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Contact Stress and Wear Study for Type Characters

Abstract: To ensure the structural integrity of solid print characters subjected to millions of stress cycles, the contact stresses encountered in service must be known. In the present paper various type geometries are explored for their influence on stress distribution. A linear programming method of stress analysis is adopted. Experimental analysis and wear characteristics are considered.

Introduction

The basic function of type characters, used for printing in typewriter and computer input-output devices, is dynamic contacting of platen-supported paper or inked ribbon to produce a print mark. Contact conditions, and primarily the resulting contact stresses, are crucial for a well-designed printer device. An important consideration is that of print quality vs contact pressure. The regional distribution of pressure within the confines of the finished print mark changes during the impact process, so that a successful transfer of ink depends both on the falloff of pressure and on the length of time in contact.

Printing is accomplished in essentially two ways, in a great variety of mechanical applications: (a) by normal impact, and (b) by sliding impact. These modes of contact thoroughly influence the printing process. It is also influenced by the mechanical supporting elements containing the type, and the structural configuration of the paper-supporting platen. Often, however, the dynamic response of the supporting elements has a long-time (low frequency) character compared to the impact-contact pulse. Therefore, a quasi-static, Hertz type contact theory can be useful, neglecting bending and wave propagation effects, and assuming that deceleration of the type is accomplished by the local (elastic) resistance of the materials to indentation. To verify the validity of such assumptions, transducer measurements are installed expediently.

Beyond achieving satisfactory print quality, the strength and structural reliability of the type character must also be safeguarded. Fracture must be avoided, and plastic deformations minimized. In characteristic repetitive impact loading (10^5–10^8 cycles is the usual range), fatigue and wear are the failure mechanisms to reckon with.

Gross fatigue results in catastrophic disintegration of the element. Wear implies a slow process, even though its gradual stage is often preceded by an "incubation period." For metal print characters, hardened steels and hard platings (e.g., chrome) are some of the traditionally used materials. For lighter printing devices, reinforced plastics have been used, and these may be metal plated for additional wear resistance. Even for the best type surface, wear must concern the designer striving for a sufficiently long cycle-life. Thus, in a sense, he must satisfy two opposite requirements: create high enough print force (for print quality), and keep print force low enough to limit wear.

Contact stress analysis for type was performed by Conway and Schaffer [1] using point-matching. A stress analysis of plated reinforced plastic type by Hiraoka and Tago [2] used the finite element technique. Impact wear considerations in general, and some for type in particular, were treated by Engel [3]; these included thermal response, viscoelasticity of the (plastic) material; and various wear mechanisms arising between pairs of impacting materials.

In this paper, contact stress results are given for various type geometries and character shapes. For the method of analysis, the linear programming method of Conry and Seyreg [4] has been found expedient. Next, experimental measurement methods are discussed, followed by some wear observations.

Nomenclature

a	mm	contact radius
a_{ij}	mm/N	influence coefficient
E	GPa	modulus of elasticity
F_i	N	elemental force
H_m	MPa	hardness
K		wear constant
p	Pa	unit pressure
P	N	total contact force
Q_i		artificial variable
R	cm	radius of curvature
t	s	time
v	m/s	sliding speed
V	m/s	normal impact speed
w	mm	elastic displacement
W	mm^3	wear
x	mm	sliding distance; coordinate
X	mm	slip
y	mm	coordinate
Y		slack variable
z	mm	original separation (prior to indentation)
α	mm	elastic approach
ν		Poisson's ratio

Contact stress analysis

The problem of determining the pressure distribution on the print character can be considered as a contact problem between two solids. If the layered nature of the platen is neglected (since the paper cover may not be adding much rigidity in the locality), then the platen may be represented by a linearly elastic, homogeneous and isotropic half-space. In the analysis to follow, the print character is assumed to be made of a much stiffer material than the platen, so that flexural deformations of the former can be neglected, with respect to local elastic deformations. Of main interest will be the evaluation of the contact area, the pressure distribution and the rigid body approach, when the system's configuration and the applied loads are known. If normal surfaces are assumed, the Hertz theory of contacts [5] would be applicable, except for numerical difficulties caused by the character boundary. To overcome that problem, the numerical method of Conry and Seyreg [4] has been applied. It will be briefly outlined, as adapted to our task.

The condition of compatible displacements of two mutually contacting bodies can be written at any point i of the contact area:

$$w_i + z_i - \alpha \geq 0. \tag{1}$$

Dividing the potential contact area into N' squares of side Δx, the total contact force P must be equal to the sum of the elemental contact forces F_j on those squares included in the contact area ($j \leq N \leq N'$):

$$\sum_{j=1}^{N} F_j = P. \tag{2}$$

The connection between F_j and w_i is obtained by utilizing the Boussinesq point-load solution [6]; if N is large enough, it is expedient to assume constant pressure within each rectangle j. On this basis the influence coefficients a_{ij} are derived by elasticity theory. There follows:

$$w_i = \sum_{j=1}^{N} a_{ij} F_j. \tag{3}$$

Introducing the slack variables Y_i in Eq. (1) so as to turn the inequality into an equality, a formal statement of the contact problem is posed as follows:

$$-\sum_{j=1}^{N} a_{ij} F_j + \alpha + Y_i = z_i; \qquad i = 1, 2, \cdots, N; \tag{4a}$$

$$\sum^{N} F_j = P; \tag{4b}$$

$$\sum^{N} F_j Y_j = 0; \tag{4c}$$

$$F_i \geq 0;\ Y_i \geq 0;\ \alpha \geq 0; \qquad i = 1, 2, \cdots, N.$$

By the device of introducing non-negative artificial variables Q_j, the above system can be formulated as a mathematical programming problem [7]:

minimize

$$Q_0 = \sum_{j=1}^{N+1} Q_j \tag{5a}$$

such that

$$\left(-\sum_{1}^{N} a_{ij} F_j\right) + \alpha + Y_i + Q_i = z_i; \qquad i = 1, 2, \cdots, N, \tag{5b}$$

and

$$\sum_{1}^{N} F_j + Q_{N+1} = P \tag{5c}$$

and subject to the condition that either $F_j = 0$ or $Y_j = 0$, and $F_j \geq 0$, $Y_j \geq 0$; $j = 1, 2, \cdots, N$; $Q_k \geq 0$; $k = 1, 2, \cdots, N + 1$.

This problem would be a linear programming problem were it not for the conditions that either $F_j = 0$ or $Y_j = 0$. The simplex algorithm for linear programming can be utilized to solve the system (5) by making a modification of the entry rules [4, 7].

In practice, the above procedure converges to a unique feasible solution in at most $3(N + 1)/2$ cycles, the majority of the cases converging in $N + 1$ cycles. A PL/I computer program was written which was executed (in cN^3 seconds approximately; $c \approx 47^{-3}$) by an IBM System/370 Model 168 computer.

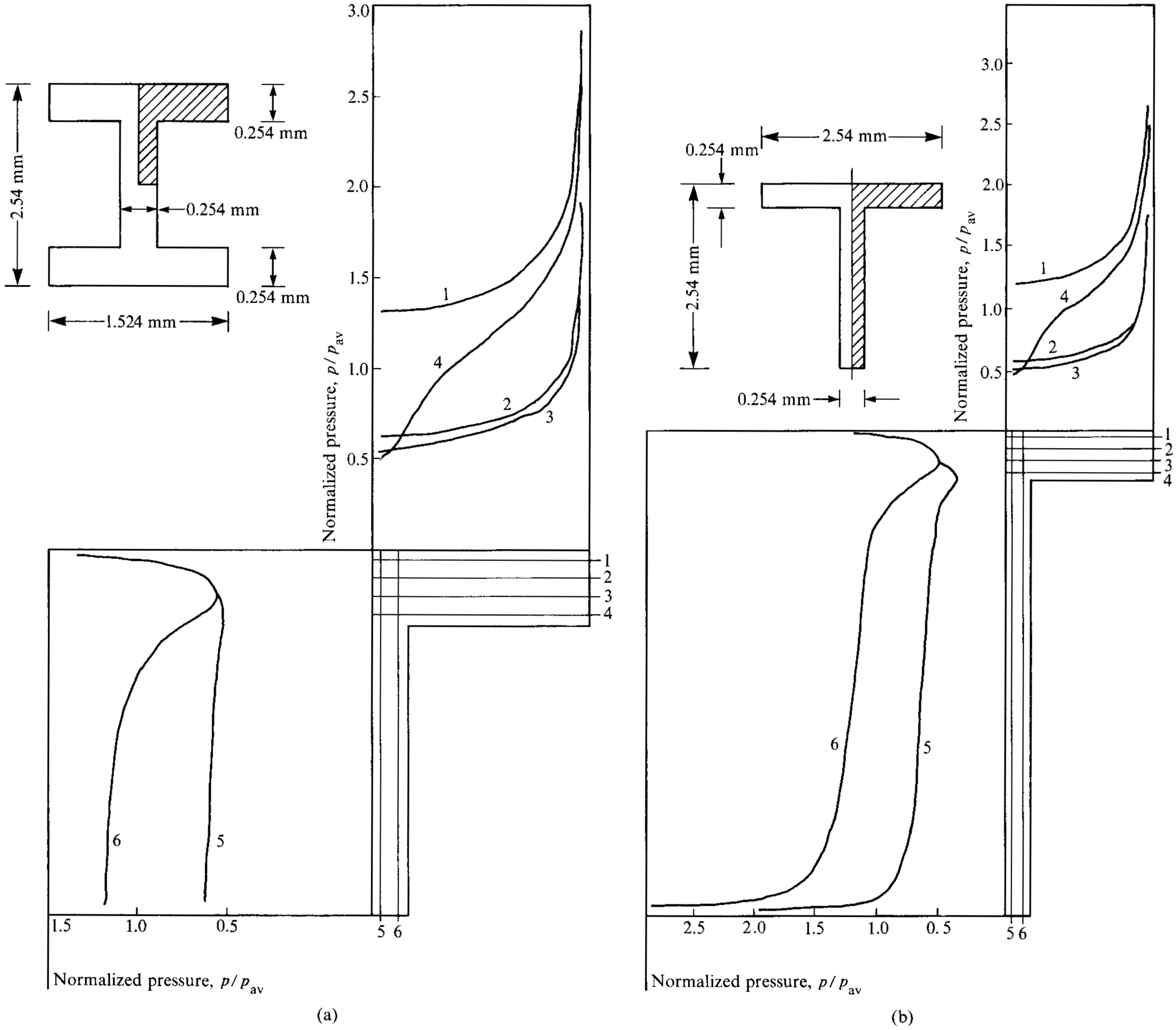


Figure 1 Pressure distribution for flat characters (a) "I", double symmetry, (b) "T", single symmetry. N = 80 elements.

• *Computation of flat contact*

When the character surface and the platen are both flat, the initial separations z_i are zero, and the contact area is known *a priori*. Thus $N \equiv N'$, and $Y_i \equiv 0$, reducing the optimization problem to an algebraic one; i.e., the solution of simultaneous equations in F_i and α.

In the numerical results to be presented, steel characters (E_1 = 207 GPa, ν_1 = 0.3) will be considered pressed against a hard rubber platen (E_2 = 690 MPa, ν_2 = 0.5). Figures 1 (a) and (b) show contact pressure profiles for a capital "I" character and capital "T" character, respectively.

It should be noted that the "I" has twofold symmetry, so that only one quarter of the character area needs to be included for contact points. The influence coefficients a_{ij} are, however, interpreted as the influence at point (k, l) due to the applied force at point $(\bar{m}_2, \bar{n}_2)$; i.e., $(\bar{m}_2, \bar{n}_2)$ includes the symmetrical points in all the four quadrants, (m, n), $(-m, n)$, $(-m, -n)$ and $(m, -n)$. The onefold symmetry of the character "T" would similarly require computation of half the contact area (split along the y axis), and the influence coefficients:

$$a_{ij} = a_{ij}(k, l; m, n) + a_{ij}(k, l; -m, n).$$

Contact pressure profiles of both "I" and "T" demonstrate the increasing of the pressure in the outward direction, towards the periphery. This is a phenomenon featured by flat punch problems in general [5], as opposed to Hertz contact of convex quadratic surfaces.

Figure 2 shows the pressure distribution for the common building block of most characters—the corner which has no symmetry. It is noted that a re-entrant corner is much lower stressed than an outside corner.

Concave and convex contacts

Depending on material and manufacturing considerations, print characters are usually made flat, concave or convex with respect to the platen. Considerations must also include the expected print force vs desired print quality, ply of paper, wear and shrinkage of the character material, and other such items.

Analyses of the "I" character, provided with a cylindrical curvature in the y, z plane, resulted in the pressure profiles of Figs. 3 (a) and (b). The former was a concave contact ($R = -1.78$ cm); the latter, convex ($R = 1.78$ cm). The contact force in both cases was $P = 133.6$ N. Total contact over the characters was made in both cases. The pressure distribution in the y direction was concave and convex, respectively, as expected.

When the total force P is not sufficient to bring about contact over the character surface, incomplete printing results. Therefore, there is great value in finding an analytical process to determine the minimum force for full contact. Such a method, based on superposition, will presently be derived; matrix notation will be used for convenience. For simplicity the character will be assumed rigid, but the plane, elastic.

Let $\mathbf{F}$ be the vector $(F_1, \cdots, F_N)$ of contact pressures and $\mathbf{W} = (w_1, \cdots, w_N)$ the vector of the resulting elastic displacements. By Eq. (3), and denoting the matrix a_{ij} of the influence coefficients by A, we have $\mathbf{W} = A\mathbf{F}$.

Now supposing that the same character (say, "I") exists in both a flat and in a curved (e.g., concave or convex) shape, we let

$\mathbf{F}_1$ = the pressure distribution which occurs when the flat character is pressed against the elastic plane with a unit force; and

$\mathbf{F}_{P0}$ = the pressure distribution resulting when the nonflat character is pressed against the plane with the minimum force P_0 achieving full contact.

If the nonflat character is pressed against the plane with a force P greater than P_0, then the displacement $\mathbf{W}$ and pressure distribution $\mathbf{F}_P$ are related as follows:

$$\mathbf{W} = A\mathbf{F}_P = A(\mathbf{F}_P - \mathbf{F}_{P0}) + A\mathbf{F}_{P0}. \tag{6}$$

Because the additional displacement due to the excess force beyond P_0 is uniform over the character surface, we can write for such a solution F_{P0}:

$$\mathbf{F}_P - \mathbf{F}_{P0} = (P - P_0)\mathbf{F}_1 = \beta\mathbf{F}_1. \tag{7}$$

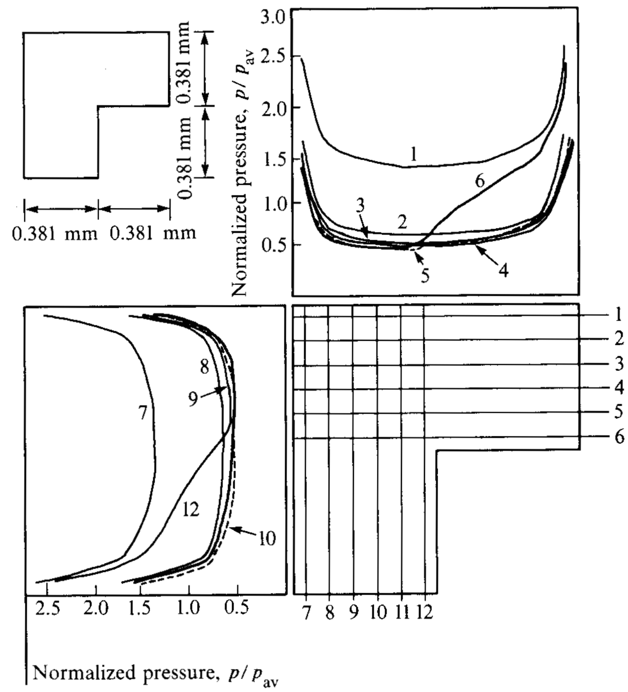


Figure 2 Flat contact of a corner.

Now β can be any positive number, leaving $F_{P0,i} \geq 0$ for all i, where $F_{P0,i}$ is the ith component of F_{P0}. This yields

$$F_{P,i} - \beta F_{1,i} \geq 0 \tag{8}$$

and

$$\frac{F_{P,i}}{F_{1,i}} \geq \beta; \tag{9}$$

thus,

$$\beta = \min_i \frac{F_{P,i}}{F_{1,i}}. \tag{10}$$

Therefore, the minimum force $P_0 = P - \beta$.

Example

For the concave letter "I" of Fig. 3(a), with an applied total force of $P = 133.6$ N; and for the flat "I", with unit applied force [Fig. 1(a)] we get

$$\operatorname*{minimum}_i \left(\frac{F_{P,i}}{F_{1,i}} \right) = \left(\frac{F_{133.6,\text{center}}}{F_{1,\text{center}}} \right)$$
$$= \frac{87.07}{7.76} = 11.22 \text{ N} = \beta.$$

Thus the minimum force to achieve full contact is

$$P_0 = P - \beta = \frac{133.6}{4} - 11.22 = 22.18 \text{ N}$$

over 1/4 of the surface, and $22.18 \times 4 = 88.7$ N over the whole surface.

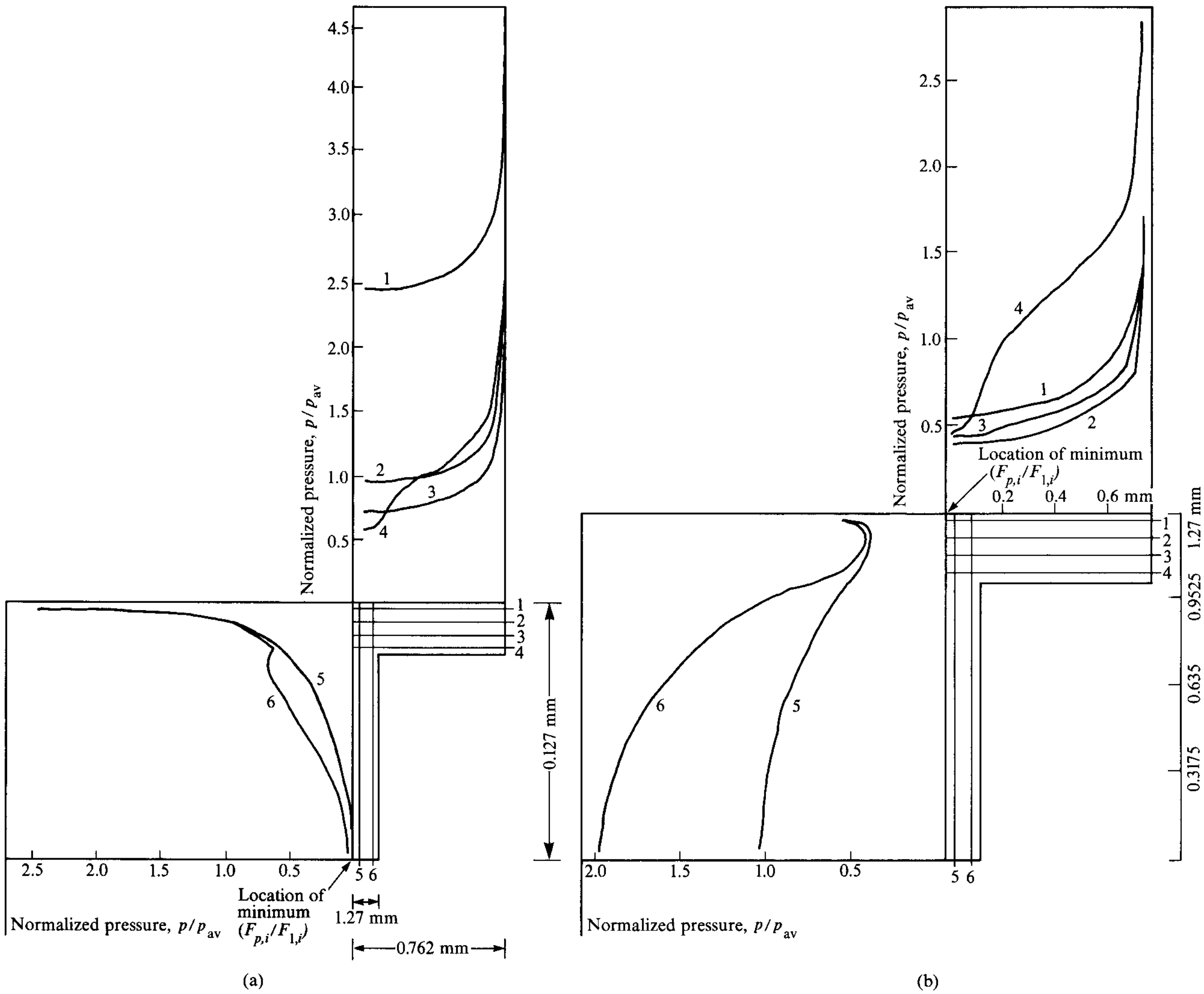


Figure 3 Pressure profiles for "I" character. For size, see Fig. 1. (a) Concave contact; (b) convex contact.

Similarly, for the convex "I" of Fig. 3(b) we get

$$\underset{i}{\text{minimum}} \left(\frac{F_{p,i}}{F_{1,i}} \right) = \left(\frac{F_{133.6,\text{top edge}}}{F_{1,\text{top edge}}} \right)$$

$$= \frac{523.7}{36.98} = 14.16 \text{ N} = \beta$$

and the minimum force,

$$P_0 = P - \beta = \frac{133.6}{4} - 14.16 = 19.24 \text{ N}$$

for 1/4 of the surface and 76.96 N for the whole surface. Both contact stress calculations were also independently made, verifying Eq. (10).

This result shows that a larger force is necessary to achieve full contact in a concave situation than in the corresponding convex one, with the absolute value of the cylindrical radius of curvature kept constant.

• *Optimal (constant-pressure) design*

It is of interest to find that initial separation z_i which would result in a uniform pressure distribution. Since the pressure distribution may be correlated (especially for normal impact printing) with the wear process and the long-term creep deformations of the surface, such an optimal shape would be expected to yield longer life and better print quality.

The contact problem is succinctly stated in Eqs. (4). Since our design objective is to have $F_1 = F^*$ for $i = 1, 2, \cdots, N$, then, from Eq. (4b), $F^* = P/N$. By knowing that $F_i = P/N$ for $i = 1, 2, \cdots, N$, the initial separation at each point minus the rigid body approach α, can be calcu-

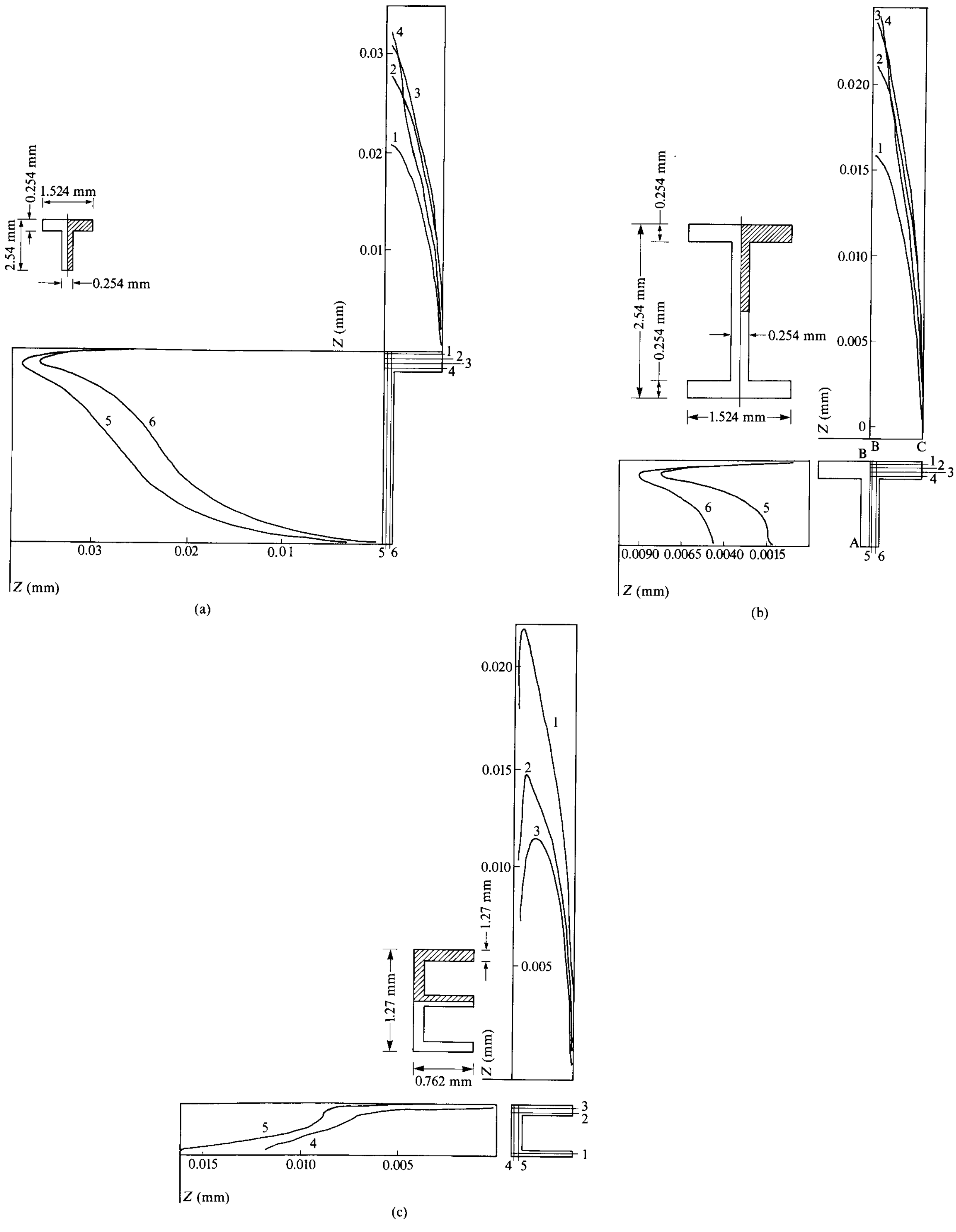


Figure 4 Optimal (constant pressure) separations for type characters: (a) "T", (b) "I", (c) "E".

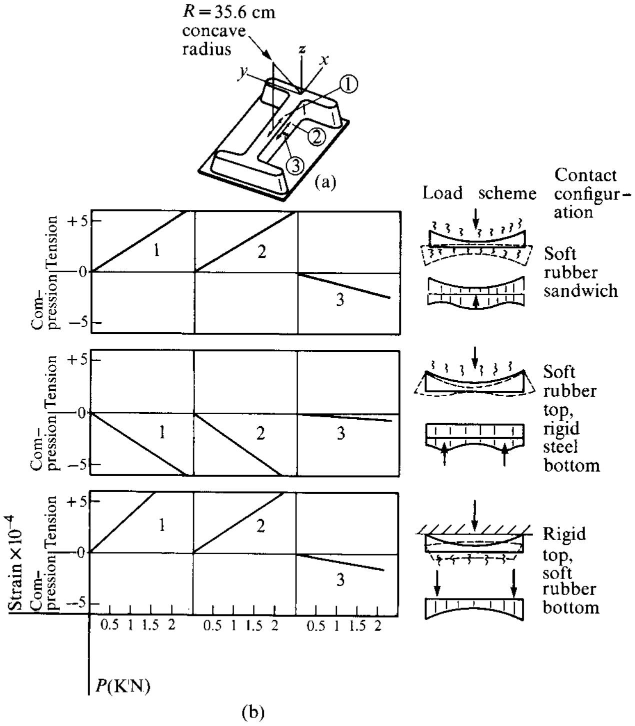


Figure 5 Strain gauge measurements of model "I" character. (a) Strain gauge locations; (b) plots of strain vs applied force.

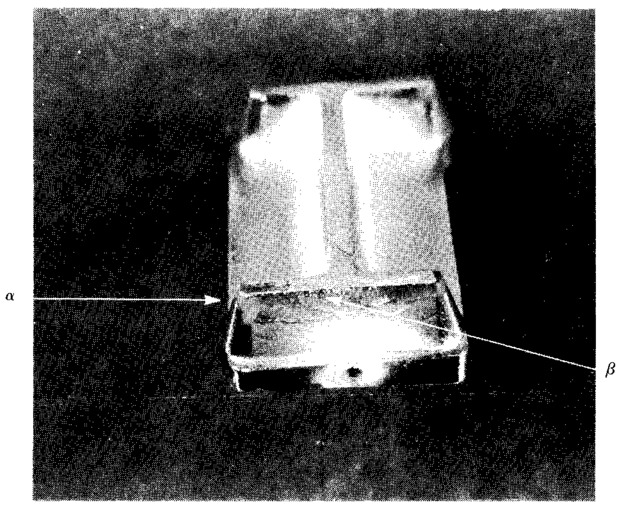


Figure 6 20× scale model of "I" character (real size), plated with thin layer of electrolytic nickel. No damage appears on the contact surface after a load of 55 000 N has been applied through a rubber slab.

lated from Eq. (4a). In this case, $Y_i = 0$ (for all i), since all N points must contact; α is just a constant. Thus Eq. (4a) becomes

$$-F^* \sum_{j=1}^{N} a_{ij} = z_i - \alpha, \qquad i = 1, 2, \cdots, N. \tag{11}$$

Then, to find the initial separation:

$$\text{minimize } \left(-F^* \sum_{j=1}^{N} a_{ij}\right) = C^* < 0. \tag{12}$$

Finally, the initial optimal separation is

$$z_i = -F^* \sum_{j=1}^{N} a_{ij} - C^* \geq 0. \tag{13}$$

The optimal shapes for capital "I", "T" and "E" characters (vs flat platen) are shown in Fig. 4. The result is an interesting double convex shape that has common characteristics for all type.

Experimental techniques

Bending stresses in statically loaded type characters were investigated by taking strain gauge measurements on a magnified scale model of a capital "I", molded of nylon with 30% glass fill reinforcement. Its size was roughly 20 times that of the character analyzed in the previous section; the location of the foil gauges is shown in Fig. 5(a). Gauge 1 was installed in a small cavity (0.1 mm deep) cut in the middle of the top surface and subsequently filled with a special epoxy. The others, 2 and 3, were bonded to the surface. Strain gauges 1 and 2 were to measure longitudinal bending effects at the top of the mid-stem area; gauge 3 would indicate the transverse pressure due to contact, near the top of the midsection.

In an Instron axial load tester (Instron Corp., Canton, MA), this model specimen was supported on its bottom by a flat steel plate. Because of the specimen's concave curvature, a stiff steel slab, loading it from above, would not achieve full contact on the top (print) surface even under 50 000 N load. Therefore the steel (top)-vs-steel (bottom) loading was omitted from the study and the following three configurations were used:

1. Soft rubber slabs (60 durometer) above and below;
2. Soft rubber slab on top and rigid steel bottom;
3. Nylon (E = 414 000 N/cm^2) slab on top and soft rubber slab on the bottom.

Figure 5(b) indicates the strain gauge response and the respective load distributions. Gauges 1 and 2 gave similar results as expected.

Visual observance of the critical bending stress system was obtained by static compression of the previous model provided with a thin, 25-μm nickel plating. No strain gauges were applied. A nylon slab on top and a steel base below were the contacting elements. The first failure of the thin plating occurred after P = 18 000 N, when the top corners at the flanges cracked, α in Fig. 6. After P = 40 000 N, the nylon slab still did not contact in the middle, so a durometer 90 hard rubber slab was substituted as an upper loading element. While this did contact over the top surface, no damage of the latter was noted even under $P = 55\ 000$ N, at which point the cracks β appeared, on the steeply sloping surfaces under the flanges.

Experimental impact force measurements for plastic type characters of the same material (nylon + 30% glass filler) were performed utilizing a hammer mechanism. The type, with its base turned upward and the character downward, was mounted at the end of a flexible cantilever beam. The character outline nearly touched a steel anvil that was bonded to the flat circular surface of a Kistler 912 force transducer (Kistler Instrument Co., Div. Sundstrand Data Control, Inc., Redwood, WA). The small anvil, provided with the desired curvature to simulate platen action, did not amplify or appreciably alter the impact response. Impact traces were recorded on an oscilloscope. By this method, not only single but also repetitive (up to 50 Hz) impact pulses were recorded (Fig. 7) facilitating a study of impact forces vs the number of cycles and the wear of the character surface.

Wear

In many printers, the normal impact between printhead and platen is accompanied by a tangential approach of those surfaces. The purpose is usually the acceleration of the printing process, so that the next character may be struck without a time-consuming stop required by purely normal impact printing. This addition of a sliding motion tends to increase wear substantially, and therefore is a prime factor in limiting the life of type [3].

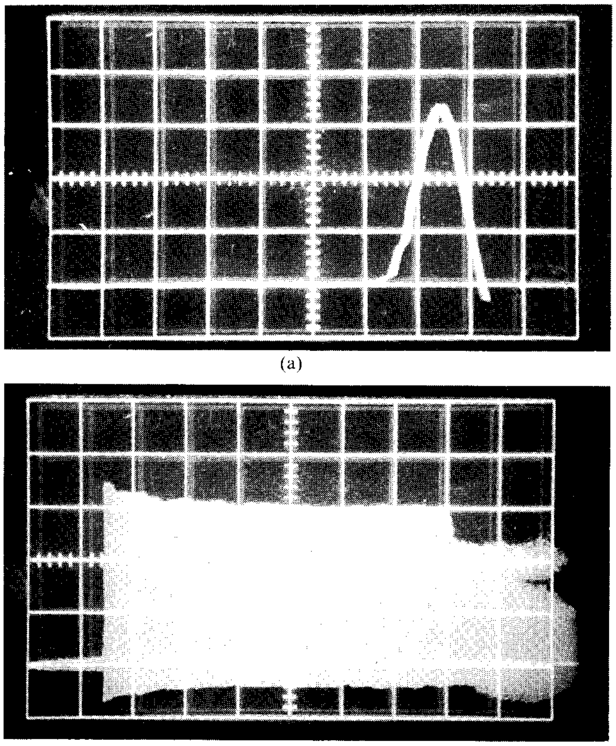
The experience with several metallic computer type systems has indicated that impacting on the fly against a paper- or ribbon-covered platen leads to an abrasive wear mechanism for the type; the wear coefficient is quite low as compared with metal-to-metal configurations, and is of the order of magnitude of 10^{-7} (see Eq. 14, [8]). The abrasive action is derived from hard spots of fibrous material in the paper and ribbon; the abrasiveness of the latter may be measured through sliding tests [9]. The abrasive wear mechanism on hardened steel and chrome plated characters brought about a greatly smoothed surface finish with respect to the original (Fig. 8).

Analytical modeling of the wear process must include a) a correct description of the impulse (i.e., the force-time relationship and the tangential slip), and b) the abrasive wear mechanism, which is conveniently cast in a differential form:

$$dW = \frac{K\,P(X)\,dX}{H_{\mathrm{m}}}, \tag{14}$$

where X denotes the slip between type and platen at any time during impact. The connection between time t and displacement x is established by

$$X = vt - x, \tag{15}$$



(a)

(b)

Figure 7 Transducer measurement of type character impact. (a) Impact force vs time (single impact); (b) impact force vs number of cycles (repetitive impact).

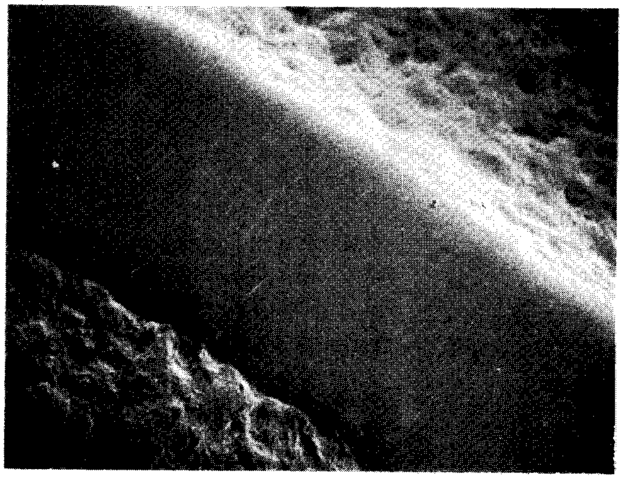


Figure 8 Type character surface, smoothed by sliding-impact wear cycles (AISI Type P5 Mold Steel, after 1.2×10^7 impacts; scanning electromicrograph, 200×).

where x and v are the tangential approach and approach speed, respectively. The relative slip dominates the wear, and only during the slipping part of the impact cycle is wear generation expected to occur.

The abrasive wear mechanism means linearity for the worn volume with respect to impact force and the number of cycles. The surface tends to be evenly worn, and the wear may be "smeared out" to give a uniform depth over the character.

When nonmetallic type is considered, wear mechanisms different from the above are often induced. Polymer type characters have been found to be abraded excessively by printing on the fly. When printing is done in a purely normal mode, reinforced plastics can be satisfactory for the 10^5–10^7 cycle range.

The principal wear mechanism for glass reinforced nylon type characters was found to be contact fatigue at the end of useful wear life. Subsurface cracks would open up in the matrix material, and large pieces of the surface would eventually depart. Other wear phenomena typically arising in polymers include those of a thermal nature; small, black charred particles of the nylon matrix were seen on the impacted surface in the above wear tests. On the other hand, an impacting hammer surface may undergo surface fatigue (pitting) due to contact with the hard glass reinforcing elements of the impacted polymer. Small tangential oscillations during impacting can also cause fretting wear of the hammers.

Plated polymer type characters have been used to delay wear of the polymer base. Plating needs to be both ductile and sufficiently strong (i.e., have a high enough yield point). Six nickel-plated reinforced polymers have been investigated as print characters [2]; cracking was found to initiate in the plating, from the polymer-plating boundary.

Discussion and conclusions

A successful method of contact stress analysis for type, using linear programming, has been described. The method compares favorably with point matching, which has been used earlier to find contact pressure distributions under flat and round-ended indenters and type [10]. The present method is automatic; it also finds the contact area for a given load by an optimization principle. This is useful when the contact force is not *a priori* known to cause a full print mark. The numerical analysis indicates that a doubly convex curvature would tend to produce minimum pressure.

The relation between contact pressure and sharpness of print [1] is known to be approximately linear in the pressure domain bordered by the threshold pressure and a saturation pressure. Thus the predictions of a theoretical pressure distribution can be checked against the optical reflectivity (inverse of sharpness) of an actual print mark. By regulating the impact force in a print mechanism, such a relation is indeed easily verified. Glass-filled nylon print characters of concave, cylindrical curvature, impacting against flat surfaces of platen-supported paper, tend to produce a mark that is sharper at the top and the bottom edges; if the peak impact force is marginal, character definition may altogether be incomplete. To aid future type designs, the analytical optimization procedure should be used alone, or perhaps be coupled with reflectivity measurements. The latter should be performed both in a regional sense (x, y plane) and in the depth of a several-part form. Thus the correct pressure distributions and print quality would be obtained.

It is apparent that the prediction of impact forces must involve complex viscothermoelastic stress analysis. Only experimental measurement by force transducer is described in this paper. The problem is important because both print quality and wear are generally affected. For a more general evaluation (including flexure of the type), the finite element method [11] seems appropriate.

Bending stresses must be considered when the platen stiffness is comparable to or greater than the type stiffness. H. C. Lee has performed such an analysis [12] in which he considered the type as a beam on an elastic foundation (i.e., the platen); the bending stresses in the type were found to decrease when the platen stiffness increased, so as to prevent excessive flexure of the type.

Type systems where superimposed sliding motion during impact is very significant are sensitive to abrasive wear. A quasi-static approximate impact analysis may be performed, decoupling the tangential and normal approach components [3]. An exact solution for spheres has been recently published by Maw, Barber, and Fawcett [13]. An abrasive impact wear analysis [8] uses the impact force variation (P vs t) during impact, but is not concerned with pressure distribution since the type character tends to wear to a conformal geometry, parallel to the platen.

When printing is accomplished through normal impact, fatigue wear assumes greater importance. The pressure distribution is now crucial. Wear phenomena have been observed to appear earlier in highly stressed regions such as the flange tips of cylindrically concave "I" characters.

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