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Digital Technique for Generating Synthetic Aperture Radar Images

Abstract: This paper describes a digital processing method applicable to a synthetic aperture radar, to be carried by the space shuttle or by satellites. The method uses an earth-fixed coordinate system in which corrective procedures are invoked to compensate for errors introduced by the satellite motion, earth curvature, and wavefront curvature. Among the compensations discussed are those of the coordinate system, skewness, roll, pitch, yaw, earth rotation, and others. The application of a Fast Fourier Transform in the numerical processing of the two-dimensional convolution is discussed in detail.

Introduction

Synthetic aperture radar (SAR) is a system capable of high resolution. It is a relatively new development which will soon join an array of other instruments in space, such as high resolution scanners, for observation, mapping, and imaging of the earth's surface. The SAR is not only a logical complement in the spectral domain to these scanners; it has its own particular application because of its all-weather capability.

In the space applications now being contemplated, a fairly high resolution is required; many corrections not needed in aircraft applications must be applied. For example, the earth's curvature must be considered, as well as curvature of the wavefront. The latter becomes particularly important when so-called multiple looks are used to rid the image of a speckle effect [1]. In that case, the size of the synthetic antenna might be increased several times in accordance with the resolution requirements.

Furthermore, the high altitude at which satellites travel will require the synthetic aperture radar to make image corrections to compensate for undesirable spacecraft motion [2, 3]. Even if no such motion is present, the system must correct for the earth's rotation.

The method followed here assumes a coordinate system fixed to the earth's surface. This leads to a treatment of the problem in the time domain. If a coordinate system had been chosen fixed to the radar, the problem would have been formulated in the frequency domain.

This paper describes a possible digital data processing method to obtain the high resolution required. After a discussion of the limitations of conventional radar, the principles of a synthetic aperture and chirp radar are briefly explained. The equivalence of the time domain formulation and the Doppler shift method is shown. A detailed derivation of the signal is given, and it is shown that a two-dimensional convolution integral may be used to obtain an approximation of the reflective properties of the surface. This approximation gives rise to a correction for wavefront curvature. Furthermore, the multiple look feature is formulated, and a method of correcting the wavefront curvature is suggested. Spacecraft motion corrections are given together with corrections for earth curvature and rotation. Finally, an estimate is made of the number of multiplications necessary for digital data processing.

Limitations of conventional radar

The prime consideration of radar as a target position measurement tool, as compared with other devices, lies in its ability to make direct measurements of radial range in terms of the round-trip time delay of the propagated signal. However, when conventional radar is used as an imaging device, its resolution is limited in both the azimuthal and the radial (i.e., along-track and cross-track, respectively) directions.

The limiting size of an object discernible on the ground is given by Rayleigh's expression [4],

$$\delta_a \approx \frac{r\lambda}{L}, \tag{1}$$

where r is the distance to the object measured from the antenna, L is a linear dimension of the antenna, and λ is the wavelength. For example, let $r = 200$ km, $\lambda = 0.23$ m, and $L = 12$ m, which is equivalent to a resolution of 4000 m. For improved resolution, it is necessary to increase the size of the antenna.

Separation of two targets in a radial direction depends on the length of the pulse. Radar receivers use filters that

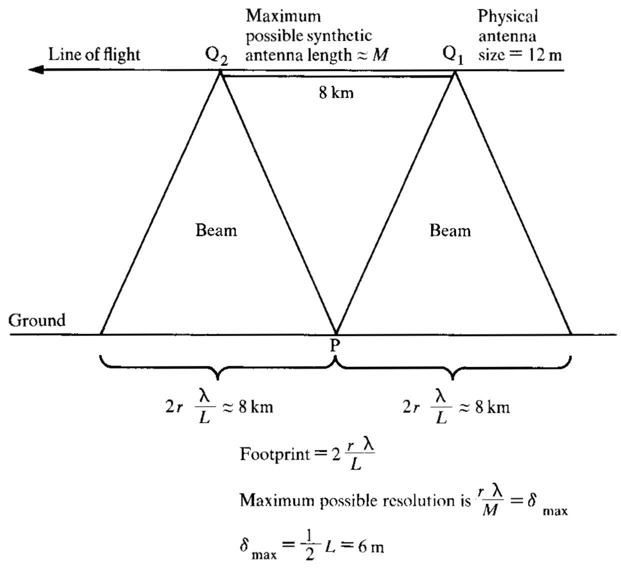

Figure 1 Synthetic aperture.

do not necessarily retain the shape of the pulse; rather, they maximize the signal-to-noise ratio. If the pulse duration is τ, the velocity of light being c (3×10^5 km/s), the radial resolution is given approximately by

$$\delta_r \approx \frac{c\tau}{2}. \tag{2}$$

For a 1-μs pulse, the radial resolution is 150 m. Since both resolutions are unsatisfactory for imaging applications, a means for improving the resolution must be found without increasing the physical size of the antenna.

Principles of synthetic aperture radar

The resolution deficiency of conventional radar systems may be overcome by the following techniques:

Azimuthal resolution may be improved by artificially increasing the length of the antenna. A simplified one-dimensional representation of this process is shown in Fig. 1. A satellite moves along the indicated line of flight. When it reaches point Q_1, the beam transmitted by its antenna commences to illuminate point P. The reflected, or returned, signal continues to return from point P until the vehicle reaches point Q_2. By properly filtering the signal recorded between points Q_1 and Q_2, the reflective properties of P can be determined with a resolution corresponding to that produced by an antenna of length $M = \overline{Q_1Q_2}$.

Radial resolution may be enhanced by a technique known as *pulse chirping*, in which conventional radar pulses are transmitted in rectangular pulse trains, with a sinusoidal signal. Pulse chirping [5] is accomplished by frequency modulation of the signal in the rectangular pulse, where each instantaneous frequency determines a particular part of the pulse. Usually linear frequency modulation is used and the pulse duration is increased somewhat. Thus,

$$f = f_b + \left(\frac{f_e - f_b}{\tau}\right) t,$$

where f indicates frequency, τ is the pulse width and t denotes time. The indices b and e refer to the beginning and end of the pulse, respectively.

Upon reception, the pulse is fed through a delay filter, where the frequency f is delayed according to

$$d = d_0 + \left(\frac{f - f_e}{f_b - f_e}\right) \tau.$$

The instantaneous frequency f_b, at the beginning of the pulse, is received first. However, it is delayed so that the end of the pulse, which arrives later from the same ground point, will coincide with the beginning of the pulse. This is true for each intermediate frequency, as is shown by the expression for the delay. The energy in the received pulse becomes concentrated, therefore, at the moment the instantaneous frequency f_e arrives (apart from a constant delay d_0).

The process described corresponds mathematically to a convolution, as is shown later. The ideal process can be approximated only because of the finite bandwidth of the receiver. If a resolution δ_r is required, it can be seen from Fig. 2 that

$$\tau = \frac{1}{\Delta f}.$$

Substitution into Eq. (2) gives

$$\Delta f \approx \frac{c}{2\delta_r}.$$

The increased resolution of the synthetic aperture is described mathematically in the same way as that obtained with the modulated chirp pulse in the radial direction. However, the frequency sweep in the former is due to Doppler shifting caused by the relative radial velocity of the spacecraft with respect to the individual ground elements. There are two different ways of describing the SAR that are completely equivalent. The first method uses the Doppler shifts, while the second method uses the difference in delay between transmission and reception of the wavelets. The latter method is used here.

The difference between the two methods is that in the Doppler description, the coordinate system moves with the spacecraft, while in the second method the coordinate system is fixed to the ground. That they are equivalent can be seen from Fig. 3, which shows the basic derivation of the Doppler effect. Suppose the spacecraft S located at point (x, y) is moving with velocity v. A reflector at point (x', y') reflects the wave transmitted from S. If the distance $\overline{SA}$ is r and the wave transmitted from S is given by $\cos \omega t$, then the received signal can be described by

$$\chi\,(x'y')\,\cos\left\{\omega\left[t - \frac{2\,r\,(x-x', y-y')}{c}\right] + \phi\right\}, \qquad (4)$$

where χ is a reflection coefficient. This is true in both coordinate systems because of the invariance of c.

(Modification of the signal due to chirping is not considered in this discussion.) The delay caused by a wave traveling with velocity c over a distance of $2r$ equals $2\,r/c$. The ϕ symbol represents the phase shift caused by reflection. Transformation to the spacecraft coordinates is accomplished by using the mapping $x - x' = vt$ and substituting this in Eq. (4).

By writing

$$r = [r_0^2 + (x - x')^2]^{\frac{1}{2}} = (r_0^2 + v^2t^2)^{\frac{1}{2}} \qquad (5)$$

and substituting this into Eq. (4), one obtains

$$\cos\left\{\omega\left[t - \frac{2}{c}\,(r_0^2 + v^2t^2)^{\frac{1}{2}}\right] + \phi\right\} = \cos\psi, \qquad (6)$$

with

$$\psi = \omega\left[t - \frac{2}{c}\,(r_0^2 + v^2t^2)^{\frac{1}{2}}\right] + \phi. \qquad (7)$$

The instantaneous frequency is defined as

$$\omega' = \frac{d\psi}{dt}. \qquad (8)$$

It follows from Eqs. (7) and (8) that

$$\frac{d\psi}{dt} = \omega - \frac{2\omega}{c}\,\frac{v^2t}{(r_0^2 + v^2t^2)^{\frac{1}{2}}} = \omega - \frac{2v\omega}{c}\,\frac{vt}{(r_0^2 + v^2t^2)^{\frac{1}{2}}}$$

$$= \omega - \left(\frac{2v\omega}{c}\right)\sin\eta, \qquad (9a)$$

which indicates that the apparent frequency is changed by an amount equal to

$$\Delta\omega = -\left(\frac{2v\omega}{c}\right)\sin\eta. \qquad (9b)$$

This is just the Doppler shift due to the relative radial velocity, which indicates that the two methods are equivalent.

This Doppler shift is twice that of the usual Doppler shift because it assumes a receiver approaching a source. This gives rise to a signal proportional to $\cos\omega[t - (r/c)]$, while the active radar shift is derived from $\cos\omega[t - (2r/c)]$. Finally, the relativistic correction factor $[1 - (v/c)^2]^{-\frac{1}{2}}$ is neglected. The coordinate system used in the following pages is the one fixed to the ground. Occasionally, however, reference is made to the Doppler point of view.

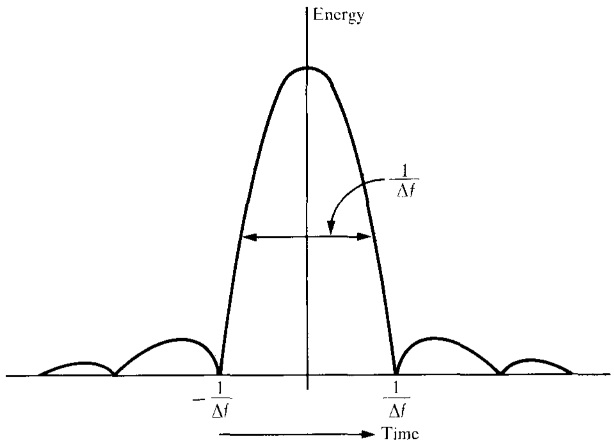

Figure 2 Contracted pulse shape.

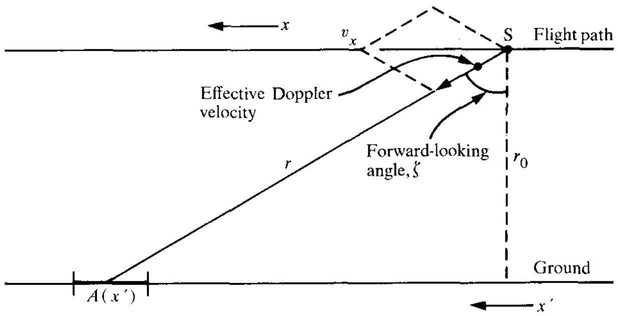

Figure 3 Effective Doppler velocity.

Transmitted and received signals

The transmitted signal is sinusoidal and is represented by $\cos\phi$ where ϕ is a function of time. In this case, the frequency is chosen to be a linear function of time as shown in Fig. 4.

During the transmission of a pulse of duration $\tau(s)$, the frequency is linearly modulated and can be represented by

$$\omega = \omega_0 + \alpha t \qquad (10)$$

and

$$jT + t_0 \le t \le jT + t_0 + \tau,$$

where j is a positive integer, $0 \le j$, and t_0 is the time at which the first pulse occurs.

Subsequently, the frequency returns to the value ω_0, remaining there for the remainder of the period T. That is, after one full period, the phase becomes

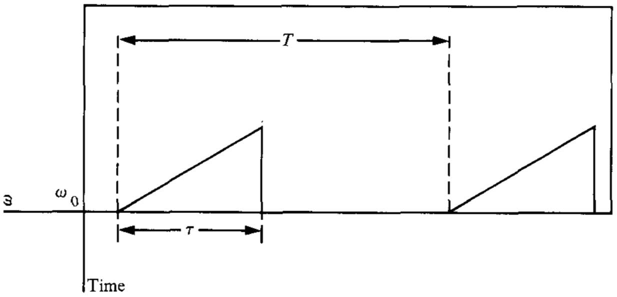

Figure 4 Chirp frequency as a function of time.

$$\phi = \phi_0 + \int_0^\tau (\omega_0 + \alpha t)\,dt + \omega_0(T - \tau)$$
$$= \phi_0 + \omega_0 T + \tfrac{1}{2}\alpha\tau^2, \qquad (11)$$

where ϕ_0 is the phase at the beginning of the frequency sweep.

At the beginning of the nth pulse, the phase is

$$\phi = \phi_0 + \omega_0(n-1)T + \tfrac{1}{2}(n-1)\alpha\tau^2, \qquad (12)$$

and at time t during the nth receiving period, the phase is

$$\phi_1 = \phi_0 + \omega_0 t + \tfrac{1}{2}n\alpha\tau^2, \qquad t \geq 0. \qquad (13)$$

The receiving period is the time during which the frequency is constant, while transmission is suppressed. At time t' during the mth transmission period, the phase is

$$\phi_2 = \phi_0 + \omega_0 t' + \tfrac{1}{2}(m-1)\alpha\tau^2 + \tfrac{1}{2}\alpha[t' - (m-1)T]^2,$$
$$t' \geq 0. \qquad (14)$$

Suppose that a signal, transmitted during the mth transmitting period at time t', is received at time t during the nth receiving period. This signal is

$$J|\chi| \cos(\phi_2 + \phi_\chi)\; dx'dy',$$

where $|\chi|dx'dy'$ is the fraction of the amplitude reflected per elementary area, ϕ_χ is the phase shift introduced upon reflection, J is a constant to be determined in a later section, and $dx'dy'$ is the elementary area. In order to register the phase of the signal, quadrature detection may be used [6]. In the quadrature detector, the signal is multiplied by $\cos\phi_1$, as well as by $\sin\phi_1$.

Upon multiplication by $\cos\phi_1$, the signal amplitude per unit area becomes

$$S^* = J|\chi| \cos\phi_1 \cos(\phi_2 + \phi_\chi)$$
$$= \tfrac{1}{2}J|\chi|[\cos(\phi_1 + \phi_2 + \phi_\chi) + \cos(\phi_1 - \phi_2 - \phi_\chi)].$$

The use of a low pass filter results in

$$S_1^* = \tfrac{1}{2}J|\chi| \cos(\phi_1 - \phi_2 - \phi_\chi)$$
$$= \tfrac{1}{2}J|\chi| \cos\{\omega_0(t - t') - \tfrac{1}{2}\alpha[t' - (m-1)T]^2 - \phi_\chi'\}, \qquad (15)$$

where

$$\phi_\chi' = \phi_\chi - \tfrac{1}{2}(n - m + 1)\alpha\tau^2.$$

If

$$\chi_c = |\chi| \cos\phi_\chi', \text{ and}$$
$$\chi_s = |\chi| \sin\phi_\chi',$$

then

$$S_1^* = \tfrac{1}{2}J\chi_c \cos\{\omega_0(t - t') - \tfrac{1}{2}\alpha[t' - (m-1)T]^2\}$$
$$- \tfrac{1}{2}J\chi_s \sin\{\omega_0(t - t') - \tfrac{1}{2}\alpha[t' - (m-1)T]^2\}. \qquad (16)$$

Similary, if the signal is multiplied by $\sin\phi_1$,

$$S_2^* = \tfrac{1}{2}J\chi_c \sin\{\omega_0(t - t') - \tfrac{1}{2}\alpha[t' - (m-1)T]^2\}$$
$$+ \tfrac{1}{2}J\chi_s \cos\{\omega_0(t - t') - \tfrac{1}{2}\alpha[t' - (m-1)T]^2\}. \qquad (17)$$

Furthermore if

$$\chi = |\chi| \exp i\phi_\chi' = \chi_c + i\chi_s,$$

Eqs. (16) and (17) can be combined to give

$$S^* = \tfrac{1}{2}J\chi \exp i\{\omega_0(t - t') - \tfrac{1}{2}\alpha[t' - (m-1)\;T]^2\}, \qquad (18)$$

where

$$S^* = S_1^* + i\,S_2^*. \qquad (19)$$

If the reflector is at a distance r from the transmitter, then

$$t - t' \approx \frac{2r}{c}, \qquad (20)$$

or, more precisely,

$$t - t' = 2\left\{\left[1 - \left(\frac{v}{c}\right)^2\right]c\right\}^{-1}\left[r + \frac{v(x' - x)}{c}\right]. \qquad (21)$$

Equation (21) takes into account the displacement of the spacecraft between transmitting and receiving times. In Eq. (21), v is the spacecraft velocity, c is the velocity of light, x is the spacecraft location at the time of reception, x' is the point on the ground at which the wave is reflected, and r is the distance between the spacecraft and the ground point at the time of reception. The Appendix shows that the difference between Eqs. (20) and (21) is usually negligible.

To recapitulate, the received signal, at time t, with a phase given by Eq. (14), is mixed in quadrature with the signal from the transmitter whose phase at that time is given by Eq. (13). After low-pass filtering, the two resulting signals are the real and imaginary parts of Eq. (18).

If Eq. (20) is used, Eq. (18) can be written as

$$s'(t) = \frac{J\chi}{2} \exp i\left\{\frac{2r\omega_0}{c} - \frac{\alpha}{2}\left[t - \frac{2r}{c} - (m-1)T\right]^2\right\}. \qquad (22)$$

As each element $dx'dy'$ in the swath reflects such a signal, this signal must be integrated over the ground area. The total signal received is

$$s(x, t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{J\chi}{2} \exp i \times \left\{2kr - \frac{\alpha}{2}\left[t - \frac{2r}{c} - (m-1)T\right]^2\right\} dx'dy', \tag{23}$$

with

$$k = \frac{\omega_0}{c} = \frac{2\pi}{\lambda}, \tag{24}$$

where λ is the wavelength of the radio wave. The left side of Eq. (23) has been written as $s(x, t)$ to indicate that a signal $s(t)$ is received for each x. Therefore, while $s(x, t)$ is a continuous signal in t, it is a sampled signal as a function of x. The sampling frequency is equal to the pulse repetition frequency.

Before evaluation of the amplitude factor J, the phase factor in Eq. (23) can be rewritten (see Fig. 5) in the form

$$2kr = 2k[r_y^2 + (x - x')^2]^{\frac{1}{2}} = \phi(x - x', r_y). \tag{25}$$

Furthermore, if

$$\frac{2r}{c} = t_r, \tag{26}$$

then t_r is equal to the delay time for a reflector at a distance r.

Substituting these expressions into the exponential of Eq. (23) gives

$$s(x, t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{J\chi}{2} \exp i\phi(x - x', r_y) \times \exp - i\left(\frac{\alpha}{2}\right)[t - t_r - (m-1)T]^2 dx'dy'. \tag{27}$$

Next the factor J is evaluated.

• *Derivation of the amplitude factor*

It is assumed that the desired result is the scattering cross-section σ (energy per unit area).

The amplitude factor depends on this scattering cross-section $\sigma(x', y')$ as well as on transmitted peak power $P(t)$, system amplification $K(t)$, antenna gain G, and antenna area A_n.

The antenna gain in the direction of the element (x', y') is also a function of the direction of the normal $\mathbf{n}$ to the antenna and the angle of rotation Ω (see Fig. 5) and can be represented by $G = G(x - x', y - y', H, \mathbf{n}, \Omega)$. The variables H, $\mathbf{n}$, Ω are added because they must be used to correct for the unwanted effects of spacecraft motion.

From the radar equation (see [7]), the power received back at the antenna from one square meter is

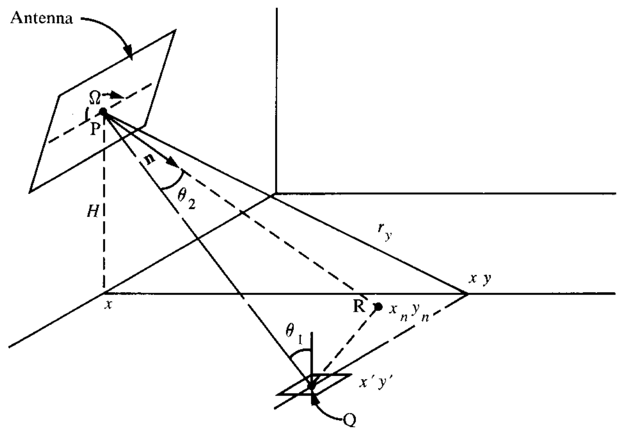

Figure 5 Antenna pointing.

$$P_r = \frac{P(t)\, A_n\, G\sigma}{(4\pi H^2)^2} \cos^6\theta_1 \cos\theta_2, \tag{28}$$

where $P(t)$ is the peak power of the pulse emitted at time t. Lambertian scattering [4] is assumed here; it accounts for $\cos^2\theta_1$, which contributes to $\cos^6\theta_1$. The remaining $\cos^4\theta$ occurs because the distance $r = H/\cos\theta_1$ appears as r^4 in the radar equation [7]. The variables H, θ_1 and θ_2 are all functions of the spacecraft position (x, y) as well as of the coordinates of the element $(x'y')$.

From Fig. 5 we see that

$$\cos\theta_1 = H[H^2 + (x - x')^2 + (y - y')^2]^{-\frac{1}{2}}. \tag{29}$$

The value of $\cos\theta_2$ can be obtained from the given direction of $\mathbf{n}$. This direction is indicated by x_n, y_n, the coordinates of the point where $\mathbf{n}$ intersects the ground plane, which are described by

$$x_n = x + \Delta x_n \text{ and}$$

$$y_n = y + \Delta y_n,$$

where Δx_n and Δy_n are the changes with respect to aircraft position. It should be pointed out that while x', y' are fixed to the ground, x_n and y_n move with the spacecraft. By appropriate choice of the origin of the coordinate system, y is usually close to zero; however, unwanted spacecraft motion may cause it to have a value different from zero. To calculate values of $\cos\theta_2$, we consider the triangle PQR. Using the cosine rule and assuming that Δx_n, Δy_n, and H are given, one can write

$$(x - x' + \Delta x_n)^2 + (y - y' + \Delta y_n)^2 = \Delta x_n^2 + \Delta y_n^2 + (x - x')^2 + (y - y')^2 + 2H^2 - 2\cos\theta_2 \times \{(\Delta x_n^2 + \Delta y_n^2)[(x - x')^2 + (y - y')^2 + H^2]\}^{\frac{1}{2}}, \tag{29a}$$

from which $\cos \theta_2$ can be obtained. The important fact is that $\cos \theta_2$ is a function of $x' - x$ and $y' - y$ only, and not of x or x', y or y' by itself, or any other function of these variables. Furthermore, $\cos \theta_2$ can be corrected for slow, unwanted spacecraft motion.

Similarly, the gain is a function of $x' - x$, $y' - y$, and Ω only. The antenna gain is a function of two angles and is assumed to be given in tabular form. One of these angles can be θ_2 (see Fig. 5), while the other angle is Ω. If the antenna rotates around the normal **n**, its pattern rotates and the gain can be corrected for the unwanted motion.

Since J in Eq. (27) is an amplitude factor while Eq. (28) describes the power, the amplitude J of the returned signal is given by

$$J = \frac{K(t)[P(t-\Delta)\, A_n G]^{\frac{1}{2}}}{2(2)^{\frac{1}{2}}\pi H^2} \cos^3\theta_1 (\cos \theta_2)^{\frac{1}{2}}, \tag{30}$$

where $K(t)$ is the system's amplitude gain between the antenna and the analog/digital (A/D) converter (inclusive).

Furthermore, Δ is the delay $2r/c$, which may be several times greater than the pulse repetition period. Also,

$$|\chi| = \sigma^{\frac{1}{2}}. \tag{31}$$

Finally, the following change of variable is made:

$$t' = \frac{2}{c}\,[H^2 + (y - y')^2]^{\frac{1}{2}}. \tag{32}$$

For a given H and y of the spacecraft, a function of y' is a function of t'. The H and y vary slowly and can be considered constant over fairly large x. By introducing t', it follows from Eq. (32) that

$$dy' = \frac{ct'dt'}{2}\left[t'^2 - 4\left(\frac{H}{c}\right)^2\right]^{-\frac{1}{2}}. \tag{33}$$

By introducing Eqs. (33) and (30) into Eq. (27), the latter can be written as

$$s(x, t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\chi}{2}(x', t')\left(\frac{ct'}{2}\right)\left[t'^2 - 4\left(\frac{H}{c}\right)^2\right]^{-\frac{1}{2}}$$
$$\times J(x - x', t') \exp i\phi(x - x', t')$$
$$\times \exp - i\frac{\alpha}{2}\,[t - t_r - (m-1)\, T]^2 dx'dt'. \tag{34}$$

In this equation, $\chi(x', y')$ has been written as $\chi(x', t')$ to indicate that χ can also be considered a function of t' instead of y'; see Eq. (32). During a radial convolution, however, H and y are to be considered constant and are allowed to vary only when x varies.

Under certain conditions, Eq. (34) can be approximated and solved by a double convolution, as shown in the next section. Since the function $t'(x)$ varies only because of unwanted spacecraft motion, it is assumed that tolerances on this motion are such that H and y can indeed be assumed to be constant over the length of the synthetic antenna. The main advantage of this double convolution is that in the frequency domain the problem is simplified to that of linear filtering.

Convolution approximation

Equation (34) can be written as a double convolution by approximating t_r with $t' + \varepsilon$ in the second exponential, where ε is a constant to be determined. Equation (34) then becomes:

$$s(x, t) = \int_{-\infty}^{+\infty} \exp - i(\alpha/2)[t - t' - (m-1)T - \varepsilon]^2 dt'$$
$$\times \int_{-\infty}^{+\infty} \chi(x', t')\, J^*(x - x', t')$$
$$\times \exp i\phi(x - x', t')dx', \tag{35}$$

with

$$J^* = J\left(\frac{ct'}{4}\right)\left[t'^2 - 4\left(\frac{H}{c}\right)^2\right]^{-\frac{1}{2}}. \tag{36}$$

The solution for x is obtained by solving first for $v(x, t')$ from

$$s(x, t) = \int_{-\infty}^{+\infty} v(x, t') \exp - i\frac{\alpha}{2}$$
$$\times [t - t' - (m-1)T - \varepsilon]^2 dt', \tag{37}$$

This is a convolution integral. Its solution is

$$v(x, t') = \frac{\alpha}{2\pi}\int_{-\infty}^{+\infty} s(x, t) \exp i(\alpha/2)$$
$$\times [t - t' - (m-1)T - \varepsilon]^2 dt. \tag{38}$$

Then $\chi(x', t')$ can be found from

$$v(x, t') = \int_{-\infty}^{+\infty} \chi(x', t')\, J(x - x', t')$$
$$\times \exp i\phi(x - x', t')dx'. \tag{39}$$

where J^* has been written as J. This is also a convolution integral, and it can be solved by using Fourier transforms. The exponential that appears in Eq. (37) is called the radial filtering function.

The solution of Eq. (37) as given by Eq. (38) is usually not practical. In general, a smoothing and a system response filter must be incorporated into the radial filter [8]. Therefore, the analytic inversion of Eq. (37) is not used here.

The accuracy of the convolution is considered in another section, but before doing this, the "multiple look" algorithms are derived.

Multiple looks

Consider an antenna with a beam width such that its synthetic antenna length is M (Fig. 6). The resolution obtainable from such an antenna is shown by $\delta = r\lambda/M$, where r is the distance from the line of flight to the strip of ground under consideration. To obtain this resolution a segment of length M of the complex signal $v(x)$ is needed. This function $v(x)$, convolved with a suitable function $h(x-x')$, results in a function $\chi(x')$. The complex signal $v(x)$ is obtained from the convolution of the received signal $s(x)$.

As seen from Fig. 6, if a resolution δ is required, instead of using the whole length M for one look at element E on the ground, it is possible to break up the synthetic length M into, say, r subsections and look at the element from different angles. In Fig. 6, M has been divided into four pieces and E is looked at from four directions. Since each look uses an antenna of length $M/4$, the resolution is now about 4δ. The data used for each look are independent. (It is possible to obtain, for instance, four looks with a resolution better than 4δ; however, the data used for each look are no longer independent in that case.) The signal received at point x by the spacecraft is $v(x, y')$, and may be considered to consist of several parts. As shown in Fig. 7, $v(x, y')$ consists of four parts (v_1 reflected from ground section 1, v_2 reflected from section 2, etc.); the problem to be solved is how to separate $v(x)$ into these four parts, $v_1(x, y')$, $v_2(x, y')$, $v_3(x, y')$ and $v_4(x, y')$, each part consisting of a different look.

In solving the problem, y' is kept constant at the beginning, but is reintroduced as a variable in the final results. The signal is separable because each section i of the ground contributes to a different part of the spectrum of $v(x)$. The Doppler signal reflected from section 1 has high positive frequencies, and the one from section 4 has high negative frequencies. Sections 2 and 3 have medium positive and negative frequencies, respectively. By finding the spectrum of $v(x)$ and dividing it into four parts, the four looks can be separated.

As an example, look 1 is considered. The spectrum of look 1 (see Fig. 7) is centered around a positive frequency because the transmitted signal is shifted up in frequency by reflections from all points of the ground in section 1 (see Fig. 8). To obtain the reflection coefficient χ, this signal v_1 must be convolved with

$$J(x-x') \exp i\phi(x-x') \approx J(u) \exp ik(u^2/r_y) \times \exp ik(2r_y),$$

where

$$\frac{M}{2} > x - x' > \frac{M}{4}.$$

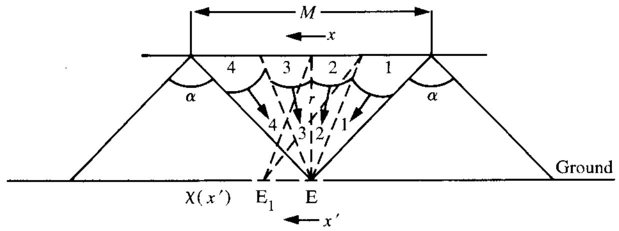

Figure 6 Four-look processing.

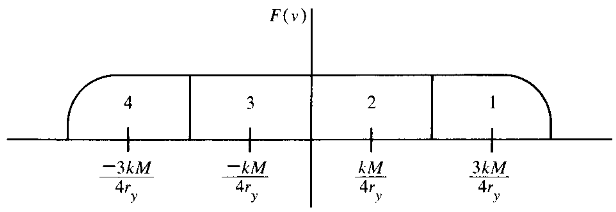

Figure 7 Spectrum of $v(x)$.

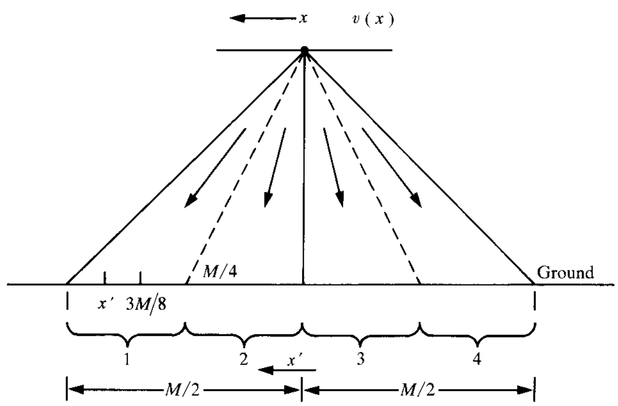

Figure 8 Four ground sections viewed.

Because of this center frequency, the sampling frequency must be high. It is shown that by heterodyning v_1, the sampling frequency can be reduced, in this case, by a factor of four.

As was shown in Eq. (39),

$$v(x) = \int_{-\infty}^{+\infty} \chi(x')\, J(x-x') \exp ik\left[\frac{(x-x')^2}{r_y}\right] dx'.$$

The phase factor $\exp 2ikr_y$ has been incorporated into χ. This does not affect its modulus which is the quantity of interest.

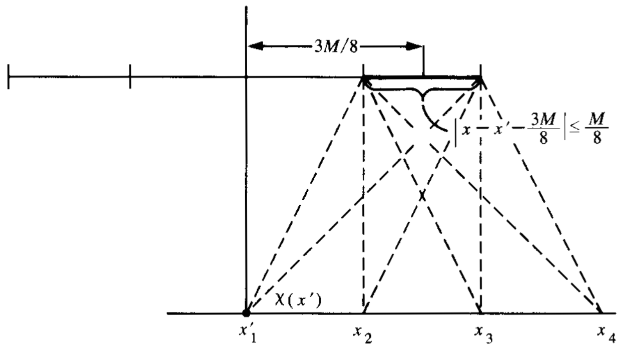

Figure 9 Synthetic antenna for look 1.

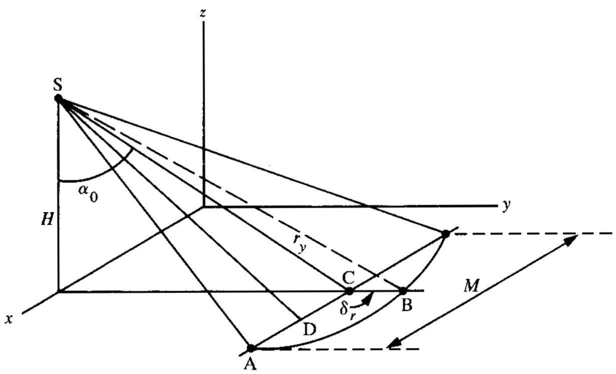

Figure 10 Limited accuracy of the chirp method.

This equation can be written as

$$v(x) = \int_{-\infty}^{+\infty} \chi(x')\ J(x-x')\ \exp i\frac{k}{r_y}\left(x - x' - \frac{3M}{8} + \frac{3M}{8}\right)^2$$

$$= \int_{-\infty}^{+\infty} \chi(x')\ J(x-x')\ \exp i\frac{k}{r_y}\left(x - x' - \frac{3M}{8}\right)^2$$

$$\times \exp i\frac{k}{r_y}\left(\frac{3M}{4}x - \frac{3M}{4}x' - \frac{9M^2}{64}\right)dx'.$$

The second term in the second exponential contributes $\exp(3ikMx'/4r_y)$ and produces only a phase shift in $\chi(x')$. It can be included in χ. Again only the modulus of χ and not its phase is of interest. The same is true for the factor $\exp(-9ikM^2/64r_y)$. It is a constant phase shift and is included in χ.

The remaining factor $\exp(+3ikMx/4r_y)$ can be removed from under the integral sign and moved to the left side. The resulting equation is

$$w_1(x) = v(x)\ \exp\left(\frac{-3iKMx}{4r_y}\right)$$

$$= \int_{-\infty}^{+\infty} \chi(x')\ J(x-x')\ \exp i\frac{k}{r_y}\left(x - x' - \frac{3M}{8}\right)^2 dx'. \qquad (40)$$

On the left of the integral is the signal v, heterodyned down with a frequency $3Mk/4r_y$. On the right is the sum of all the reflections from the ground. They can be regarded as a collection of Doppler waves with instantaneous frequency $(2kv/r_y)[x - x' - (3M/8)]$ (see Fig. 9). It can be seen that if $(x - x')$ is kept between $M/2$ and $M/4$, these frequencies are around zero and are limited. The frequencies on the right side of Eq. (40) are limited by limiting $|x - x' - (3M/8)| \le (M/8)$; band filtering and smoothing can therefore be accomplished by a suitable filter $g[x - x' - (3M/8)]$ on the right, which is essentially 0 for $|x - x' - (3M/8)| > (M/8)$, and 1 for $|x - x' - (3M/8)| \le (M/8)$. Equation (40) is then modified to

$$v(x)\ \exp\left(\frac{-3ikMx}{4r_y}\right)$$

$$= \int_{-\infty}^{+\infty} \chi_1(x')\ J(x-x')\left[\exp\frac{ik}{r_y}\left(x - x' - \frac{3M}{8}\right)^2\right]$$

$$\times g\left(x - x' - \frac{3M}{8}\right)dx', \qquad (41a)$$

and similar expressions for the three other looks. They are

$$v(x)\ \exp\left(\frac{-ikMx}{4r_y}\right)$$

$$= \int_{-\infty}^{+\infty} \chi_2(x')\ J(x-x')\left[\exp\frac{ik}{r_y}\left(x - x' - \frac{M}{8}\right)^2\right]$$

$$\times g(x - x' - \frac{M}{8}\Big)dx', \qquad (41b)$$

$$v(x)\ \exp\left(\frac{+ikMx}{4r_y}\right)$$

$$= \int_{-\infty}^{-\infty} \chi_3(x')\ J(x-x')\left[\exp\frac{ik}{r_y}\left(x - x' + \frac{M}{8}\right)^2\right]$$

$$\times g\left(x - x' + \frac{M}{8}\right)dx', \text{ and} \qquad (41c)$$

$$v(x)\ \exp\left(\frac{+3ikMx}{4r_y}\right)$$

$$= \int_{-\infty}^{+\infty} \chi_4(x')\ J(x-x')\left[\exp\frac{ik}{r_y}\left(x - x' + \frac{3M}{8}\right)^2\right]$$

$$\times g\left(x - x' + \frac{3M}{8}\right)^2 dx. \qquad (41d)$$

In the numerical procedure heterodyning is accomplished by assigning lower frequencies to the different parts of the spectrum. Before proceeding it is necessary to consider the limitations of the convolution as described in the previous sections.

- *Accuracy of approximation*

Replacing t_r by $t' + \varepsilon$ means using the approximation

$$[H_0^{\,2} + (y - y')^2 + (x - x')^2]^{\frac{1}{2}} \approx r_y + \frac{(x - x')^2}{2r_y}, \tag{42}$$

where

$$r_y = [H^2 + (y - y')^2]^{\frac{1}{2}},$$

and $(x - x')^2/2r_y$ is approximated by $\varepsilon c/2$.

The value of $\Delta r_y = (x - x')^2/2r_y$ varies from 0 to $M^2/8r_y$ where M is the synthetic antenna length. With no further correction, this quantity must be smaller than the resolution (see Fig. 10). In the case of multiple looks, the situation is worse, because the same condition holds for each quarter of the synthetic antenna. Suppose a four-look process is required. The radar is at a height H and has a minimum side viewing angle α. The resolution is δ; thus

$$\Delta r = \overline{\mathrm{SA}} - \overline{\mathrm{SD}} \approx \frac{3M^2}{32r_y}.$$

By eliminating M with the help of $\delta = 4r_y\lambda/M$, and applying the constraint $\Delta r < \delta$, one obtains

$$\delta > \left(\frac{3\lambda^2 H}{2 \sin \alpha}\right)^{\frac{1}{3}}. \tag{44}$$

For example, if $H = 2.40 \times 10^5$ m, $\theta_{\min} = 20°$, and $\lambda = 0.2$ m, one achieves the resolution $\delta > 35$ m. Therefore, it is necessary to correct for this effect due to wavefront curvature, if a resolution better than 35 m is required. For the shuttle radar, as well as another proposed radar, the required resolution is about 25 m; in those cases, a correction is required.

• *Correction for wavefront curvature*

The correction for wavefront curvature is made by dividing the azimuthal swath into a number of sections such that over each section the distance to the radar does not change more than δ_r.

First the swath width is divided, in the azimuthal direction, into a number of sections, with several adjacent sections constituting a look. The intention is to obtain $v_1(x, t')$ from section 1, $v_2(x, t')$ from section 2, etc., and then combine these contributions for the calculation of $\chi(x', y')$ by using the azimuthal convolution on each section separately. The general equation was [see Eq. (37)]

$$s(x, t) = \int_{-\infty}^{+\infty} v_\varepsilon(x, t') \times \exp - i\frac{\alpha}{2}[t - t' - (m-1)\,T - \varepsilon]^2\, dt',$$

with $v(x, t')$ replaced by v_ε to indicate that the solution v depends on ε. By denoting Fourier transforms with capital letters, Eq. (37) can be solved by

$$S(x, n) = v_\varepsilon(x, n)\, H(n, \varepsilon), \tag{46}$$

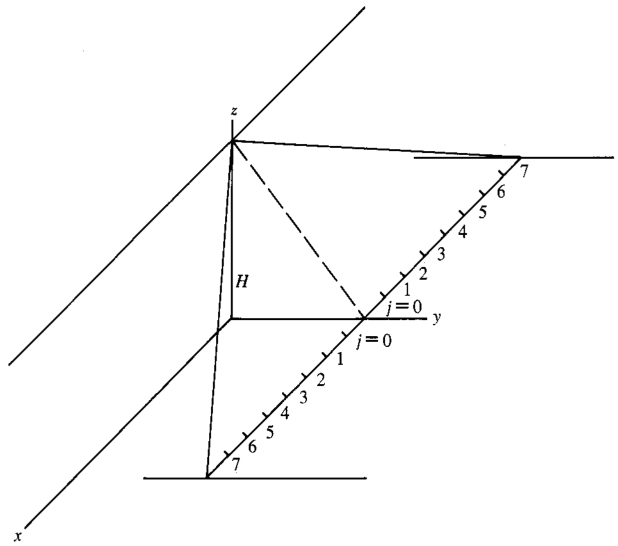

Figure 11 Sections with $\Delta r < 25$ m.

where n stands for the angular frequency, and by

$$H(n, \varepsilon) = \int_{-\infty}^{+\infty} \exp - i\frac{\alpha}{2}[u - (m-1)\,T - \varepsilon]^2 \times \exp(inu)\, du = H(n) \exp(in\varepsilon), \tag{47}$$

where $H(n)$ is the transform for $\varepsilon = 0$.

By substituting Eq. (47) into Eq. (46) and applying the inverse transform, we obtain

$$v_\varepsilon(x, t') = \int_{-\infty}^{+\infty} [\exp(-in\varepsilon)]\frac{S(x, n)}{H(n)} \exp(-int')\, dn = \int_{-\infty}^{+\infty} v(x, n) \exp[-in(t' + \varepsilon)]dn = v(x, t' + \varepsilon), \tag{48}$$

where

$$v(x, n) = \frac{S(x, n)}{H(n)}.$$

It appears that v_ε can be obtained from v by interpolation, provided v is available for several different values of t'. Since v is calculated for all ranges within the swath, this interpolation can be performed as shown in the following computational scheme.

Suppose the ground swath is divided in the azimuthal direction into J strips, as shown in Fig. 11, where $J = 16$. For each section an ε_j is chosen to approximate $(x - x')^2/r_y c$ for the jth strip; i.e.,

$$\varepsilon_j = \frac{1}{r_y c}\left[(2j + 1)\frac{M}{32}\right]^2, \tag{49}$$

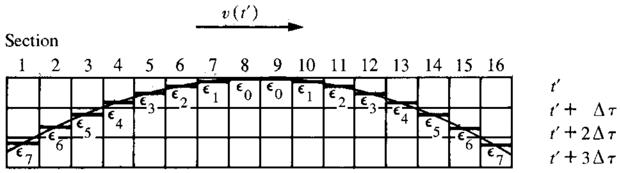

Figure 12 Spectra for interpolation.

which corresponds to the value at the center point of each strip. In that case, $|\Delta r| \leq 10$ m for each of the 16 sections. (The values $H = 794$ km, $\delta = 25$ m, $\alpha = 20°$, and four looks have been used, corresponding to the values of a proposed radar.)

Next, the signal of each of the 16 looks is isolated by using the difference in Doppler shift from the different ground sections. Each strip corresponds to a band in the Doppler spectrum.

The following computational scheme is adopted, with $J = 16$.

1. Compute $v(x, t')$ with a radial convolution for a number of radii at least equal to the number of points required for the azimuthal convolution.
2. Obtain the azimuthal spectrum $v(l, t')$ in the usual manner.
3. The spectra of several previous $v(l, t')$ are kept in memory so that v is available for, say, four different t' (see Fig. 12).
4. The spectrum is divided into 16 sections. If $v(l, t')$ is obtained in, say, 16384 points, each section must have 1024 frequencies.
5. To obtain the first section of the first look, the first sections of the four spectra are interpolated to obtain $v(l, t' + \varepsilon_7)$, where ε_7 is obtained using Eq. (49). Figure 12 indicates roughly where the ε_j are on the time scale.
6. The values of l are reassigned the proper values in the total spectrum of the first look. In the case under consideration, the first look consists of 4096 frequencies. The assigned frequencies are $4095 \geq l \geq 3072$. The spectrum is zero everywhere else.
7. This spectrum is used for the spectrum of the left side of Eq. (41a), and similarly for the spectrum of the kernel

   $$J\left[\exp\frac{ik}{r_y}\left(x - x' - \frac{3M}{8}\right)^2\right] g\left(x - x' - \frac{3M}{8}\right),$$

   where only the high frequency part (one-fourth of the spectrum) needs to be used.
8. Items 5, 6, and 7 are repeated for sections 2, 3, and 4, using ε_6, ε_5 and ε_4, respectively, to interpolate the spectra. Furthermore, the proper values of l are assigned to the spectra and the appropriate part of the spectrum of the kernel is used.
9. The four solutions of χ so obtained are added, and the resulting χ is the first-look solution.
10. The same is done for looks 2, 3, and 4.

The method just described is applicable if the Fast Fourier Transform (FFT) method is used to perform the convolution [9]. Because the four sections of each look have the proper frequencies, the four-look resolution is retained.

If each of the 1024 frequencies for a section is assigned to be around 0, the resulting resolution would be the one obtained with 16 independent looks, with loss in resolution of a factor of 16. This might be desirable as a possible option.

Finally, there is no reason to divide the antenna into 16 equal pieces, as was done here to simplify the argument. On the contrary, it is advantageous to divide the antenna into unequal parts, with the smallest sections on the outside and the largest ones in the center, where the correction will be negligible. For example, the eight sections dividing $M/2$ can be, starting at the end, four sections each $M/32$ long, two sections each $M/16$ long, and finally two sections each $M/8$ long.

Correction for spacecraft motion

Calculations of $\chi(x', y')$ are performed in a right-hand coordinate system that is fixed to the ground. The x axis is defined as the line in which the ground plane is intersected by the plane formed by the velocity vector of the spacecraft and the normal to the antenna opening. The y axis is perpendicular to the x axis. In this section, we discuss the corrections needed to compensate for the errors introduced by the motion of the spacecraft and the rotation of the earth. In the next section, corrections to compensate for the curvature of the earth are considered.

Corrections are of three different types: a) correction of the exponential factors in Eq. (35); b) correction of $J(x - x', t')$ in Eq. (35); and c) correction of the coordinate system.

• *Exponentials*

From Eq. (25) we know that

$$\exp i\phi \approx \exp ik\frac{(x - x')^2}{r_y} \exp 2ikr_y, \text{ and}$$

$$r_y = [H^2 + (y - y')^2]^{\frac{1}{2}}.$$

Suppose r_y is a function of t (or x) due to spacecraft motion or noncircular orbit. In that case, the exponential

$$\exp 2ikr_y = \exp 2ik\left[r_y(0) + \frac{\partial r_y}{\partial H}\frac{\partial H}{\partial t}\Delta t + \frac{\partial r_y}{\partial y}\frac{\partial y}{\partial t}\Delta t + \cdots\right].$$

When considering a particular synthetic antenna length, $r_y(0)$ represents the value of r_y when the spacecraft is in the center of the (synthetic) antenna. The two other terms are due to changes in H and y as a function of time and will vary with $x = vt$, where v is the velocity of the spacecraft. They have the form $\exp 2ik\alpha_1 x$ where

$$\alpha_1 = \frac{+v_H}{v}\frac{H}{r_y(0)} + \frac{v_y}{v}\frac{y-y'}{r_y(0)}.$$

Here v_H and v_y are the unwanted velocities in directions H and y, respectively.

The factor $\exp 2ikx(v_H H)/[vr_y(0)]$ causes a shift in the frequencies, as can be seen by moving it to the left side of Eqs. (41), and thereby introduces a heterodyning frequency that can vary as a function of time if v_H varies with time. The numerical procedure is not affected (except for the assignment of frequencies). The factor $\exp -2ik(2v_y/v)[(y-y')/r_y(0)]$ equals one because $v_y = 0$.

This follows from the way in which the coordinate system is defined; i.e., the x axis is defined as parallel to the velocity vector and must be redefined constantly due to earth rotation. Subsequent terms in the series of Eq. (50) must be small. (The term is of the order of $ik\Delta H^2/r$; if $\Delta H \approx 25$ m, $k = 2 \times 10^{-1}$ m, and $r = 2.5 \times 10^5$ m, this term equals 10^{-3}.)

Next the factor $\exp ik[(x-x')^2/r_y]$ must be considered. As before,

$$\exp ik\frac{(x-x')^2}{r_y} = \exp ik\left\{\frac{(x-x')^2}{r_y(0)} + \frac{(x-x')^2}{r_y^2(0)} \times \left[\frac{-H}{r_y(0)}\Delta H - \frac{y-y'}{r_y(0)}\Delta y\right] + \cdots\right\}.$$

Since

$$\frac{H}{r_y(0)} < 1, \text{ and } \frac{y-y'}{r_y(0)} < 1,$$

the condition

$$|\Delta| \leq \frac{k(x-x')^2}{r_y^2}\Delta H \ll \pi$$

must be satisfied, and the same is true for the second term for all expected spacecraft motion. For a typical radar $\lambda = 0.20$ m, after assuming $|x-x'| \leq 5$ km, while $r_y \approx 250$ km; and if

$$\frac{2\pi}{0.2}\left(\frac{5}{250}\right)^2 \Delta H \ll \pi,$$

then $\Delta H \ll 250$ m. Since the 5 km are covered in roughly 0.5 s, the velocity must be kept well within 500 m/s.

If the vertical velocity is due to eccentricity of the orbit, the maximum allowable eccentricity can be calculated. The vertical velocity in an elliptic orbit is [10]

$$r = \kappa\left[\frac{\mu}{r_p(1+\kappa)}\right]^{\frac{1}{2}} \sin\theta,$$

where κ represents eccentricity, $\mu = 4.01 \times 10^{14} m^3 s^{-2}$, and $r_p = 6580$ km and is the radius vector from the center of the earth to the satellite (perigee distance).

The maximum radial velocity is then

$$\dot{r}_{\max} = \kappa\left[\frac{\mu}{r_p(1+\kappa)}\right]^{\frac{1}{2}}.$$

Suppose that $V_{\max} = 7.8$ m/s $\ll 500$ m/s; then $\kappa < 0.001$.

- *Amplitude factor*

The value of J as given in Eq. (36) must be corrected for slow variations in H and y. It is assumed that J does not change much over the length of the synthetic antenna so that in Eq. (39) J changes only when x changes. We have to consider, however, that the amplitude factor J does change slowly from convolution to convolution.

- *Ground coordinate system*

The ground coordinate system is defined as follows. At time $t = 0$, a plane through the velocity vector of the radar-carrying satellite intersects the horizontal ground plane in a line parallel to the x axis. The x axis itself is the intersection if the plane is inclined an angle $(90-\alpha_0)$ with the ground plane, where α_0 is the side viewing angle.

The velocity vector is in a coordinate system fixed to the earth. The origin is defined by erecting a normal to the velocity vector in the plane containing the x axis. The point at which this normal intersects the x axis is the origin. The y axis is defined as the normal to the x axis erected in the ground plane at the origin.

The ground reference grid is a set of points formed by the intersection of sets of horizontal and vertical lines at distance δ from each other, parallel to the x and y axis, with δ, as known, as the resolution.

In this earth-fixed coordinate system, the velocity vector does not remain constant. This requires a periodic redefinition of the coordinate system, as well as certain corrections between redefinitions. Changes in position, velocity, and acceleration of the radar antenna with respect to its position and velocity vector at time $t = 0$ are assumed to be known in the coordinate system just defined. The changes may be directly measured and registered on a magnetic tape together with the timing signal and received signal. For example, if a pulse is transmitted at time $t = 0$, the position of the radar (antenna feed) is given by the height H_0 and the side viewing angle α_0.

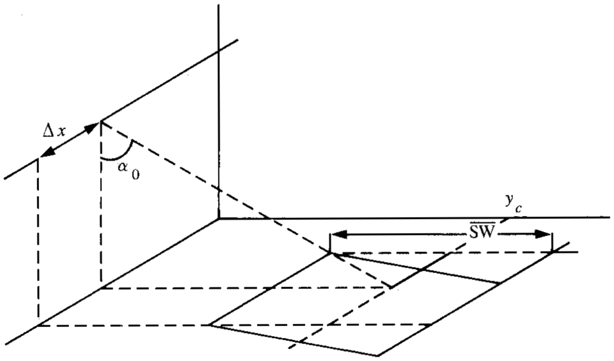

Figure 13 Skewness due to spacecraft motion.

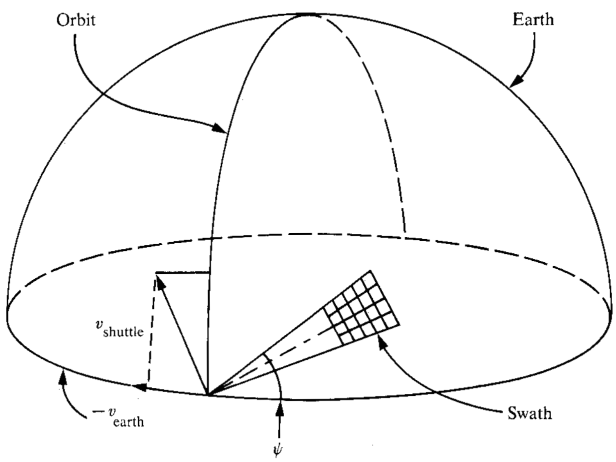

Figure 14 Coordinate system at the equator.

The sample points of $v(x, t')$ on the ground vary with x due to changes in H and y, because sampling occurs a fixed time after the transmission of the pulse. At time $t = 0$, the sampling points are

$$y'_n(0) = \left[\frac{c^2(t'_0 + n\Delta t')^2}{4} - H_0^2\right]^{\frac{1}{2}} - \left(\frac{c^2 t_0'^2}{4} - H_0^2\right)^{\frac{1}{2}}, \qquad (51a)$$

where $t'_0 = 2H_0/c \cos\alpha_0$, and the integer $n \le \overline{\mathrm{SW}}/2\delta$ while Δt is the sampling time.

For the nth point,

$$y'_n(x_m) = y'_n(0) + \Delta y_m - [H_0/y'_n(0)]\, \Delta H_m, \qquad (51b)$$

where Δy_m and ΔH_m are the changes in y and H_0 of the spacecraft at the time of transmission of the mth pulse (or halfway between receiving and transmitting). The sampling of $v(x)$ occurs at the location

$$x'_{m+1} = x'(t = 0) + \Sigma\Delta x_m, \qquad (52a)$$

with

$$\Delta x_m = v_{m+\frac{1}{2}}\, \Delta t_m, \qquad \Delta x_0 = 0. \qquad (52b)$$

The sampling points in the x direction are the x coordinates of the spacecraft midway between receiving and transmitting of a pulse. If higher accuracy is required, skewness of the grid must be taken into account.

Skewness

In Fig. 13, the swath is shown, indicating the delineation of the area from which the signal is received. The swath length is $\overline{\mathrm{SW}}$, and displacement of the spacecraft during reception is given by

$$\Delta x = \frac{(2\overline{\mathrm{SW}} \sin \alpha_0)\ v}{c}. \qquad (52)$$

For $\overline{\mathrm{SW}}$ = 100 km, $\alpha_0 = 40°$, and v = 7800 m/s, Δx = 3.5 m.

This is taken into account in the interpolation in the x direction by assuming that the signal $v(x)$ arrives from

$$x = x_p + \frac{2[H^2 + (y - y')^2]^{\frac{1}{2}}}{c}\, v, \qquad (53)$$

with x_p the x coordinate at which the pulse was transmitted. The effect of the antenna pattern is negligible.

Roll, pitch and yaw

The amplitude factor J is directly affected by roll, pitch and yaw. As indicated before, J depends on the direction of the normal to the antenna, the angle Ω, etc. The signal $v(x, y')$ to be convolved in the azimuthal direction may have shifted to higher frequencies because the forward-looking angle has increased; however, the frequency return of an element for which $x - x' = 0$ is still zero, independent of the beam locus (assuming $v_H = 0$).

One final effect may occur if, as generally happens, the antenna feed is not in the center of the mass. In that case, rotational motion causes changes in x, y and H that must be taken into account.

Two corrections remain, i.e., those for earth rotation and earth curvature. Earth rotation necessitates a periodic redefinition of the coordinate system. If a continuous swath is required, certain problems involving translation from one set of grid points to another must be solved.

One particular problem connected with earth rotation is that at the equator, the grid becomes so skewed that edge effects become significant and too many points must be dropped, thereby reducing the effective swath width. This can be prevented by the method discussed in the next section.

Correction for earth rotation

In order to correct for the rotation of the earth, the antenna beam can be steered an angle ψ forward. If

v_x and v_y are measured in an inertial system, then $\psi = \arctan(v_y/v_x)$. Corrections are therefore made by redefinition of the coordinate system, for example, every 100 km. In that case the velocity in the y direction, in the earth-fixed coordinate system, is zero; however, since v_y and v_x vary, in general, with the position of the spacecraft, the angle ψ changes slowly and must be modified accordingly. Figure 14 shows a radar in polar orbit at the time it crosses the equator. Also shown is the direction of the coordinate system at that moment.

Correction for earth curvature

The orthogonal coordinates used to describe the synthetic aperture radar on a spherical earth are a set of longitude and latitude circles as shown in Fig. 15.

The coordinates of a point Q are the arcs $\zeta_x' = \xi_x - \xi_x'$ and $\zeta_y' = \xi_y - \xi_y'$, with respect to the nadir point N with coordinates ξ_x and ξ_y. The signal can be written with the same approximation as before:

$$s(\xi_x, t) = \int\int J\chi(\xi_x', \xi_y')\, d\xi_x'\, d\xi_y' \times \exp i\{2kp - \tfrac{1}{2}\alpha[t - t' - (m-1)\ T]^2\}, \quad (54)$$

where the integration is performed over sufficiently large arcs.

The distance ρ (See Fig. 15) is given by

$$\rho = [(H + r_0 - r_0 \cos\beta \cos\gamma)^2 + (r_0 \sin\beta)^2 + (r_0 \cos\beta \sin\gamma)^2]^{\frac{1}{2}}, \quad (55)$$

where

$$\beta = (\xi_y' - \xi_y)/r_0, \quad (56)$$

and

$$\gamma = (\xi_x' - \xi_x)/r_0 \cos\beta. \quad (57)$$

In the same way that $r = [r_y^2 + (x - x')^2]^{\frac{1}{2}}$ was expressed as a power series, ρ in Eq. (55) can be expanded in a power series in $(\xi_x - \xi_x')$. The coefficients are functions of r, the distance in the $z - y$ plane between the radar and the ζ_x' arc.

The correction for curvature consists therefore of three parts:

1. The kernel in the azimuthal convolution is changed to the one resulting from the expansion of Eq. (55).
2. By setting $\xi_x - \xi_x' = 0$, Eqs. (55) - (57), together with Eqs. (56) and (57), provide the transformation from equally spaced time points (or equally spaced r points) to the corresponding $(\xi_y - \xi_y')$, analogous to Eq. (51).
3. Distances between points on the x arcs are y-dependent. They are closer at greater distances.

In the following section, digital processing considerations are described and the necessary number of multiplications is estimated.

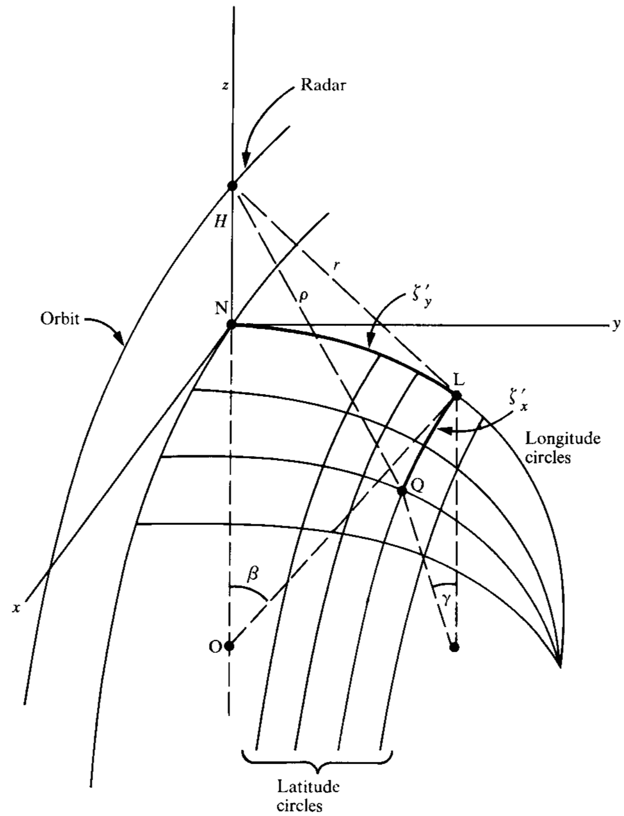

Figure 15 Curved coordinates of point Q.

Digital processing considerations

The numerical process that converts the signals $s(x, t)$, received from the ground, into a measure for the scattering cross section $\sigma = |\chi|^2$ of a ground element, consists of six basic operations, performed in the following order:

1. Normalization;
2. Convolution in the radial (y' or t') direction [solution of Eq. (37)];
3. Interpolation in the y direction;
4. Interpolation in the x direction;
5. Convolution in the x direction [solution of Eq. (39)]; and
6. Computation of the scattering cross section.

In addition to these operations, spacecraft motion compensations are necessary. A simple diagram representing the interaction among parameters, data and operations is shown in Fig. 16.

The signal $s^*(x, t)$ is a complex signal, sampled and digitized in the x as well as the t direction. It is assumed that there are N resolution elements in the radial direction. If the resolution is δ_r, the swath width will be $N\delta_r$.

• *Calibration, normalization and sampling of the signal*

The incoming signal $s^*(x, t)$ depends on the amplitude of the transmitted pulse $P(t)$, which is monitored and regis-

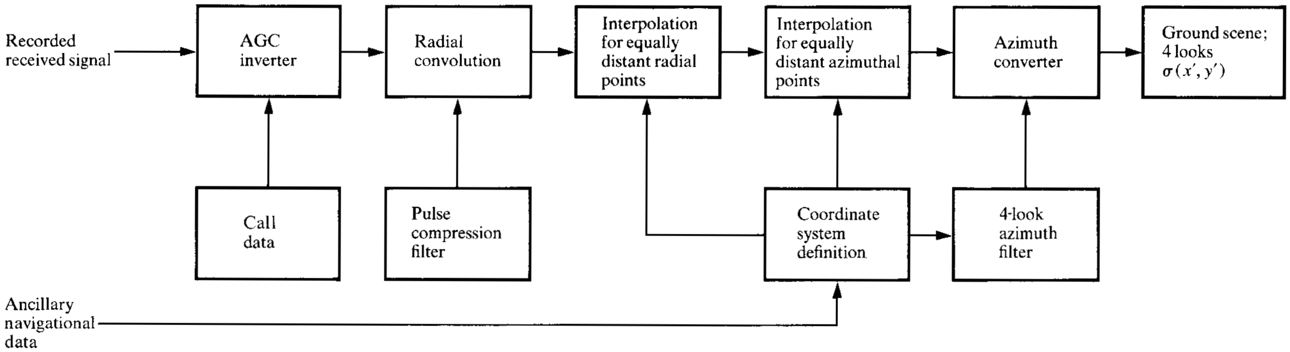

Figure 16 Computational scheme.

tered on the raw flight tape. It is also proportional to the gain of the receiver at time of reception (also registered). Thus,

$$s(x, t) = s^*(x, t) \left[\frac{P_0}{P(t-\Delta)}\right]^{\frac{1}{2}} \frac{K_0}{K(t)},$$

where $K(t)$ is the system's amplification factor, or gain, $P(t-\Delta)$ is the peak power of the pulse being received at time t, and P_0 and K_0 are constants.

The power P in the pulse is monitored. The gain K is varied and registered. Variation of K is desirable in order to accommodate greatly varying conditions with a relatively small word length. In other words, the low frequencies in the signal are registered with the help of the gain variations.

• *Radial convolution*

It is known from the sampling theorem [8] that the signal s must be sampled every $2\delta_r \sin\alpha_0/c$ seconds if the radial resolution is δ_r. If the pulse length is τ_0, the number of points in the radial filter equals

$$n_f = \frac{\tau_0 c}{2\delta_r \sin \alpha_0}. \tag{58}$$

Each point has a weighting factor, which is given by a complex number. Its value is $\exp\,[-i\,\alpha/2(n\Delta\tau - \varepsilon)^2]$, where n is an integer and $0 \le n \le (\tau_0 c)/(2\delta_r \sin\alpha_0)$.

Equation (38) gives limits of integration from $-\infty$ to $+\infty$. In reality, the pulse length equals τ_0 and the integration cannot be performed over an interval longer than τ_0. However, it is possible to integrate over a shorter interval, all other factors being the same. This results in a reduced resolution, although, when combined with a smoothing of the data, it reduces computation time and may therefore be of interest. The computation time reduction can be demonstrated by noting that

$$v(t') = \int_{t'}^{t'+\tau} s(t) \exp i\frac{\alpha}{2}(t-t')^2 dt$$

can also be written as

$$v(t') \exp i\frac{\alpha}{2}t'^2 = \int_{t'}^{t'+\tau} s(t) \exp i\frac{\alpha}{2}t^2 \exp(-i\alpha t t')\, dt,$$

where the function $[s(t) \exp i(\alpha/2)t^2]$ is frequency analyzed, and each frequency can be determined with an accuracy $1/\tau_0$.

If the integration time is shorter than τ_0, say $\tau < \tau_0$, the frequencies can be determined with, at most, an accuracy $\approx 1/\tau$. Two adjacent ground elements, separated in radial direction by a distance $\delta_r' \sin\alpha_0$, are separated in time by (see Fig. 17)

$$\Delta\tau = \frac{2\delta_r' \sin \alpha_0}{c}, \tag{59}$$

and in frequency by

$$\alpha\Delta\tau = \frac{2\alpha\delta_r' \sin \alpha_0}{c}. \tag{60}$$

If the frequency is accurate to $1/\tau$, one can write

$$\frac{2\alpha\delta_r' \sin \alpha_0}{c} = \frac{1}{\tau},$$

from which

$$\delta_r' = \frac{c}{2\alpha\tau \sin \alpha_0};$$

by using

$$\delta_r = \frac{c}{2\alpha\tau_0 \sin \alpha_0},$$

one can obtain

$$\delta_r' = \delta_r \frac{\tau_0}{\tau}. \tag{61}$$

In other words, the resolution decreases (δ_r increases) as

τ decreases. Prefiltering decreases the number of necessary sample points in direct proportion to δ_r'.

The total number of points in the radial direction is inversely proportional to δ, i.e.,

$$n_r = \frac{\overline{\mathrm{SW}}}{\delta_r'}, \tag{62}$$

where $\overline{\mathrm{SW}}$ is the swath width.

A reduced number of points in the radial direction can be obtained by limiting the receiver bandwidth and lowering the A/D conversion rate. The reduction can also be obtained by numerical smoothing, though part of the advantage of a reduction in points will be lost in that case. The number of points in the filter can be found as follows: The sample points are separated in time by $\Delta\tau$, as given by Eq. (59). The number of points in the filter is n_f, and can be obtained from Eq. (61) by writing

$$n_f = \frac{\tau}{\Delta\tau} = \frac{c\tau}{2\delta_r' \sin \alpha_0} = \frac{c\tau^2}{2\delta_r \tau_0 \sin \alpha_0}. \tag{63a}$$

Furthermore,

$$n_f = \frac{c\tau_0 \delta_r}{2 \sin \alpha_0 \delta_r'^2}. \tag{63b}$$

From Eq. (63b) it is apparent that the number of points in the filter can be reduced quadratically with the required resolution.

- *Number of arithmetic operations for the radial convolution*

Assume that the convolution is performed by using FFT techniques [3], that the number of points in the filter is n_f, and that the total number of points in the radial direction is N_r. The number of points used in the FFT is denoted by n_r, and the number of points in the filter n_f must be enlarged to n_r by adding zeros to the data.

The convolution is performed by determining the Fourier transform of $s(x, t)$ in n_r points, multiplying it by the spectrum of the filter and transforming the result back to the time domain. If the Fourier transform of the filter is known, the total operation consists of two FFT computations in n_r points and n_r multiplications. Increasing the number of points in the filter by adding zeros does not change the shape of the spectrum, but it does divide the spectral domain into smaller Δk, where Δk is the spacing between adjacent wave numbers.

Also, by using the numerical Fourier transform, the functions are made into periodic functions and the result gives the correlation coefficient correctly in $(n_r - n_f)$ points only [9]. Sufficient overlap must therefore be used to obtain correct values in all points lying on a radius. The total number of convolutions for N_r points is therefore the nearest integer greater than or equal to

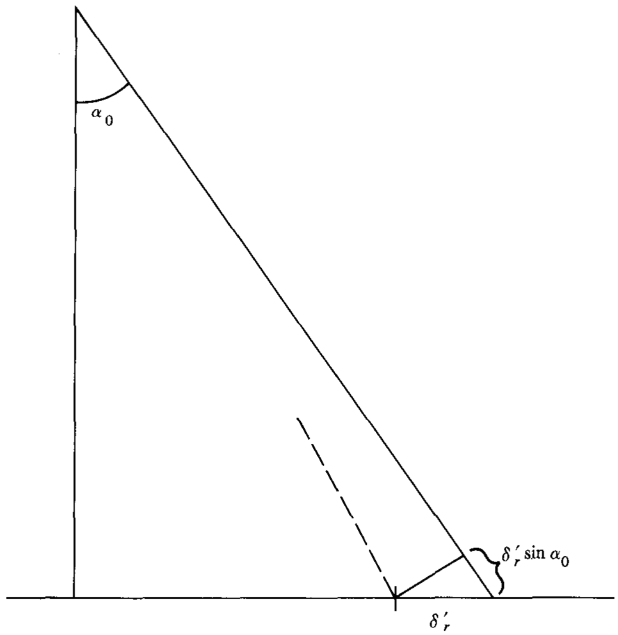

Figure 17 Spatial distance translated into time difference.

$$\left(\frac{N_r}{n_r - n_f}\right).$$

To obtain the total number of operations, the number of operations per radius must be multiplied by the total number of radii.

Finally, the number of complex multiplications, additions and subtractions of an FFT (with n_r a power of two) is given by $\frac{1}{2}n_r \log_2 n_r$, where n_r is the number of points and $\log_2$ indicates the logarithm with base two.

Example

The following data are used to compute the total number of multiplications in the radial convolution for an area 100×100 km.

$\overline{\mathrm{SW}} = 100$ km (swath width);
$\delta_r = 20$ m (resolution);
$p = 1750$ (pulse rate);
$\tau = 17.5$ μs (pulse length);
$\alpha = 40°$; and
$v = 7800$ m/s.

From Eq. (58), we know that

$$n_f = \frac{\tau_0 c}{2\delta_r \sin \alpha_0} \approx 200.$$

If we choose $n_r = 256 (> n_f)$, a frame length of 100 km gives $100.000\ p/v = 22400$ radii. (The pulse rate accommodates a four-look process.) The total number of multiplications $n_\times$ is, for $n_r = 256$,

$$n_\times = \text{int}_+\left(\frac{5000}{256-200}\right) \times (8 \times 256 + 256) \times 22400$$

$$= 4.65 \times 10^9 \text{ multiplications,}$$

where the operator int_+ preceding a value indicates that the nearest higher integer is substituted for the actual value.

For $n_r = 512$,

$$n_\times = \text{int}_+\left(\frac{5000}{512-200}\right) \times (9 \times 512 + 512) \times 22400$$

$$= 1.95 \times 10^9 \text{ multiplications.}$$

For $n_r = 1024$,

$$n_\times = \text{int}_+\left(\frac{5000}{1024-200}\right) \times (10 \times 1024 + 1024) \times 22400$$

$$= 1.77 \times 10^9 \text{ multiplications.}$$

For $n_r = 2048$,

$$n_\times = \text{int}_+\left(\frac{5000}{2048-200}\right) \times (11 \times 2048 + 2048) \times 22400$$

$$= 1.66 \times 10^9 \text{ multiplications.}$$

For $n_r = 4096$,

$$n_\times = \text{int}_+\left(\frac{5000}{4096-200}\right) \times (12 \times 4096 + 4096) \times 22400$$

$$= 2.39 \times 10^9 \text{ multiplications.}$$

It appears that $n_r = 2048$ gives the minimum number of multiplications. The choice of n_r depends, however, not only on the number of multiplications, but also on the available hardware, on memory limitations, and to some extent on the software being used.

Furthermore, there is about an equal number of complex additions, as well as complex subtractions. The number of arithmetic operations in the azimuthal convolution, which will be calculated in the next section, must also be considered.

• *Convolution in the azimuthal direction*

The convolution is given in Eq. (39) with

$$\phi(x - x', t') = 2k[r_y^2 + (x - x')^2]^{\frac{1}{2}}. \qquad (25)$$

The integration is usually performed over a length smaller than the effective length of the synthetic antenna. This is accomplished by using a suitable smoothing filter. Suppose a resolution of $\delta = 25$ m is required with four independent looks. The synthetic antenna length is then

$$M = \frac{4r\lambda}{\delta},$$

and the number of points in the azimuthal filter becomes

$$n_f' = \frac{Mp}{v} = \frac{4r\lambda}{\delta v}. \qquad (64)$$

If $r_{max} = 280$ km,
$\lambda = 0.2$ m,
$v = 7800$ m/s,
$\delta = 25$ m,

Eq. (64) gives $n_f' = 2000$ points.

The antenna length M can be varied by using a suitable filter length; however, the sampling frequency p is determined by the highest frequency in the signal, which is determined in turn by the maximum synthetic antenna length. This maximum length is given by

$$M_{max} \approx \frac{2.2r\lambda}{L},$$

where it is assumed that the side lobes of the antenna pattern are sufficiently suppressed, and therefore that the main lobe has been broadened by ten percent. This causes the 2.2 factor and roughly corresponds to, say, the 30 dB points of the assumed antenna pattern, including a 10 percent widening effect due to side lobe suppression. By using Eq. (9), this gives for the maximum frequency

$$|\Delta\nu|_{max} \approx \frac{v}{c}\left(\frac{2.2M_{max}}{r_{max}}\right) \approx \frac{2.2v}{L}.$$

For v = 7800 m/s and L = 10.5 m this results in a sampling frequency larger than $2\Delta\nu = 3280$ per second for each component of the complex signal.

The usual assumption is that only the center portion of the synthetic antenna (between the 3 dB points) needs to be taken into account. This results in a maximum sampling frequency of 1560 per second. In some cases this results in aliasing problems because the azimuthal signal is sampled before filtering. On the other hand, because of the antenna pattern and the smoothing filter, the high frequencies have more noise and are less useful. The aliasing problem might be serious in the case of four-look processing. The first and fourth looks might be of limited use only.

Returning to the azimuthal convolution, suppose that the FFT of the complex function $v(x)$ is obtained over $n_f = 2048$ points. The spectrum obtained is divided into four bands of 512 frequencies each. Similarly, the spectra of the filters, as shown in Eq. (64), are determined in 2048/4 points. (Zeros are added to obtain this number.)

The four looks, therefore, require one FFT of 2048 points, four FFT's of 512 points and 1048 multiplications. This results in a total of $N \log N$ complex multiplications where N is the number of points. Again the convolution gives the correct result in $N - n_f$ points so that the total number of convolutions is given by $\text{int}_+[22400/(N - n_f)]$.

For a 2048-point FFT for 5000 different radii,

$$n_a = \mathrm{int}_+\left(\frac{22\,400}{2048 - 1195}\right) \times (2048 \times 11) \times 5000$$

$$= 3.38 \times 10^9 \text{ multiplications.}$$

For a 4096-point FFT,

$$n_a = \mathrm{int}_+\left(\frac{22\,400}{4096 - 1195}\right) \times (4096 \times 12) \times 5000$$

$$= 1.96 \times 10^9 \text{ multiplications.}$$

For a 8192-point FFT,

$$n_a = \mathrm{int}_+\left(\frac{22\,400}{8192 - 1195}\right) \times (8192 \times 13) \times 5000$$

$$= 2.13 \times 10^9 \text{ multiplications.}$$

The minimum is at 4096 points.

The total number of complex multiplications in the convolution is the sum of the radial and azimuthal convolutions. This sum is $(1.66 + 1.96) \times 10^9 = 3.62 \times 10^9$ complex multiplications. To this must be added the number of multiplications required to calculate the corrections, interpolations, etc. No estimate of this is attempted here; the total number, however, is not greatly increased.

Summary

The synthetic aperture radar system that, according to existing plans, will operate aboard the space shuttle and on several satellites has been described. Experience with this type of radar has so far been limited to applications in conventional aircraft. Since the literature on this subject is rather scarce, and in most cases the subject is treated in the frequency domain rather than in the time domain, this paper provides a new approach. The much greater altitude at which the shuttle and satellites will operate requires corrections not normally necessary in aircraft applications, such as the earth curvature correction that has been described in the paper. As seen, the contemplated space applications require images of fairly high quality with high resolution, necessitating the introduction of multiple looks, a technique in which a ground element is looked at from several slightly different angles. As illustrated, this requires a synthetic antenna several times longer than the one used in the simple one-look case.

In this paper a derivation of the equation describing the signal received by the radar has been given. The multiple look process has been formulated and a digital solution of the resulting equations presented. This solution provides also the necessary correction for the wavefront curvature. Corrections that compensate for spacecraft motion, other than the orbital velocity, and for the effects of earth rotation have also been considered.

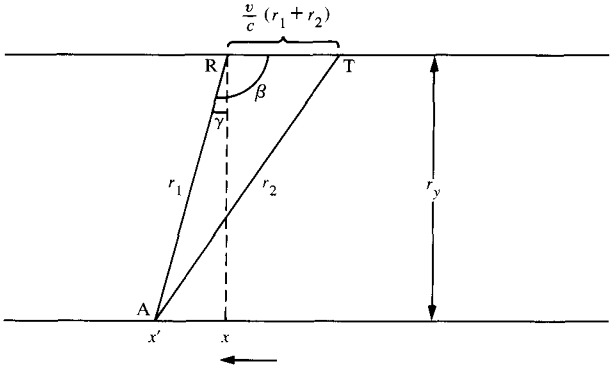

Figure A1 Spacecraft motion during signal reception.

The digital data processing requirements have been estimated and the required number of multiplications given to a reasonable approximation. The Fast Fourier Transform has been employed to solve the convolution integrals occurring in the problem.

One conclusion that seems obvious is that, to prepare an image of an area 100×100 km, 16×10^9 real multiplications are required. Therefore, real time processing, on a general purpose computer, appears to be prohibitive, if one assumes that it takes about 13 seconds to gather the data. Special array processors, or a scheme offering similar specialized performance, seem to be required to provide the necessary turnaround time.

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Appendix: Correction for spacecraft motion during wave travel time

Since the height at which the SAR flies is great, the effect of the finite travel time of the electromagnetic wave, and the subsequent displacement of the spacecraft during this time, must be investigated. Let the position of reception be x, the position of the reflector x' (Fig. A1). If the wave is transmitted at point T and received at point R, total travel time is $(r_1 + r_2)/\mathrm{c}$, and the displacement of the spacecraft is

$$\overline{\mathrm{RT}} = \left(\frac{r_1 + r_2}{c}\right) v = \frac{v}{c}(r_1 + r_2) = (r_1 + r_2) f, \qquad \text{(A1)}$$

where $f = v/c$.

By using triangle ART and the fact that

$$\cos\beta = \frac{x - x'}{r_1}, \tag{A2}$$

it follows that

$$r_1 = \left(\frac{1+f^2}{1-f^2}\right) r_2 + \left(\frac{2f}{1-f^2}\right)(x - x'),$$

and

$$r_1 + r_2 = \frac{2r_2}{1-f^2} + \frac{2f(x - x')}{1-f^2}.$$

The equation of importance is Eq. (25), which now becomes

$$k(r_1 + r_2) = \frac{2kr_2}{1-f^2} + \frac{2fk(x - x')}{1-f^2}. \tag{A3}$$

The first change is that $2kr_2$ is changed to $2kr_2/(1-f^2)$, and can therefore be corrected by assuming the wave number not to be k but $k/(1-f^2)$. Since $f \approx 3 \times 10^{-5}\ \mathrm{s}^{-1}$, the correction is an extremely small one ($\approx 10^{-9}$) and can be neglected. The second change is the additive term $[2f/(1-f^2)](x - x')$; when added in this exponential, it shifts all frequencies by an amount

$$\Delta\omega = \frac{2fkv}{1-f^2}. \tag{A4}$$

This is a constant term; it can be compensated for numerically by changing the heterodyning frequency in the four-look processing if necessary.

With $f \approx 3 \times 10^{-5}\ \mathrm{s}^{-1}$, $k = 2\pi/0.2$, and $v = 7800$ m/s,

$\Delta\omega \approx 1$ Hz,

which usually can be neglected. If necessary, it can be taken into account, since the only effect is a shifting of the frequency spectrum.

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