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# Steady Axisymmetric Solutions for Pressurized Rotating Flexible Disk Packs

**Abstract:** Steady solutions are obtained for undisturbed pressurized flexible disk packs of various configurations. Simplified fluid equations of motion are coupled with the equations governing the deflection of flexible membranes in order to arrive at the differential equations for the spacing between the disks of the pack. A simple asymptotic-solution formula is derived for computing the dependence of the spacing between two neighboring disks on air density and viscosity, the volume rate of flow of air between the two disks, the angular speed of rotation, and the radial position on the disks.

## Introduction

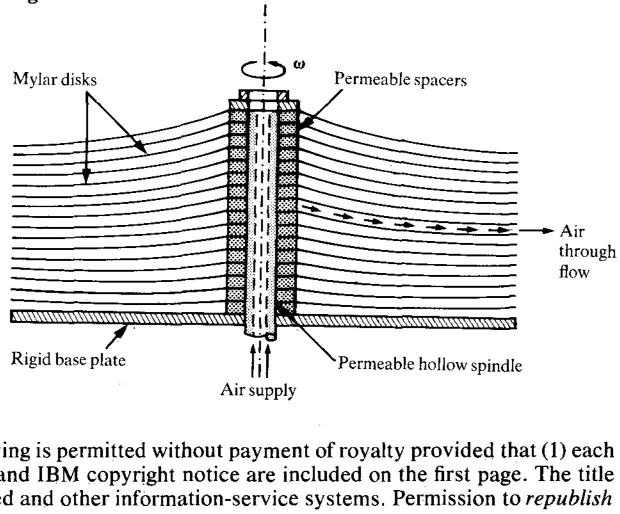
A potentially useful configuration for high-density data storage is the stacked flexible disk file configuration, consisting of a large number of concentrically mounted disks on a hollow spindle next to a flat, rigid base plate. The spacing between the disks, which rotate with constant angular velocity $\omega$, can be controlled by permeable spacers through which a controlled quantity of air is delivered between each of the disks. A cross-sectional sketch of such a configuration is shown in Fig. 1 [1].

During operation the disk pack must be entered at a preselected disk by a read-write element suspended on a suitably controlled inserter. Such a device will open the pack, thereby disturbing the steady rotating configuration. Since the steady spacing of the disks determines the location of the particular disk to be addressed, it is necessary for the operation of the disk file to know accurately the position of each disk as a function of flow rate and rotational speed.

In this paper, attention is confined to the study of the steady configuration of the undisturbed pack. Fortunately, the fundamental problem of a single disk rotating next to a fixed base plate has been studied both theoretically and experimentally by Pelech and Shapiro [2]. They treated the air between the base plate and disk as a viscous, incompressible fluid and employed the appropriate form of the Navier-Stokes equations of motion. These equations were linearized for certain limits on the physical variables by use of arguments based on dimensional analysis. The disk was modeled as an elastic membrane (i.e., having no bending stiffness), and the fluid and disk equations of motion were coupled through the fluid pressure. A single nonlinear differential equation was derived which governs the spacing between the membrane and the base plate, and this equation was solved numerically for several chosen values of the parameters in the problem.

The methods of analysis employed by Pelech and Shapiro are applied here to several different configurations occurring in connection with the stacked flexible disk file. After reviewing the analysis in [2] for a single membrane with a fixed base, the problem of a single membrane with a rotating base plate is considered. Thereafter, two rotat-

**Figure 1** Cross-sectional view of stacked flexible disk file.



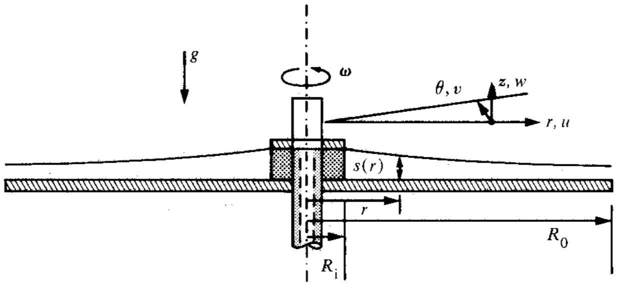



**Figure 2** Single membrane adjacent to rigid wall, rotating on a thin air film.

ing membranes with no base plate are studied, and, finally, the multimembrane configuration of the stacked flexible disk pack is analyzed for both fixed and rotating base plates.

## Single membrane, fixed base plate

Here we review the solution obtained in [2] for a single membrane rotating on a thin air film next to a fixed base plate, as shown in Fig. 2. Let $r$, $\theta$, $z$ be cylindrical polar coordinates and let $u$, $v$, $w$ be the components of the fluid velocity vector in these directions. The solution to be obtained is axisymmetric and therefore has no dependence on $\theta$. Let $\omega$ denote the angular speed and $s(r)$ the spacing between the membrane and the base plate at radial position $r$. Let $R_i$ and $R_0$ denote the radius at which the membrane is fixed to the spindle and the outer membrane radius.

The Navier-Stokes equations of motion of the fluid under conditions of axisymmetry and incompressibility appear then as

$$\rho\left(u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial r} + \mu\left[\frac{\partial^2 u}{\partial r^2} + \left(\frac{1}{r}\right)\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right]; \tag{1a}$$

$$\rho\left(u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right) = \mu\left[\frac{\partial^2 v}{\partial r^2} + \left(\frac{1}{r}\right)\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right]; \tag{1b}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial r^2} + \left(\frac{1}{r}\right)\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right], \tag{1c}$$

and the corresponding incompressibility constraint is

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

where $\rho$, $\mu$ and $P$ denote the fluid density, viscosity and fluid pressure, respectively. In [2] it is shown for this configuration that if $s(r)$, $R_0$, $\omega$, $\rho$, $\mu$ and the net volume rate of outflow $Q$ satisfy the inequalities

$$\frac{s}{R_0} \ll 1, \left(\frac{\rho\omega R_0^2}{\mu}\right)^2\left(\frac{s}{R_0}\right)^4 \ll 1, \frac{Q}{\omega R_0^2 s} \ll 1, \tag{3}$$

then the pressure $P$ can be considered to be independent of $z$, the velocity component $w$ is negligible compared to $u$, $v$ and (1a), (1b) can be replaced by

$$-\rho\frac{v^2}{r} = -\frac{dP}{dr} + \mu\frac{\partial^2 u}{\partial z^2};$$

$$\frac{\partial^2 v}{\partial z^2} = 0. \tag{4}$$

The volume rate of flow $Q$ is given in terms of $u$ by

$$Q = 2\pi r\int_0^{s(r)} u\,dz. \tag{5}$$

If the membrane is considered to be elastic and its deviation from a flat configuration is assumed to be infinitesimal, then the equation of motion for the membrane stress is the same as that for a rotating flat disk, i.e.,

$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta + \omega^2 r^2\rho_D = 0, \tag{6}$$

where $\sigma_r$ and $\sigma_\theta$ are the radial and tangential stresses, respectively, while $\rho_D$ is the disk density. The solution satisfying zero radial displacement at $r = R_i$ and zero radial stress at $r = R_0$ is obtained from (6) with use of the appropriate stress-strain relations and yields

$$\hat{\sigma}_r = \left(\frac{3+\nu}{8}\right)\frac{1}{\hat{r}^2}[\alpha + (1-\alpha)\hat{r}^2 - \hat{r}^4];$$

$$\hat{\sigma}_\theta = \left(\frac{3+\nu}{8}\right)\frac{1}{\hat{r}^2} \times\left[-\alpha + (1-\alpha)\hat{r}^2 - \left(\frac{1+3\nu}{3+\nu}\right)\hat{r}^4\right], \tag{7}$$

in which

$$\alpha = \frac{1 - [(1+\nu)/(3+\nu)](R_i^2/R_0^2)}{[(1+\nu)/(1-\nu)](R_0^2/R_i^2) + 1};$$

$$\hat{\sigma} = \frac{\sigma}{\omega^2\rho_D R_0^2};\ \hat{r} = \frac{r}{R_0}, \tag{8}$$

where $\nu$ represents the Poisson ratio of the membrane. Equation (7) gives the nonuniform tensions in the membrane, which is loaded transversely by gravity forces and the pressures differential across it. The equation governing the transverse steady, axisymmetric deflection of the membrane under these conditions is (see, for example, Simmonds [3] or Eversman [4])

$$\left(\frac{1}{r}\right)\frac{d}{dr}\left(r\sigma_r\frac{ds}{dr}\right) = -q, \tag{9}$$

where $q$ is force/volume in the $z$ direction applied to the membrane. By use of (6) in (9) we obtain in dimensionless form

$$-\hat{\sigma}_r\frac{d^2\hat{s}}{d\hat{r}^2} + \left(\hat{r} - \frac{\hat{\sigma}_\theta}{\hat{r}}\right)\frac{d\hat{s}}{d\hat{r}} = \Delta\hat{P} - \hat{g}, \tag{10}$$

in which

$$\hat{s} = s/R_0;\ \Delta\hat{P} = \frac{\Delta P}{\omega^2\rho_D R_0 t};\ \hat{g} = \frac{g}{R_0\omega^2}, \tag{11}$$

and $\Delta P$ denotes the pressure below minus the pressure above the membrane and $t$ is the membrane thickness. For the case under consideration it is pointed out in [2] that the pressure above the rotating membrane is essentially ambient so that $\Delta P$ in (11) is the same as $P$ in (1), i.e., the fluid pressure above ambient. [Note: (Eqs. (28) of [2] and (2) of [5] appear to be in error in that they do not contain the $\hat{\sigma}_\theta/r$ coefficient of $d\hat{s}/d\hat{r}$ in (11).]

The boundary conditions for the fluid between a fixed base plate and rotating membrane are

$$\begin{aligned} u &= 0 \qquad \text{at } z = 0,\ s(r); \\ v &= 0 \qquad \text{at } z = 0, \\ v &= \omega r \qquad \text{at } z = s(r). \end{aligned} \tag{12}$$

The integration of (4) subject to (12) yields

$$\begin{aligned} u &= \left(\frac{s^2}{2\mu}\right)\frac{dP}{dr}\left[\left(\frac{z}{s}\right)^2 - \left(\frac{z}{s}\right)\right] \\ &\quad - \rho\frac{\omega^2 rs^2}{12\mu}\left[\left(\frac{z}{s}\right)^4 - \left(\frac{z}{s}\right)\right]; \\ v &= \left(\frac{z}{s}\right)\omega r. \end{aligned} \tag{13}$$

By use of (4) the pressure gradient and flow rate $Q$ of this velocity field are related by

$$\frac{dP}{dr} = \frac{3}{10}\rho\omega^2 r - \frac{6\mu Q}{\pi r s^3}. \tag{14}$$

The coupling of the fluid equations and membrane deflection equation is accomplished by substituting (14) into $d/dr$ of (10), which yields

$$\begin{aligned} &\hat{\sigma}_r\frac{d^3\hat{s}}{d\hat{r}^3} + \left(\frac{d\hat{\sigma}_r}{d\hat{r}} - \hat{r} + \frac{\hat{\sigma}_\theta}{\hat{r}}\right)\frac{d^2\hat{s}}{d\hat{r}^2} \\ &\quad + \left[\left(\frac{1}{\hat{r}}\right)\frac{d\hat{\sigma}_\theta}{d\hat{r}} - \frac{\hat{\sigma}_\theta}{\hat{r}^2} - 1\right]\frac{d\hat{s}}{d\hat{r}} \\ &= \frac{6\mu Q}{\pi\rho_D\omega^2 R_0^5\,\hat{t}\hat{r}\hat{s}^3} - \frac{3\rho\hat{r}}{10\rho_D\hat{t}}. \end{aligned} \tag{15}$$

Equation (15) is a third order nonlinear ordinary differential equation for determining $s(r)$. The boundary conditions are obtained from knowledge of $s(R_i)$, $P(R_0)$, and the requirement that $d^2s/dr^2$ must remain bounded. In [2] it is shown that these conditions lead alternatively to an initial value problem for (15). The initial conditions are placed on $s$, $ds/dr$ and $d^2s/dr^2$ at $r = R_0$. By use of (7), (10), and (14) these conditions can be written in the form

$$\begin{aligned} \hat{s}(1) &= \text{prescribed}; \\ \hat{s}'(1) &= \frac{\hat{P}(1) - \hat{g}}{1 - \hat{\sigma}_\theta(1)} \\ &= \frac{\hat{P}(1) - \hat{g}}{1 - [(3+\nu)(1-2\alpha) - (1+3\nu)]/8}; \\ \hat{s}''(1) &= \frac{[1 + \hat{\sigma}_\theta(1) - \hat{\sigma}_\theta'(1)]\hat{s}'(1) - \hat{P}'(1)}{\hat{\sigma}_r'(1) - 1 + \hat{\sigma}_\theta(1)} \\ &= \frac{\left\{\dfrac{8 + [(3+\nu)(2-5\alpha) + (1+3\nu)]}{8 - [(3+\nu)(1-2\alpha) - (1+3\nu)]}\right\}[\hat{P}(1) - \hat{g}]}{-1 - [(3+\nu)(1+4\alpha) + (1+3\nu)]/8} \\ &\quad + \frac{\dfrac{6\mu Q}{\pi^2\rho_D\hat{t}\hat{s}^3(1)R_0^5} - \dfrac{3\rho\hat{r}}{10\rho_D\hat{t}}}{-1 - [(3+\nu)(1+4\alpha) + (1+3\nu)]/8}. \end{aligned} \tag{16}$$

Once $s(r_0)$ is chosen and $Q$, $\omega$, and $P(r_0)$ are fixed, all the initial data in (16) are determined. Then (15) can be integrated numerically for $R_i < r < R_0$. The value of $s(R_i)$ obtained in this way can be compared with the known value. Numerical solutions were presented in [2] based on this technique for many different values of the parameters $Q$, $\omega$, and $R_0$. These solutions generally showed excellent agreement with their experimental results.

It was observed in [2] that there is a self-regulating tendency for the spacing between the membrane and the base plate for sufficiently large values of $r$ and $Q$. Such an asymptotic solution results from (15) under these conditions when the terms on the right-hand side are of larger order than those on the left-hand side. The solution obtained is the same that results from setting $dP/dr$ equal to zero in (14), i.e.,

$$s = \left(\frac{20\mu Q}{\pi\rho\omega^2}\right)^{\frac{1}{3}} r^{-\frac{2}{3}}, \tag{17}$$

which, from (15), should be valid provided that, in addition to (3), the inequality

$$\left(\frac{\rho_D\omega^2 R_0^5}{\mu Q}\right)\left(\frac{s_0}{R_0}\right)^4\left(\frac{t}{R_0}\right) \ll 1, \tag{18}$$

is also satisfied. The similarity solution (17) predicts that $s \propto Q^{\frac{1}{3}}$ at fixed $r$ and $s \propto r^{-\frac{2}{3}}$ at fixed $Q$. The experimen-

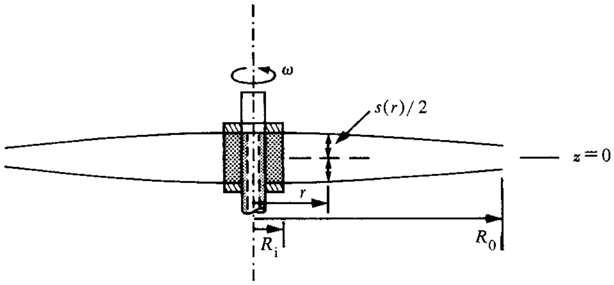



**Figure 3** Two rotating membranes adjacent to each other.

tal results in [2] showed good agreement also with these relationships in the outer portions of the membrane. We note that the solution (17) is independent of the gravity force.

## Single membrane, rotating base plate

This configuration is the same as in Fig. 2, but now the membrane and the base plate are assumed to rotate with the same angular speed $\omega$. Equations (1)–(8) apply here also but the boundary conditions for the fluid become

$$u = 0,\ v = \omega r \qquad \text{at } z = 0,\ s(r) \tag{19}$$

in place of (12). The integration of (4) subject to (19) yields

$$u = \frac{s^2}{2\mu}\left(\frac{dP}{dr} - \rho\omega^2 r\right)\left[\left(\frac{z}{s}\right)^2 - \frac{z}{s}\right], \qquad v = \omega r, \tag{20}$$

which, with (5), gives

$$\frac{dP}{dr} = \rho\omega^2 r - \frac{6\mu Q}{\pi r s^3}. \tag{21}$$

When (21) is compared to (14), we see that it differs only in that $3\rho/10$ has been replaced by $\rho$. Likewise, the equations that replace (15), (16) for this case can be obtained from (15), (16) by substituting $\rho$ for $3\rho/10$. Furthermore, the asymptotic solution that replaces (17) is

$$s = \left(\frac{6\mu Q}{\pi\rho\omega^2}\right)^{\frac{1}{3}} r^{-\frac{2}{3}}. \tag{22}$$

Based on the similarity solutions (17), (22) we should expect

$$s_{\text{rotating base}} = \left(\frac{3}{10}\right)^{\frac{1}{3}} s_{\text{fixed base}}, \tag{23}$$

provided (3), (18) are all satisfied. These asymptotic solutions are restricted to the outer regions of the membrane since near the hub the boundary conditions predominate instead of the differential equation (15).

## Two membranes, no base plate

A sketch of this configuration is shown in Fig. 3. The spacing between the two membranes is $s(r)$, and there is symmetry about the plane $z = 0$ if gravity forces are neglected. Equations (4) again govern the fluid velocity provided inequalities (3) remain satisfied. The fluid boundary conditions now become

$$u = 0,\ v = \omega r \qquad \text{at } z = \pm s/2, \tag{24}$$

and (13) is replaced here by

$$u = \frac{s^2}{8\mu}\left(\frac{dP}{dr} - \rho\omega^2 r\right)\left[\left(\frac{2z}{s}\right)^2 - 1\right], \qquad v = \omega r. \tag{25}$$

We then obtain from (5) with limits $-s/2$ to $s/2$, using this flow field,

$$\frac{dP}{dr} = \rho\omega^2 r - \frac{6\mu Q}{\pi r s^3}, \tag{26}$$

which is exactly the same as (21). Thus, the treatment from here on for obtaining the spacing between two rotating membranes is the same as that for the spacing between one membrane and a rotating base plate. In particular, the asymptotic solution here is also

$$s = \left(\frac{6\mu Q}{\pi\rho\omega^2}\right)^{\frac{1}{3}} r^{-\frac{2}{3}}, \tag{27}$$

i.e., the same as (22). Based on this solution we expect

$$s_{2\text{ membranes}} = s_{\text{rotating base}} = \left(\frac{3}{10}\right)^{\frac{1}{3}} s_{\text{fixed base}}. \tag{28}$$

## *N* membranes, fixed base plate

A cross-sectional sketch of this configuration appears in Fig. 4.

We assume, in addition to small displacements, that the inequalities

$$\frac{s_\alpha - s_{\alpha-1}}{R_0} \ll 1;$$

$$\left(\frac{\rho\omega R_0^2}{\mu}\right)^2\left(\frac{s_\alpha - s_{\alpha-1}}{R_0}\right)^4 \ll 1;$$

$$\frac{Q_\alpha}{\omega R_0^2\,(s_\alpha - s_{\alpha-1})} \ll 1, \tag{29}$$

are satisfied for $\alpha = 1, 2, \cdots, N$. Then (4) holds for each fluid region, or

$$-\rho\frac{v_\alpha^2}{r} = -\frac{dP_\alpha}{dr} + \mu\frac{\partial^2 u_\alpha}{\partial z^2},$$

$$\frac{\partial^2 v_\alpha}{\partial z^2} = 0, \qquad \alpha = 1, 2, \cdots, N. \tag{30}$$

The fluid boundary conditions are

$$u_\alpha = 0 \qquad \text{at} \quad z = s_\alpha, s_{\alpha-1}, \alpha = 1, 2, \cdots, N;$$

$$v_\alpha = \omega r \qquad \text{at} \quad z = s_\alpha, s_{\alpha-1}, \alpha = 2, 3, \cdots, N;$$

$$v_1 = \omega r \qquad \text{at} \quad z = s_1. \tag{31}$$

The integration of (30) subject to (31) gives

$$u_\alpha = \frac{1}{2\mu}\left(\frac{dP_\alpha}{dr} - \rho\omega^2 r\right)[z^2 - (s_\alpha + s_{\alpha-1})z + s_\alpha - s_{\alpha-1}],$$

$$\alpha = 2, 3, \cdots, N;$$

$$v_\alpha = \omega r;$$

$$u_1 = \left(\frac{s_1^2}{2\mu}\right)\frac{dP_1}{dr}\left[\left(\frac{z}{s_1}\right)^2 - \left(\frac{z}{s_1}\right)\right] - \frac{\rho\omega^2 r s_1^2}{12\mu}\left[\left(\frac{z}{s_1}\right)^4 - \left(\frac{z}{s_1}\right)\right];$$

$$v_1 = \frac{z}{s_1}\,\omega r. \tag{32}$$

Use of (32) in

$$Q_\alpha = 2\pi r \int_{s_{\alpha-1}}^{s_\alpha} u_\alpha dz \tag{33}$$

yields

$$\frac{dP_\alpha}{dr} = \rho\omega^2 r - \frac{6\mu Q_\alpha}{\pi r(s_\alpha - s_{\alpha-1})^3}, \qquad \alpha = 2, 3, \cdots, N;$$

$$\frac{dP_1}{dr} = \frac{3}{10}\rho\omega^2 r - \frac{6\mu Q_1}{\pi r s_1^3}. \tag{34}$$

Now each membrane continues to obey (10) with $\sigma_r$ given by (7), (8) while $\Delta P$ represents the pressure below minus the pressure above. Therefore, (10) becomes

$$-\hat{\sigma}_r \frac{d^2\hat{s}_\alpha}{d\hat{r}^2} + \left(\hat{r} - \frac{\hat{\sigma}_\theta}{\hat{r}}\right)\frac{d\hat{s}_\alpha}{d\hat{r}} = \hat{P}_\alpha - \hat{P}_{\alpha+1} - \hat{g},$$

$$\alpha = 1, 2, \cdots, N, \tag{35}$$

where $P_{N+1}$ is ambient. Use of (34) in $d/dr$ of (35) yields

$$\hat{\sigma}_r \frac{d^3\hat{s}_\alpha}{d\hat{r}^3} + \left(\frac{d}{d\hat{r}}\hat{\sigma}_r - \hat{r} + \frac{\hat{\sigma}_\theta}{\hat{r}}\right)\frac{d^2\hat{s}_\alpha}{d\hat{r}^2} + \left[\left(\frac{1}{\hat{r}}\right)\frac{d\hat{\sigma}_\theta}{d\hat{r}} - \frac{\hat{\sigma}_\theta}{\hat{r}^2} - 1\right]\frac{d\hat{s}_\alpha}{d\hat{r}}$$

$$= \begin{cases} \dfrac{6\mu}{\pi\rho_D\omega^2 R_0^5 \hat{r}\hat{t}}\left[\dfrac{Q_N}{(\hat{s}_N - \hat{s}_{N-1})^3}\right] - \dfrac{\rho}{\rho_D}\left(\dfrac{\hat{r}}{\hat{t}}\right), & \alpha = N \\ \left(\dfrac{6\mu}{\pi\rho_D\omega^2 R_0^5 \hat{r}\hat{t}}\right)\dfrac{Q_\alpha}{(\hat{s}_\alpha - \hat{s}_{\alpha-1})^3} - \dfrac{Q_{\alpha+1}}{(\hat{s}_{\alpha+1} - \hat{s}_\alpha)^3}, & \alpha = 2, 3, \cdots, N-1 \\ \dfrac{7\rho\hat{r}}{10\rho_D\hat{t}} + \dfrac{6\mu}{\pi\rho_D\omega^2 R_0^5 \hat{r}\hat{t}}\left[\dfrac{Q_1}{\hat{s}_1^3} - \dfrac{Q_2}{(\hat{s}_2 - \hat{s}_1)^3}\right], & \alpha = 1. \end{cases} \tag{36}$$

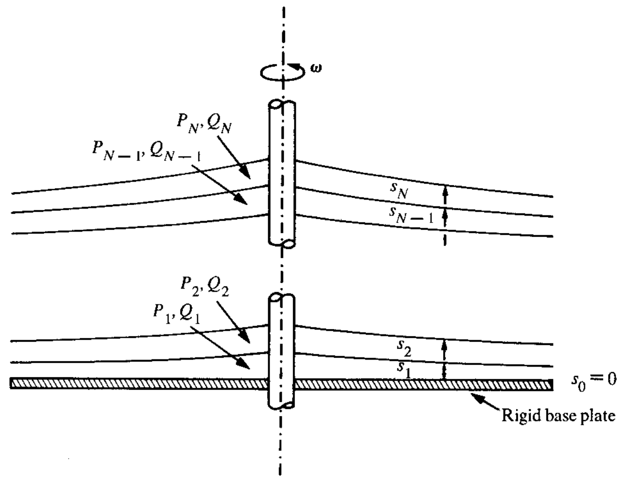



**Figure 4** Configuration of stacked flexible disk pack with fixed base plate.

The initial conditions analogous to (16) could also be listed. We would then have a coupled system of $N$ third-order nonlinear ordinary differential equations with proper initial data for determining $\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_N$. The numerical task of solving such a system for large $N$ ($N = 100$, say) would be formidable. We can, however, get an asymptotic solution from (36) also. Assuming the terms on the left-hand side are all of order

$$\frac{s_\alpha(R_0) - s_\alpha(R_i)}{R_0 - R_i}$$

and that the terms on the right-hand side are of higher order, we obtain from (36)

$$s_N - s_{N-1} = \left(\frac{6\mu}{\pi\rho\omega^2 r^2}\right)^{\frac{1}{3}} Q_N^{\frac{1}{3}};$$

$$s_\alpha - s_{\alpha-1} = (s_{\alpha+1} - s_\alpha)(Q_\alpha/Q_{\alpha+1})^{\frac{1}{3}},$$

$$\alpha = 2, 3, \cdots, N-1; \tag{37a}$$

$$\frac{Q_1}{s_1^3} = \frac{Q_2}{(s_2 - s_1)^3} - \frac{7\pi\rho\omega^2 r^2}{60\mu}, \tag{37b}$$

provided

$$\left(\frac{s_N^0 - s_N^i}{R_0 - R_i}\right)\left(\frac{s_N^0 - s_{N-1}^0}{R_0}\right)^3 \frac{\rho_D\omega^2 R_0^4 t}{\mu Q_N} \ll 1;$$

$$\left(\frac{s_\alpha - s_\alpha^i}{R_0 - R_i}\right)\rho_D\omega^2 R_0^4 t \times \left[\frac{\mu Q_\alpha R_0^3}{(s_\alpha^0 - s_{\alpha-1}^0)^3} - \frac{\mu Q_{\alpha+1} R_0^3}{(s_{\alpha+1}^0 - s_\alpha^0)^3}\right]^{-1} \ll 1,$$

$$\alpha = 2, 3, \cdots, N-1;$$

$$\left(\frac{s_1^0 - s_1^i}{R_0 - R_i}\right)\rho_D\omega^2 R_0^4 t\left[\frac{\mu Q_1 R_0^3}{(s_1^0)^3} - \frac{\mu Q_2 R_0^3}{(s_2^0 - s_1^0)^3}\right]^{-1} \ll 1, \tag{38}$$

where $s_\alpha^0$, $s_\alpha^i$ stand for $s_\alpha(R_0)$, $s_\alpha(R_i)$. By use of (37a) with (37b) successively, we obtain the simpler result

$$s_\alpha - s_{\alpha-1} = \left(\frac{6\mu}{\pi\rho\omega^2 r^2}\right)^{\frac{1}{3}} Q_\alpha^{\frac{2}{3}}, \qquad \alpha = 2, 3, \cdots, N;$$

$$s_1 = \left(\frac{20\mu}{\pi\rho\omega^2 r^2}\right)^{\frac{1}{3}} Q_1^{\frac{2}{3}}. \tag{39}$$

By comparing (39) with (22), (17) we see that the similarity solutions predict that the spacing between the first membrane and the fixed base is the same for one membrane as for a stack of $N$ membranes and that the spacing between adjacent membranes is the same as that between one membrane and a rotating base plate.

### *N* membranes, rotating base

The configuration is the same as that shown in Fig. 4, except that we now assume that the base plate rotates with the stack of membranes. Under assumptions (29) the fluid equations again become (30) and the boundary conditions are as in (31), except that $v_1 = \omega r$ at $z = 0$ also. The terms $u_\alpha$ and $v_\alpha$ are now given by $(32)_{1,2}$ for $\alpha = 1, 2, \cdots, N$ and $dP_\alpha/dr$ is given by $(35)_1$ for $\alpha = 1, 2, \cdots, N$. The membrane deflection equations are given by $(36)_1$ for $\alpha = N$ and $(36)_2$ for $\alpha = 1, 2, \cdots, N - 1$. Finally, if inequalities $(38)_1$, $(38)_2$ are valid for $\alpha = 1, 2, \cdots, N$, then the similarity solution is

$$s_\alpha - s_{\alpha-1} = \left(\frac{6\mu}{\pi\rho\omega^2 r^2}\right)^{\frac{1}{3}} Q_\alpha^{\frac{1}{3}}, \qquad \alpha = 1, 2, \cdots, N. \tag{40}$$

### Discussion and conclusions

The asymptotic solution (40) essentially results from setting $dP_\alpha/dr$ in (34) equal to zero. This is a statement that the spacing $s_\alpha - s_{\alpha-1}$ regulates itself so that the centrifugal pressure increase in $r$ is balanced by the pressure drop in $r$ associated with the volume through flow $Q_\alpha$. Since the pressure (from ambient) at $r = R_0$ is zero, the asymptotic solution is a statement that the pressure remains very small in the outer regions of the pack. Thus, while the pack thickness may be increasing with $r$ close to the hub if enough air is forced through, the asymptotic solution predicts that the pack thickness will be decreasing in $r$ according to $r^{-\frac{2}{3}}$ in the outer regions of the pack. Another prediction of the solution (40) is that the pack thickness should remain constant for changing $Q_\alpha$, $\omega$ provided $Q_\alpha/\omega^2$ remains constant.

It should be emphasized that the validity of Eq. (15) [or (36)] is restricted to the ranges of the physical parameters as expressed by the inequalities (3) [or (29)]. Furthermore, the validity of the similarity solution (17) [or (40)] rests on the satisfaction of the additional inequalities (18) [or (38)]. The asymptotic solution (40) should be applicable for the prediction of steady pack profiles in the outer circumferential regions of the pack.

### References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