaced from their ideal positions, even if the machined slots which constitute the Brewster window seats are perfect, by amounts well in excess of one degree. A thickness difference in the

seal of 0.5 mm between the top and the bottom of a Brewster window introduces a tilt of about 2° .

The fixture used to hold the various components together during sealing was therefore modified. The new fixture applied a stronger and more uniformly distributed pressure to the Brewster windows so as to put them in immediate contact with the seats.

Brewster windows as partial polarizers and linear retarders

After the previous conclusions were reached, examination of many lasers soon made it apparent that some lasers whose Brewster windows had been ascertained to be well within the one-degree angular tolerance mentioned still exhibited considerable losses. Misalignment of the Brewster windows was therefore ruled out as an explanation for these losses. A puzzling fact was that these losses were due to strong reflections at the Brewster windows themselves as before. Another unexpected finding was that the output of these lasers exhibited a very slight degree of ellipticity. This implied that the eigenvalues of the corresponding resonators were complex. Earlier in this study it had been proved that the eigenvalues of a resonator including only partial polarizers (i.e., the simple misalignment case) were real. Assuming that the Brewster windows were also linear retarders was a logical step.

This decision was also motivated by the knowledge that, according to the glass manufacturer, a slight amount of residual birefringence might be present in the glass. Also, examination of all sealed parallel plate lasers in a polariscope revealed the existence of stress in the regions surrounding the Brewster windows.

Actual computations for the partial polarizers and linear retarders case

The matrices $\mathbf{M}$, $\mathbf{M}'$, and $\mathcal{M}$ are given by (7), (8), and (9), the matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ by (13) and (14). Since $\theta = 0$ and $i = i' = i_B$, $\mathcal{M}$, $\mathbf{M}_1$, and $\mathbf{M}_2$ are now expressed by

$$\mathcal{M} = \mathbf{M}\mathbf{M}'^2\mathbf{M}, \quad (22)$$

$$\mathbf{M}_1 = \begin{pmatrix} n & 0 \\ 0 & \frac{2}{1+n^2} \end{pmatrix}, \quad (23)$$

$$\mathbf{M}_2 = \begin{pmatrix} \frac{1}{n} & 0 \\ 0 & \frac{2n^2}{1+n^2} \end{pmatrix}. \quad (24)$$

The angles φ and φ' which enter into the definition of M and M' and which characterize the birefringence of the Brewster windows have been assumed to be small. This assumption makes an analytic solution possible. After many manipulations, not reproduced here, one finds finally for the polarization loss coefficient,

$$L = (\varphi \sin 2\alpha + \varphi' \sin 2\alpha')^2. \quad (25)$$

Glass suppliers' catalogs give for the residual stress birefringence of precisely annealed optical glass [3] a maximum value of 10 nm/cm. This is a very conservative estimate and it is generally much less. Considering the thickness of the Brewster windows that were used (typically 3 mm), the contribution to the angles φ and φ' that can be ascribed to residual stress birefringence is likely to be about 1° (but could in extreme cases reach 2°).

The presence of birefringence in Brewster windows on which seal glass had been applied was confirmed by ellipsometric measurements performed by Hauge. (Little or no birefringence was apparent in unsealed Brewster windows.) On the basis of these measurements one can reasonably expect the contribution to the angles φ and φ' that can be ascribed to stress birefringence to be of the order of several degrees (typically 3° , extreme cases 5°). No information is available as to the sign of the combined birefringences nor to the orientations α and α' of the principal stress axes with respect to the reference axes of each Brewster window.

Assuming for the variables involved in the expression of L the extreme values $\alpha = \alpha' = 45^\circ$ and $\varphi = \varphi' = 4^\circ$, one finds $L = 2.6\%$. Such a large value for the polarization loss coefficient would be absolutely catastrophic and very likely would prevent lasing action altogether, when combined with the other loss factors, in a low gain helium neon laser. On the other hand, if the relationship $\varphi \sin 2\alpha = -\varphi' \sin 2\alpha'$ is fulfilled, one can have $L = 0$ no matter how "large" the angles φ and φ' might be. Self-compensation of the birefringence in the Brewster windows is a possibility. Evidently every intermediate situation between the extreme cases is possible.

Experimental verification

Birefringence in the Brewster windows appears to be the only plausible explanation which can account for the strong reflections at the Brewster window surfaces of parallel plate lasers in otherwise geometrically correct resonators. However, *in situ* measurements of the birefringence in Brewster windows affixed to the plasma structure did not seem feasible. Also, as established previously, apparently satisfactory performance without reflection losses is no guarantee of the absence of birefrin-

gence. Another method that was used to ascertain unambiguously whether or not the Brewster windows are birefringent was to search for beats in the laser output. If the windows are birefringent, to each longitudinal mode will correspond two frequencies, ν and $\nu' = \nu + \Delta\nu$. The frequency change $\Delta\nu$ may be derived from the resonance conditions for ν and ν' ,

$$\Delta\nu = \nu \frac{\sum t \Delta n}{S}, \quad (26)$$

where S is the length of the resonator and $\sum t \Delta n$ is the sum of the changes in the optical path in the cavity that can be attributed to birefringence. With $\nu \approx 5 \times 10^{14}$ Hz ($\lambda = 0.6328 \mu\text{m}$), $S = 300$ mm, and $\sum t \Delta n$ typically falling between 3 and 12 nm, one has $5 \text{ MHz} \leq \Delta\nu \leq 20 \text{ MHz}$, which is easily measurable. This experiment was carried out on three lasers, including one with negligible Brewster window losses. The observed beat frequencies were, respectively, 12, 8, and 13 MHz.

Conclusion

The low power output of some lasers whose poor performance cannot be ascribed to other factors (such as gas composition, pressure, electrical and geometrical characteristics, scattering, misalignment, and absorption in the Brewster windows) may be due to stress birefringence. This birefringence results from the superposition of a pre-existing stress birefringence in the Brewster window (a very small factor) and of an induced stress birefringence which results from the sealing operation (a critical factor). This effect generally manifests itself as strong reflections emanating from the Brewster window. However, it has been shown that appreciable birefringence may be present without having harmful results. Because the latter phenomenon is essentially unpredictable with the manufacturing process used up to this point, this process cannot be relied upon to obtain satisfactory lasers on a consistent basis. A practical solution to the birefringence problem turned out to be proper annealing of the laser structure following a suitable thermal procedure.

Acknowledgments

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The author is located at the IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598.