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Walsh Functions for Digital Impedance Relaying of Power Lines

Abstract: Impedance distance relaying for fault-protection of a plurality of high-voltage lines has not been accomplished with a minicomputer because of the burden of time placed upon these computers. A new method for computing impedance from data samples is proposed, which would employ only the computer operations of Add, Subtract, and Shift. This is valuable because these operations are one to two orders of magnitude faster on present day minicomputers than the operations of Multiply, Divide, Square, and Square Root. The new method is based upon the use of Walsh functions. When compared with the best competitive method, this new method shows superiority in speed and an accuracy that meets proposed objectives.

Introduction

Digital relaying is a new branch of the science of protective relaying of a power system. It attempts to perform, with a digital computer, many and perhaps all of the functions currently performed by electromechanical and static (solid state) relays. One of the most difficult and important of these functions is distance relaying in which the apparent impedance of the line is sensed by the relay. When a fault hits one or more of a plurality of high-voltage lines protected by a bank of relays in a substation, all the relays process the data simultaneously, and a decision to trip the appropriate circuit-breakers is made in about one cycle. The difficulty in doing this with any single digital computer [1] suitable for substation application lies in the sequential nature of all such computers and the consequent burden of time placed upon them.

Recently, for instance, the use of a single minicomputer has been considered for the protection of a small number of high-voltage lines [2]. It must be programmed to carry out a long sequence of operations which serve to detect the fault; classify it as to type, severity, and line number; calculate the appropriate impedances; and finally make a decision whether or not to issue the trip-signals. Mann and Morrison [3] did this for a single, three-phase line and achieved a time of 5.4 ms from fault to trip. That is, they almost achieved quarter-cycle relaying in a laboratory test. Furthermore, they did this with standard computer techniques. Inasmuch as 4 ms of this was used for detection and classification algorithms, it is problematic as to whether they could still do this for six lines, as they so stated. There is even the question of whether their method would suffice for protecting six lines in one cycle under field conditions. Rockefeller and Udren [4], using a modification of the method of Mann and Morrison on a dedicated minicomputer similar to Mann and Morrison's, were only able to protect a single three-phase line under field conditions in times ranging from one to three cycles. This experience illustrates the burden of time in protecting only one three-phase line. Consequently, we may regard the multiple-line problem as far from solved at the present time, even with a dedicated computer.

However, even if Mann and Morrison's algorithms could be speeded up by hardware improvements, for example, they would still be questioned because of their assumption of pure sine wave conditions. During the immediate post-fault period, a dc offset which invalidates the sine wave assumption generally occurs. Mann and Morrison would minimize this dc offset by compensatory impedance. Rockefeller and Urden minimized dc offset, with apparent success, by using first and second derivatives of the measured quantity in formulas in which Mann and Morrison used its value and a first derivative. Although one would have thought that the use of high-order numerical derivatives by Rockefeller and Udren would, because of noise, generate spurious signals and cause false tripping, interestingly enough, their installation did not experience false-tripping during a period when 70 faults were correctly detected. In spite of this success, it is the purpose of this paper to present a method which does not assume sinusoidal conditions, but which is nevertheless exceptionally fast because it requires only addition and subtraction operations of the computer. (There is no particular advantage for cases in which addition and subtraction offer no significant time reduction over multiplication and division, but in present-day minicomputers the difference is typically one to two orders of magnitude.) This method makes use of Walsh functions which, in the past five years, have attained considerable prominence in communications theory and in image processing.

Through Walsh analysis, which is analogous to Fourier analysis, the fundamental 60 Hz component of current and voltage signals can rapidly be extracted and measured for amplitude and phase. The apparent impedance of the line is then calculated from these four quantities. The speed and accuracy with which this is done will be compared with a competitive algorithm based upon 12-point Fourier analysis. Relevance of the method to multiple-line relaying will be touched upon.

Fourier-Walsh theory

Walsh analysis decomposes a function into a set of waves which are square-waves and square-wavelike. Since impedance of a linear system is defined in terms of fundamental-frequency voltage and current sine waves, it is necessary—if one works in the Walsh domain—to establish a connection between the results of the Walsh analysis and the fundamental sine waves of Fourier analysis. In the following section we shall establish this connection.

The methodology is called the Fourier-Walsh theory. Note that it is perfectly proper for us to seek to calculate impedance pre-fault and post-fault because the power system has made a transition from one linear system to another at the time of the fault. Of course, there are instances in which the increase in fault-current drives certain components into nonlinear regimes, but these cases will not be considered here.

First, we consider the reason that Fourier analysis is too slow for relaying on a real-time basis; i.e., the computation of impedance as each new sample of data is received will be briefly discussed. In what follows, we shall confine our attention to one-cycle relaying primarily because exposition of both Fourier and Walsh methods is most convenient when the time interval chosen for analysis equals the period of the signal. Also, one-cycle relaying appears to be adequate for several large power systems in the United States.

Ramamoorty [5] was the first to propose the use of Fourier analysis rather than the method of Mann and Morrison. Sampling the waveform at 20 points per cycle he finds the (best) sine wave through this data (42 Multiplies and 39 Adds are required). On the IBM System/7, for example, this takes 4.6 ms. Hence, such computers cannot make this fit in real time because the sampling interval is 0.8 ms.

The method is based on a Fourier analysis of the waveform. The ensemble of samples over a period of one cycle is assumed to repeat periodically, and Fourier analysis is performed on the ensemble of samples. The amplitude and phase-angle of the fundamental component are obtained as follows:

$$a_1 = \frac{2}{m}\left(\frac{f_0}{2} + f_1 \cos x + f_2 \cos 2x + \cdots f_{m-1} \cos (m-1)\, x + \frac{f_m}{2}\right),$$

$$b_1 = \frac{2}{m}\left(f_1 \sin x + f_2 \sin 2x + \cdots f_{m-1} \sin (m-1)\, x\right), \quad (1)$$

where $f_0, f_1, f_2, \cdots, f_m$ are sampled values of input signal over a period of one cycle, and x is the sampling interval given by $2\pi/m$. The factors $\cos x$, $\sin x, \cdots \cos mx$, $\sin (m-1)x$ are constants and can be calculated and stored in the computer a priori, as weighting functions on the sample values. The fundamental quantity is given by

$$g(t) = \sqrt{a_1^{\,2} + b_1^{\,2}}\, \sin[wt + \arctan\,(b_1/a_1)].$$

If this computation is made for both voltage and current, the impedance and phase-angle can be evaluated. Ramamoorty claims excellent impedance results (better results than using the Mann and Morrison method) for faults on a model line in the laboratory. However, this is only to be expected because he employs 20 points (compared with 5 for Mann and Morrison) and uses a longer sampling interval (0.8 ms to their 0.5 ms). Ramamoorty is simply willing to trade time for accuracy. According to the sampling theorem, 20 points per cycle should yield good information at about the tenth harmonic. However, this appears to be unwarranted detail. The fifth harmonic could provide enough; i.e., it would be sufficient to determine whether there was an arcing fault. Hence, 10 points per cycle should be more than enough for this method. Reducing the number of calculations by half [6] can be achieved by noting that $\cos x = \cos (m-1)\, x$, etc., and that $\sin x = -\sin (m-1)\, x$; hence, we have for a_1 and b_1, with m even.

$$a_1 = \frac{2}{m}\left[\frac{f_0}{2} + (f_1 + f_{m-1}) \cos x + (f_2 + f_{m-2}) \cos 2x + \cdots \left(f_{\frac{m}{2}-1} + f_{\frac{m}{2}+1}\right) \cos \left(\frac{m}{2}+1\right)x + \frac{f_m}{2}\right];$$

$$b_1 = \frac{2}{m}\left[\frac{f_0}{2} + (f_1 - f_{m-1}) \sin x + (f_2 - f_{m-2}) \sin 2x + \cdots \left(f_{\frac{m}{2}-1} + f_{\frac{m}{2}+1}\right) \sin \left(\frac{m}{2}-1\right)x\right]. \quad (2)$$

This becomes a fairly fast algorithm for fitting the best sine wave to the data. One might use 10 points per cycle; compute a_1 and b_1, with 5 Multiplies each and 5 Adds each, plus one more Add for a_1. This takes about 10 ×

wal(0, t′)	++++++++++++++++
wal(1, t′)	++++++++--------
wal(2, t′)	++++--------++++
wal(3, t′)	++++----++++----
wal(4, t′)	++----++++----++
wal(5, t′)	++----++--++++--
wal(6, t′)	++--++----++--++
wal(7, t′)	++--++--++--++--
wal(8, t′)	+--++--++--++--+
wal(9, t′)	+--++--+-++--++-
wal(10, t′)	+--+-++--++-+--+
wal(11, t′)	+--+-++-+--+-++-
wal(12, t′)	+-+--+-++-+--+-+
wal(13, t′)	+-+--+-+-+-++-+-
wal(14, t′)	+-+-+-+--+-+-+-+
wal(15, t′)	+-+-+-+-+-+-+-+-

t' 0 ½ 1

Figure 1. The first sixteen Walsh functions of integral index k.

$100 = 1$ ms on the IBM System/7 with its software multiply/divide time of 100 μs. These times now seem more practical. Significant improvements could be achieved, especially for the System/7 and similar small computers, if multiplications could be avoided. This can be done by analyzing the function in terms of its Walsh functions, wal(k, t), instead of its sine and cosine functions. The a_1 and b_1 can be calculated from the appropriate Walsh coefficients, W_k.

The time-consuming multiplications, and even the squaring and square-rooting operations, in the Fourier analysis algorithm used by Ramamoorty, can be almost entirely eliminated if the function $g(t)$ is analyzed into its Walsh functions, wal(k, t).

This is possible because wal(k, t) has only two values, ±1, and so Walsh analysis can be performed by the operations of addition and subtraction. The first 16 of these functions are shown in Fig. 1. These functions resemble "squared up" sine and cosine functions, and form a complete orthonormal set. They are undefined at the points at which they change from ±1 to ∓1, but as these points are a set of measure zero, this is of no consequence.

We shall use the notation and development of N. Blachman [7] in what follows. Let $t' = t/T$. Let us define a Fourier expansion of $g(t)$ in the interval $(0, T)$ as

$$g(t) = F_0 + \sqrt{2}\, F_1 \sin\frac{2\pi t}{T} + \sqrt{2}\, F_2 \cos\frac{2\pi t}{T} + \sqrt{2}\, F_3 \sin\frac{4\pi t}{T}\, T \cdots, \tag{3}$$

and a Walsh expansion as

$$g(t) = \sum_{k=0}^{\infty} W_k \,\mathrm{wal}(k, t/T). \tag{4}$$

In these expansions

$$F_0 = \frac{1}{T}\int_0^T g(t)\, dt,$$

$$F_1 = \frac{\sqrt{2}}{T}\int_0^T g(t)\, \sin\frac{2\pi t}{T}\, dt,$$

$$F_2 = \frac{\sqrt{2}}{T}\int_0^T g(t)\, \cos\frac{2\pi t}{T}\, dt,$$

and

$$W_k = \frac{1}{T}\int_0^T g(t)\, \mathrm{wal}(k, t')\, dt. \tag{5}$$

The set of components F_k form a vector in Hilbert space and so does the set W_k. The two vectors are related by the orthogonal matrix A; thus,

$$\mathbf{W} = A\mathbf{F}.$$

Since $A^{-1} = A^t$ where A^t is the transpose of A, we also have

$$\mathbf{F} = A^t\mathbf{W}.$$

The matrix A has been found to be, in part,

$$A = \begin{bmatrix}
1 & & & & & & & & & & \\
 & 0.900 & & & & 0.300 & & & & 0.180 & \\
 & & 0.900 & & & & -0.300 & & & & 0.180 \\
 & & & 0.900 & & & & & & & \\
 & & & & 0.900 & & & & & & \\
 & -0.373 & & & & 0.724 & & & & 0.435 & \\
 & & +0.373 & & & & 0.724 & & & & -0.435 \\
 & & & & & & & 0.900 & & & \\
 & & & & & & & & 0.900 & & \\
 & -0.074 & & & & -0.484 & & & & 0.650 & \\
 & & -0.074 & & & & 0.484 & & & & 0.650
\end{bmatrix}.$$

Now suppose $g(t)$ is $g(t) = \sqrt{2}\, F_1 \sin 2\pi t/T$. If we were to pass this through a "Walsh filter" to determine its set of Walsh coefficients $\{W_k\}$, $k \le 10$, we should measure

$$W_1 = 0.900\, F_1, \qquad W_5 = -0.373\, F_1,$$

$$W_9 = -0.074\, F_1.$$

All other W_k, $k \le 10$, are zero. Now, given $\mathbf{W}$ what is $\mathbf{F}$?

It is $\mathbf{F} = A^t\mathbf{W}$. We want only the sine and cosine coefficients which are F_1 and F_2, respectively. These immediately are found to be

$$F_1 = 0.900W_1 - 0.373W_5 - 0.074W_9,$$

$$F_2 = 0.900W_2 + 0.373W_6 - 0.074W_{10}. \tag{6}$$

Inserting the measured values of W_k we find

$$F_1 = 0.810F_1 + 0.139F_1 + 0.0055F_1,$$

$$F_2 = 0. \tag{7}$$

Consequently, if we use only W_1, and drop W_5 and W_9, we see from Eq. (7) that one makes an error of 19 percent in calculating F_1. If we use W_1 and W_5, dropping W_9, we make an error of 5 percent in calculating F_1. Hence, only W_0, W_1, W_2, W_5, and W_6 seem to be needed to represent (to 5 percent accuracy) an arbitrary sinusoid, plus some dc component, in the interval $(0, T)$. Very fast algorithms can be devised to calculate the W_k.

Let us sample $(m+1)$ times in the interval $(0, T)$ at $t = 0, \cdots, t = j(\Delta t), \cdots$, and at $t = m(\Delta t) = T$. From the definition of W_k in Eq. (5), we must perform a numerical integration which closely approximates the integral. We shall use the trapezoidal rule, although Simpson's rule, which is more accurate, could also have been chosen. Let $I(j_1, j_2)$ denote the trapezoidal rule integral of the integrand $f(j\Delta t) \equiv g(j\Delta t)\ \mathrm{wal}(k, j\Delta t')$ from $t_1 = j_1 \Delta t$ to $t_2 = j_2 \Delta t$, in which interval $f(t)$ is continuous and has continuous derivatives. Then, from Eq. (5),

$$W_k = \sum_{j_1 < j_2} I(j_1, j_2), \qquad j_1 = 0, \cdots, j_2 =, \cdots, m, \tag{8}$$

which simply means that we sum up the "pieces." Each piece is, according to the trapezoidal rule,

$$I(j_1, j_2) = \tfrac{1}{2} f(j_1) + f(j_1 + 1) + \cdots f(j_2 - 1) + \tfrac{1}{2} f(j_2) \tag{9}$$

Let us simplify the notation for discrete $g(t)$ and $\mathrm{wal}(k, t')$ as follows. Define

$$g_j \equiv g(j\Delta t),$$

$$w_j(k) \equiv \mathrm{wal}(k, j\Delta t'),$$

so that

$$f_j = g_j \cdot w_j(k).$$

Let us evaluate W_k as each new sample of $g(t)$ is taken. We must drop off g_0 (the "oldest" value) and add or subtract g_{m+1} (the "newest" value). As a general procedure, we imagine $g(t)$ to be stepped leftwards s steps corresponding to a time displacement $s\Delta t$. The integrand now becomes

$$f_{j+s} = g_{j+s}\, w_j(k), \qquad s = 0, 1, 2, \cdots.$$

As an example, let $m = 8$, corresponding to double the intervals shown in Fig. 1. Then, with the help of Fig. 1, it is easy to show that

$$W_0(s) = \tfrac{1}{8}\left(\tfrac{1}{2} g_s + g_{1+s} + \cdots + g_{m-1+s} + \tfrac{1}{2} g_{m+s}\right).$$

Hence, recursively,

$$8W_0(s+1) = 8W_0(s) - \tfrac{1}{2} g_s - \tfrac{1}{2} g_{1+s} + \tfrac{1}{2} g_{m+s} + \tfrac{1}{2} g_{m+1+s}, \qquad s = 0, 1, 2, \cdots.$$

Also, recursively, it follows from Eqs. (5), (8), and (9) that

$$8W_1(s+1) = 8W_1(s) - \tfrac{1}{2} g_s - \tfrac{1}{2} g_{1+s} + g_{4+s} + g_{5+s} - \tfrac{1}{2} g_{8+s} - \tfrac{1}{2} g_{9+s}, \qquad s = 0, 1, 2 \cdots,$$

and

$$8W_2(s+1) = 8W_2(s) - \tfrac{1}{2} g_s + \tfrac{1}{2} g_{1+s} + g_{2+s} + g_{3+s} - g_{6+s} - g_{7+s} + \tfrac{1}{2} g_{8+s} + g_{9+s},$$
$$s = 0, 1, 2, \cdots.$$

This is the same number of Adds as would be used when calculating W_k directly as

$$8W_2(s) = \tfrac{1}{2} g_s + g_{1+s} - g_{3+s} - g_{4+s} - g_{5+s} + g_{7+s} + \tfrac{1}{2} g_{8+s}.$$

We find by direct application of the trapezoidal rule that

$$8W_5(s) = \tfrac{1}{2} g_s - g_{2+s} + g_{6+s} - \tfrac{1}{2} g_{8+s}, \qquad s = 0, 1, 2, \cdots,$$

and, directly also, that

$$8W_6(s) = \tfrac{1}{2} g_s - g_{4+s} + \tfrac{1}{2} g_{8+s}, \qquad s = 0, 1, 2 \cdots.$$

Note that except for the end points of the interval $(0, T)$, application of the trapezoidal rule has the interesting and valuable results of "setting-to-zero" that value of the integrand at which a jump occurs. The example shows that 34 Adds serve to compute W_0, W_1, W_2, W_5, and W_6. Division by 2 is a Shift Right operation. On the System/7, it takes 50 ns for each shift of one position; however, the Shift instruction takes an instruction cycle of 0.4 μs. If we increase the number of samples taken from 9 to 17, only 8 more Adds occur (in W_5 and W_6). Since addition on the System/7 takes 0.8 μs, these coefficients are ready in $42 \times 0.8 = 34$ μs.

- *Impedance algorithm*

In this study a one-cycle sampling interval or "window" was used. It is not until the window is sampling one cycle post-fault that a meaningful impedance value is calculated. At this time, if sinusoidal behavior is reestablished (there may be a dc component), Walsh analysis is very simple. It is only necessary to calculate W_1 and W_2 in order to know F_1 and F_2, the sine and cosine functions, respectively. As we have shown, the only computer operations needed in this calculation are addition, subtraction, and shift. If there is decaying "dc offset" post-fault (and there generally is), higher order Walsh coefficients may be necessary to attain the desired degree of accuracy in impedance; also if third and fifth harmonics are present (characteristic of arcing faults) the same requirement may apply. The number of Walsh coefficients needed to get F_1 and F_2 accurately is an important subject. However, since this relies upon the field conditions and the speed of relaying desired (one cycle, or two cycles, etc.), we will not pursue it further here. Eventually all post-fault transients do die out; sinusoidal conditions are reestablished, and then W_1 and W_2 are sufficient to cal-

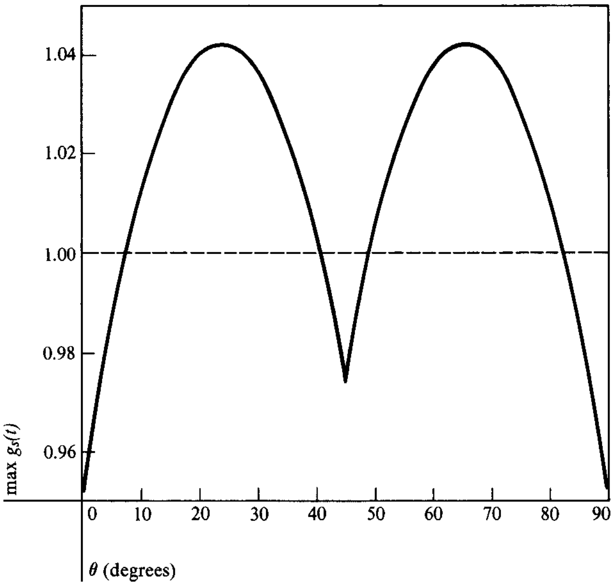

Figure 2. Accuracy of Amplitude and Phase Theorem.

culate F_1 and F_2. Our rule would be: calculate W_1 and W_2, and then calculate as many $W_k (k > 2, \leq 15)$ as there is time for.

Let us calculate the Walsh coefficients W_0, W_1, W_2, W_5, W_6, W_9, and W_{10} of the signal, $g(t)$ in the time interval $(t, t + T)$. (In practice, all of these may not be needed, as was noted in the preceding paragraph.) We can then calculate its fundamental sine and cosine components of amplitude, $\sqrt{2}\, F_1$ and $\sqrt{2}\, F_2$ respectively, according to Eq. (6). The amplitude of the sinusoid is $\sqrt{2}\sqrt{F_1^2 + F_2^2}$. The impedance is then, by definition,

$$|Z| \equiv \frac{\sqrt{F_1^2 + F_2^2}_{\text{voltage}}}{\sqrt{F_1^2 + F_2^2}_{\text{current}}}. \tag{10}$$

On the System/7, multiplication takes 100 μs; consequently, even $|Z|$ cannot be calculated in real time if the sampling interval is 500 μs (4 multiplies plus one divide $= 5 \times 100\ \mu\text{s} = 500\ \mu\text{s}$). Also, it would take even more time to do a phase-of-Z = arctan $(F_1/F_2)_{\text{voltage}}$ − arctan $(F_1/F_2)_{\text{current}}$.

Another approach to this problem, which *avoids* the square-root and squaring operations, is as follows. Given the Walsh coefficients, we can represent the fundamental sine wave $g_s(t)$ approximately as

$$g_s(t) = \sum_{k=1}^{10} W_k \,\text{wal}(k, t/T).$$

The maximum of this function may then be expressed as a sum of terms $\pm W_k$, since wal$(k, t/T) = \pm 1$. If $g(t)$ can be assumed sinusoidal, the question is, what is this sum precisely? The solution to this problem is given in the following theorem.

Amplitude and Phase Theorem Given a sinusoid $g_s(t)$ with period T, let the interval T be divided into 16 equal sub-intervals of duration Δt, and let these sub-intervals be numbered 1 − 16. Within T let $g_s(t)$ be expanded in the set of Walsh functions wal$(1, t')$, wal$(2, t')$, wal$(5, t')$, wal$(6, t')$, wal$(9, t')$, wal$(10, t')$; thus, with $t' = t/T$,

$$g_s(t) = \sum_{k=1}^{10} W_k \,\text{wal}(k, t').$$

Then to an accuracy of $(+4.28\%, -4.8\%)$, the maximum value of $g_s(t)$, max $g_s(t)$ is given in terms of $|W_1|$ and $|W_2|$ as

$$\max g_s(t) = 1.0822\,(|W_1| + |W_2|) + 0.414\big||W_2| - |W_1|\big|. \tag{11}$$

The amplitude of the sine wave equals max $g_s(t)$.

The time of occurrence of the maximum is that mth sub-interval within which the functions in the set [wal$(1, t')$, wal$(2, t')$, wal$(6, t')$, wal$(10, t')$] assume values given in terms of W_1 and W_2 according to the following formulas:

$$\begin{aligned}
\text{wal}(1, m\Delta t') &= S(W_1),\\
\text{wal}(2, m\Delta t') &= S(W_2),\\
\text{wal}(6, m\Delta t') &= S(W_2)\ S(|W_2| - |W_1|),\\
\text{wal}(10, m\Delta t') &= -S(W_2),
\end{aligned} \tag{12}$$

where $S(x)$ denotes a function which is "+" when x is positive and "−" when x is negative; i.e., $S(x)$ is the "sign of x," $x > 0$, $x < 0$. For $x = 0$ both + and − signs are to be used, and the maximum is not unique.

Since the proof [8] is somewhat lengthy for inclusion in this paper, we shall not give it. Furthermore, it is straightforward to prove it numerically for all cases to which it applies. Equation (11) can be checked against the true value, true max $g_s(t)$, which is the amplitude of the sinusoid:

$$\text{true max } g_s(t) = \text{Amplitude} = \sqrt{2}\sqrt{F_1^2 + F_2^2}. \tag{13}$$

Again, for the sinusoid, it will be found that

$$W_1 = 0.900\ F_1,$$

$$W_2 = 0.900\ F_2.$$

These can be substituted into Eq. (11) to check against Eq. (13).

The deviation from true max varies from +4.2 to −4.8 percent depending on the ratio of $F_1{:}F_2$. This ratio, in turn, depends upon the time t of the first sample relative to the first (positive slope) zero-crossing of a sine wave. The value of max $g_s(t)$ from Eq. (11) is plotted against $\theta = 360° \times (t/T)$ in Fig. 2.

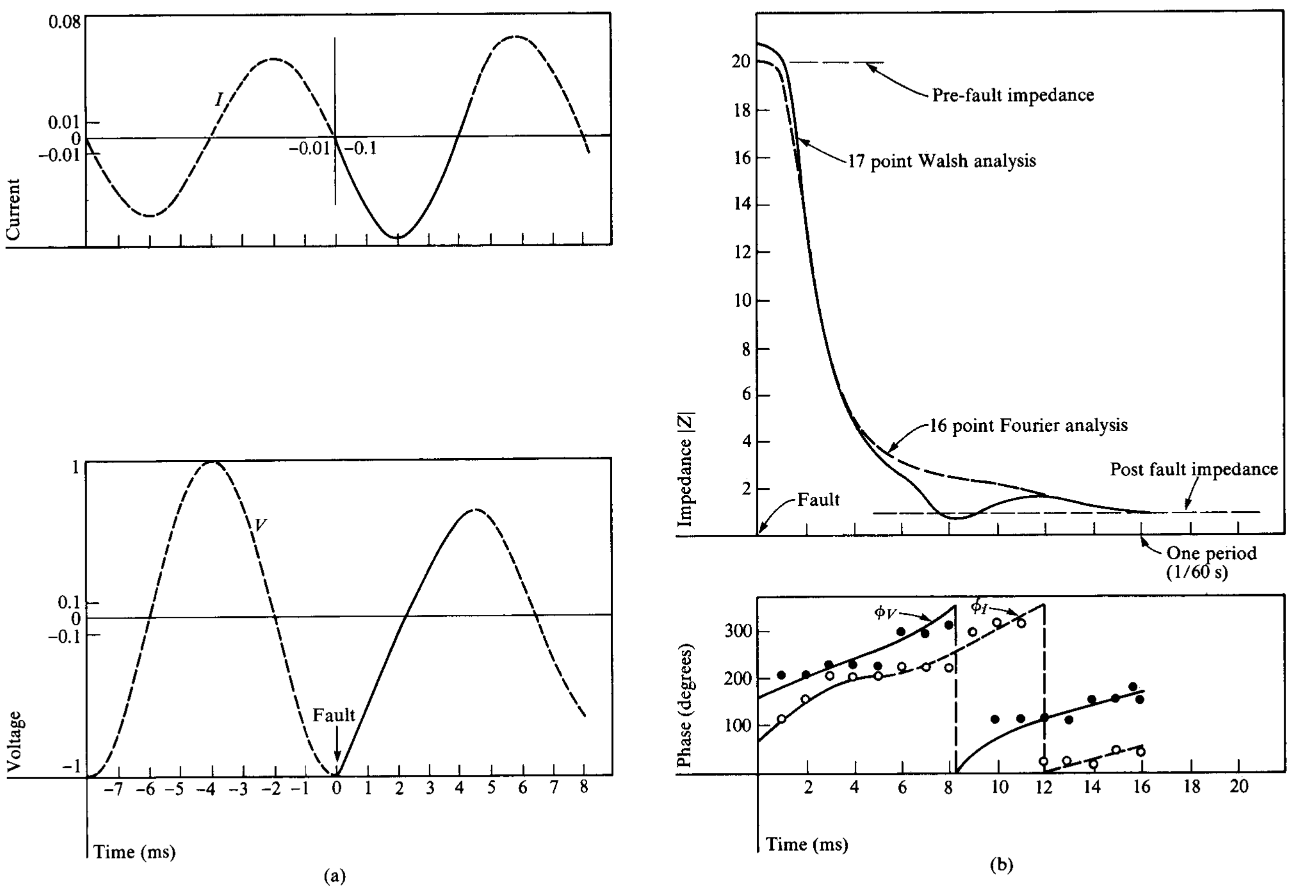

Figure 3 (a) Data for fault A; (b) impedance calculation for fault A

The accuracy of the expression in the Amplitude and Phase Theorem for max $g_s(t)$ can be increased by increasing the number of samples and the number of Walsh functions, as shown in subsequent sections. In the derivation of Eq. (11), Walsh functions up to and including wal$(9, t')$ and wal$(10, t')$ were used to approximate the sinusoid. Greater accuracy required higher sequency Walsh functions and therefore further subdivision of the period T.

Using Eq. (10), the impedance is

$$|Z| = \frac{\{1.0822(|W_1| + |W_2|) + 0.414||W_2| - |W_1||\}_{\text{volt.}}}{\{\text{same}\}_{\text{current}}}. \tag{14}$$

It is important to note here that W_1 and W_2 are coefficients of $g_s(t)$, which is the best sine wave fit to $g(t)$; Eq. (14) presumes that $g_s(t)$ is somehow available, which is to say that F_1 and F_2 are somehow known. Then, $W_1 = 0.900\ F_1$, and $W_2 = 0.900\ F_2$. We propose, using Eq. (6) to find F_1 and F_2, keeping only needed terms in W_k. In Eq. (6) these are coefficients of $g(t)$, the raw data. The two sets of W_k should not be confused.

Two useful forms of Eq. (11) that accurately represent Eq. (11) to an accuracy better than 1/2 percent are

$$\begin{aligned} \max g_s(t) &\doteq \tfrac{2}{3}|W_1| + \tfrac{3}{2}|W_2| \qquad |W_2| > |W_1|, \\ &\doteq \tfrac{3}{2}|W_1| + \tfrac{2}{3}|W_2| \qquad |W_1| > |W_2|. \end{aligned}$$

$$\begin{aligned} \max g_s(t) \doteq \frac{1}{(0.414)} [& (2 + \tfrac{1}{16})(|W_1| + |W_2|) \\ & + ||W_2| - |W_1||]. \end{aligned}$$

The last formula is especially suited for performing the multiplication by Shift Left and Shift Right instructions, which take only 0.4 μs each on the System/7.

Test of theory

Figure (3a) shows fault data taken from the literature ([3], Fig. 7). We took two 60-Hz periods and performed first a 16-point Fourier analysis and then a 17-point Walsh analysis. We calculated the following quantities: a) F_1 and F_2 in

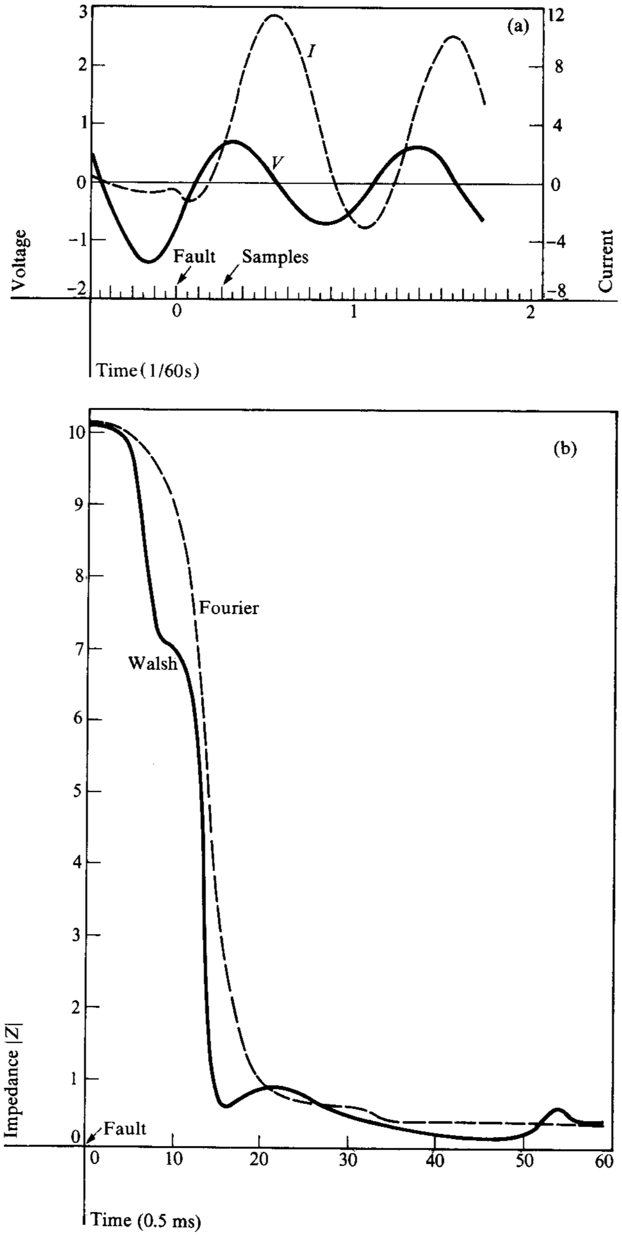

Figure 4 (a) Data for Fault B; (b) impedance calculation for fault B.

$$g(t) = F_0 + \sqrt{2}\,F_1 \sin\frac{2\pi t}{T} + \sqrt{2}\,F_2 \cos\frac{2\pi t}{T} + \cdots$$

from a 16-point Fourier analysis, and b) W_1, W_2, W_5, and W_6 from a 17-point Walsh analysis. The formulas for W_1, W_2, W_5, and W_6 are given in Appendix A.

For a), $|Z|$ is calculated from Eq. (10), and for b) $|Z|$ is calculated from Eq. (14) using Eq. (6) without the terms W_9 and W_{10} to get F_1 and F_2.

The impedance value is calculated each time the one-period "window" is stepped to the right, one step at a time. At time t, the window is the interval $[t - T, t]$. The results are plotted in Fig. (3b). We see that the initial and final impedances, which were arranged to be 20.0 and 1.00, are correctly calculated. The Fourier values decrease monotonically, whereas the Walsh values oscillate, going below the final value by 30 percent. It is not surprising that there is an oscillation since digital filters such as the Walsh algorithm behave much like electrical filters in their response to a transient signal. This, in the present case, is the discontinuity at the fault. The interesting feature is that the Fourier method produces a monotonically decreasing $|Z|$, because this is a more desirable—perhaps requisite—kind of behavior when thresholds of $|Z|$ are set for tripping. A second example is shown in Figs. 4(a) and (b). Data were taken from [9]. Here, note the dc offset.

• *Phase of impedance*

So far we have considered only the modulus of Z. However, the Walsh function method can be used to get the phase-angle, depending upon the number of points used per cycle. For 16-sub-intervals per cycle (17 points), the time of the maximum of the sinusoid can be obtained to within $360°/16 = 22.5°$. Although requirements on the phase-angle have not been discussed in relaying literature, it would seem that an accuracy to within 7° (achieved by Mann and Morrison) is a better target.

A very fast method of getting phase to within 22.5°, is to use the Amplitude and Phase Theorem, Eq. (12), which yields the values of wal$(1, t')$, wal$(2, t')$, wal$(6, t')$ and wal$(10, t')$ when $g_s(t)$ assumes its maximum value. Reference to Fig. 1 will permit one to decode this information into a time, or times, of occurrence. A 16-element table can be built up in this way. Table look-up can be done in a few microseconds on most minicomputers; hence the approximate phase determination is extremely fast. However, reference to Fig. 3(b), in which ϕ_V and ϕ_I are plotted for a simulated fault, shows that there is too much scatter in the results to make this method useful. The reason for this was investigated.

We have found that by analyzing sine waves by the Walsh method, and then plotting the inverse Walsh transform, the original sine wave is poorly recovered, even though the maximum value comes out quite well (as seen also in Fig. 2). An example of this is shown in Fig. 5, where Walsh functions through wal$(10, X)$ are used to represent $\sin x$. The trouble is that [10] we must take higher order Walsh functions. In fact, analysis shows that if 16 points are sampled, 16 Walsh functions wal$(0, t') \cdots$ wal$(15, t')$ must be used in the transform and in the inverse transform. This is generally true for

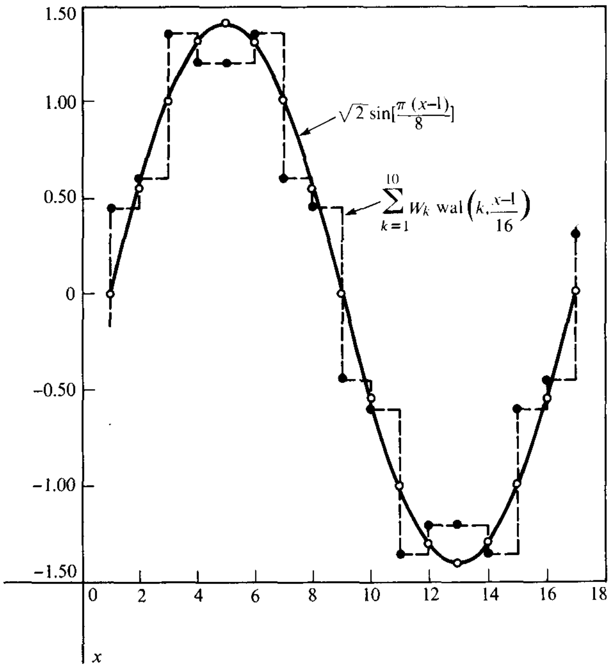

Figure 5. Approximate representation of the sine function by Walsh functions.

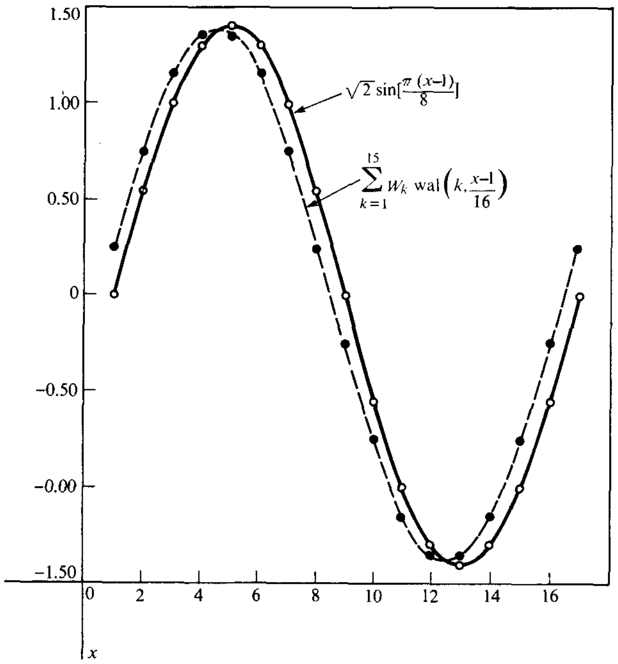

Figure 6. Exact representation of the sine function by Walsh functions.

any orthonormal set of functions and not just for Walsh functions. The success of this is shown in Fig. 6, wherein the inverse transform is seen to plot smoothly as a sine wave. Interestingly enough, however, it is phase shifted by 11.25° because the trapezoidal rule is used in evaluating the W_k; also, its amplitude appears to be slightly reduced. Nevertheless by taking all 16 Walsh functions, the original data set is essentially recovered and it seems that a table look-up method for phase would be successful, given the algorithm appropriate to the full set of Walsh functions. This is supplied in the following theorem.

Second Amplitude and Phase Theorem Given a sinusoid $g_s(t)$ with period T, let the interval T be divided into 16 equal sub-intervals. Within T let $g_s(t)$ be expanded in the set of Walsh functions wal$(1, t')$, wal$(2, t') \cdots$ wal$(15, t')$, so that

$$g_s(t) = \sum_{k=1}^{15} W_k \operatorname{wal}(k, t'). \tag{15}$$

Then to an accuracy ±2.6 percent, the maximum value, max $g_s(t)$, is given in terms of W_1 and W_2 by two different formulas, $\max_1 g_s(t)$ and $\max_2 g_s(t)$, depending upon which region of the $(|W_1|, |W_2|)$-space the point $(|W_1|, |W_2|)$ lies within,

$$\max_1 g_s(t) = (\alpha - \gamma)|\Delta| + (1 + \beta)\Sigma$$
$$\text{when } (\alpha - \gamma)|\Delta| \leq \beta\Sigma,$$

or

$$\max_2 g_s(t) = (\alpha + \gamma)|\Delta| + (1 - \beta)\Sigma$$
$$\text{when } (\alpha - \gamma)|\Delta| > \beta\Sigma. \tag{16}$$

The maximum lies in the sub-interval for which

$$\operatorname{wal}(1, m\Delta t') = S(W_1),$$
$$\operatorname{wal}(2, m\Delta t') = S(W_2),$$
$$\operatorname{wal}(6, m\Delta t') = S(W_2)\, S(\Delta),$$
$$\operatorname{wal}(10, m\Delta t') = -S(W_2), \qquad \text{if } (\alpha - \gamma)|\Delta| \leq \beta\Sigma,$$

but

$$\operatorname{wal}(10, m\Delta t') = S(W_2), \qquad \text{if } (\alpha - \gamma)|\Delta| > \beta\Sigma.$$

Here

$$\Delta \equiv -|W_1| + |W_2|, \qquad \Sigma \equiv |W_1| + |W_2|,$$

and

$$\alpha = 0.414, \qquad \gamma = 0.198, \qquad \beta = 0.0823$$

The function $S(x)$ has the meaning already given to it in the previously given theorem. Again, it would take too much space to give the proof here, but this is not necessary since it may be adequately checked by moving a sine wave through the sampling window in 16 equal phase increments of 22.5° each.

A scheme for getting an accurate phase angle would then be to use the above theorem to get the approximate time(s), and hence phase(s), of the maximum and then calculate $g_s(t)$, the sinusoid that best fits $g(t)$, near the expected zero-crossing. The time of this is known immediately by incrementing or decrementing t_m by a quarter-period. For example, we seek the value of τ when

$$g_s\left(t_m \pm \frac{T}{4} + \tau\right) = \sum_{k=1}^{15} W_k \operatorname{wal}\left(k, \frac{t_m}{T} \pm \tfrac{1}{4} + \frac{\tau}{T}\right) = 0. \quad (17)$$

One way of doing this is to compute $g_s(t)$ on both edges of the sub-interval $[t_a, t_b]$ in which the zero-crossing is expected. By comparing $(g_s(t_a) + g_s(t_b))/(2$ and $4)$ with $g_s(t_a)$ and $g_s(t_b)$, the crossing can be localized to one-quarter of the interval, hence within 5.6°. This requires 18 Adds, four Multiplies, and four Compares.

• *The 12-point Fourier analysis*

By deciding to divide the Fourier analysis period T into 12 parts, data-weighting factors such as $\sin x$, $\sin 2x$, etc. and $\cos x$, $\cos 2x$, become either $\pm 1/2$ or $\pm\sqrt{3}/2$. Since fast (20 μs on the System/7) multiplying programs can be devised for fixed constant multiplication, and since division by 2 is a Shift Right operation (requiring only 400 ns on the System/7), this method could be competitive with Walsh analysis. The method analyzes the waveforms as [11]

$$g(t) = a_0 + a_1 \cos x + a_2 \cos 2x + \cdots a_6 \cos 6x$$
$$+ b_1 \sin x + b_2 \sin 2x + \cdots b_5 \sin 5x,$$
$$x = 0, \frac{\pi}{6}, \frac{2\pi}{6}, \cdots \frac{11\pi}{6}.$$

Data points are denoted as $u_0, u_1, u_2, \cdots u_{11}$. We are interested in a_0, a_1, and b_1. These are

$$12a_0 = p_0 + p_1 + p_2 + p_3$$
$$6a_1 = q_0 + \frac{\sqrt{3}}{2} q_1 + \tfrac{1}{2} q_2,$$
$$6b = \tfrac{1}{2} r_1 + \frac{\sqrt{3}}{2} r_2 + r_3, \quad (18)$$

where

$$p_0 = u_0 + u_6, \quad q_0 = u_0 - u_6, \quad r_1 = w_1 + w_5,$$
$$p_1 = v_1 + v_5, \quad q_1 = v_1 - v_5, \quad r_2 = w_2 + w_4,$$
$$p_2 = v_2 + v_4, \quad q_2 = v_2 - v_4, \quad r_3 = w_3, \quad (19)$$

and where

$$v_1 = u_1 + u_{11}, \quad w_1 = u_1 - u_{11},$$
$$v_2 = u_2 + u_{10}, \quad w_2 = u_2 - u_{10},$$
$$v_3 = u_3 + u_9, \quad w_3 = u_3 - u_9,$$
$$v_4 = u_4 + u_8, \quad w_4 = u_4 - u_8,$$
$$v_5 = u_5 + u_7, \quad w_5 = u_5 - u_7.$$

The important point to notice is that there are only two multiplications by $\sqrt{3}$; each would take about 20 μs on the System/7. All other operations take less than 1 μs. Time to do Eqs. (18) and (19), "preparation time," is 45.2 μs. Time required for $6a_1$ is 23.6 μs (this includes one multiplication by $\sqrt{3}$), and time for $6b_1$ is also 23.6 μs.

Hence, the time to go from data $\{u_1\}$ to $6a_1$ and also $6b_1$ is 82.8 μs. A Walsh analysis requires the following:

$0.460\, F_1 = [(2 + \tfrac{1}{16})\, W_1 - W_5]$: 2.8 μs;

$0.460\, F_2 = [(2 + \tfrac{1}{16})\, W_2 + W_6]$: 2.8 μs.

To get W_1, W_2, W_5, and W_6 from data requires:

for both W_1 and W_5: 12 μs, and

for both W_2 and W_6: 14.8 μs.

Hence, to get $0.460\, F_1$ and $0.460\, F_2$ requires:

$(0.460)\, F_1$: $2.8 + 12 = 14.8$ μs;

$(0.460)\, F_2$: $2.8 + 14.8 = 17.6$ μs.

Thus, the difference between the Walsh- and Fourier-analysis times on the System/7 is

Walsh: $0.460\, F_1$ and $0.460\, F_2$: 32.4 μs;

Fourier: $6a_1$ and $6b_1$: 84.8 μs.

If, in the Fourier method, $\frac{\sqrt{3}}{2}$ is represented as

$$\sqrt{3}/2 \doteq \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} - \tfrac{1}{128}$$

and Shift Right and Add operations are used in the System/7, then a multiplication time closer to 4 μs than 20 μs will be obtained. Also, instead of 84.8 μs, the time for $6a_1$ and $6b_1$ would be 52.8 μs. Hence, the Walsh method is at least 50 percent faster than the Fourier method. However, it may be more inaccurate if there are significant deviations from the sinusoid. The Walsh algorithm should be tested when there are some third and fifth harmonics present, as for an arcing fault.

An impedance calculation on a System/7 using the amplitude and phase formula, Eq. (11), for both Walsh and Fourier methods (i.e., obtaining F_1 and F_2 by Fourier analysis and then using $W_1 = 0.9\, F_1$ and $W_2 = 0.9\, F_2$ in (11)) takes

Impedance 17-point Walsh:

35.2 (numerator) + 35.2 (denominator)

+ 100 (divide) = 170.4 μs,

12-point Fourier:

$$55.6\text{ (numerator)} + 55.6\text{ (denominator)} + 100\text{ (divide)} = 211\ \mu\text{s},$$

accuracy: $\pm$ 8 percent in $|Z|$ (Walsh).

Hence, both these methods are competitive. The 17-point Walsh accuracy is not very good; $\pm$5 percent is a better target figure. The $\pm$8 percent inaccuracy is assigned on the basis of the stated inaccuracy of the amplitude and phase formula, Eq. (11), used to obtain the voltage and current amplitudes.

To better illustrate the speed advantages of the new method, it is instructive to a) calculate a_1 and b_1 (proportional to F_1 and F_2, respectively) from 12-point Fourier analysis, as we have just done, and then to use these values in the usual impedance formula, Eq. (10); and b) to obtain F_1 and F_2 by 17-point Walsh analysis, as we have just done, but to use the more accurate ($\pm$5 percent) formulas for $|Z|$ given in Appendix B, which are derived from the Second Amplitude and Phase Theorem. On a machine having a 1.6 μs Add (Subtract), 20 μs Multiply, 30 μs Divide, the straight Fourier method takes 410 μs, whereas our method takes 160 μs. We presume that for the former, the square-root-of-c iteration, $x_{n+1} = (x_n + c/x_n)/2$, takes four steps.

Early warning of a fault

For early warning, the fastest impedance algorithm we know, in the absence of a fast hardware multiply and divide, is the following. Since we are monitoring an impedance threshold $|Z_{\text{Ref}}|$ and will signal an Alert when $|Z| \leq |Z_{\text{Ref}}|$, we can then monitor the voltage amplitude $\hat{V}$ and signal when

$$\hat{V} \leq \hat{I}|Z_{\text{Ref}}|. \tag{20}$$

$|Z_{\text{Ref}}|$ is a fixed constant, and its multiplication by $\hat{I}$ can be done with a relatively fast (20 μs) program involving Shift Operations on $\hat{I}$. With the use of the Amplitude and Phase Theorem for $\hat{V}$ and $\hat{I}$, Eq. (20) can be verified in about 40 μs. Also, with the phase-algorithm given in the Second Amplitude and Phase Theorem, the phase of Z can be found to $\pm$11.25° in about 10 μs. Now we ask, how many lines can be monitored using this algorithm alone?

• *Multiple-line problem*

Assume all corresponding voltage buses for n lines are common. There are now three voltages to be measured. There are six impedances to be calculated for each line, and the data must be converted into digital form and stored for quick access. Each line should be looked at every ms to catch the fault at once. The computation and processing times add as follows:

(Convert and store) + Calculate impedance = 1000 μs.

Let CS_1 denote the time to convert and store one data point, then

$$(3 + 3n)\ CS_1 + 6 \times 40 \times n = 1000,$$

and therefore

$$n = \frac{333 - CS_1}{80 + CS_1}.$$

For the System/7, the conversion time alone is at least 50 μs; hence n, at most, is 2. However, it would be more desirable if $n = 4$ or 6.

We could speed up the impedance calculation and decrease the conversion time. By using a Direct Memory Access (DMA) channel and a 1-μs ADC unit and commutator, the conversion and storage time per point might be brought down to a few microseconds, in which case $n = 4$. Thus, we would not need to alter the cycle time of the computer nor find new algorithms for $\hat{V}$, $\hat{I}$, and $\hat{I}|Z_{\text{Ref}}|$. Nevertheless, some improvement here would be necessary to reach $n = 6$.

It appears, then, that the early warning aspect of the multiple-line problem can perhaps be solved with presently available equipment and algorithms. A device that could both scan analog input lines and convert the analog value to a digital number in 2 μs is not really a "standard" feature, but it is available. A DMA channel is available only on some machines. The cost of these devices is well justified, however, considering the improvement from $n = 2$ to $n = 4$.

Using the Walsh and 12-point Fourier algorithms (given in the preceding paragraphs), and high-speed data-input devices, we can begin to feel that the multiple-line problem can be solved with a single minicomputer, and without special-purpose peripheral gear.

Discussion

The objective of this paper has been to perform an impedance calculation from raw data using only the operations of Add, Subtract, and Shift. Reference to Eq. (14) for impedance modulus, and to Eq. (16) ff. for phase of impedance shows that this plan has been quite successful. Why this is so, and when it may cease to be so is made clearer with reference to Fig. 7, a flow chart of our method. Along the top of the diagram the customary Fourier method of obtaining F_1 and F_2, the fundamental sine and cosine components of $g(t)$, is indicated. The Walsh "route" to F_1 and F_2 proceeds through the Walsh coefficients W_k, then through the transformation $\mathbf{F} = A^t\mathbf{W}$ represented by the vertical arrow. What we have shown in this paper is that the "long-cut," via the Walsh coefficients, can be much faster than the fastest Fourier method in obtaining the modulus of impedance, with

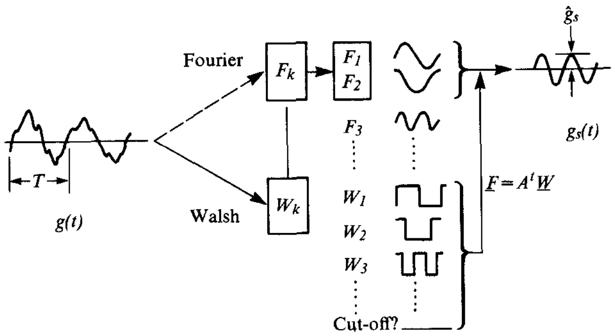

Figure 7. Flow chart of Fourier and Walsh methods for obtaining the fundamental of a function with period T.

sufficient accuracy. However, it is clear from Fig. 6 that this result is contingent upon not having to take too many W_k terms because the transformation to **F** and **W** requires non-integer type multiplications, which are usually much slower than additions and subtractions. In this paper, we have used W_1 and W_5 for F_1, and W_2 and W_6 for F_2. The number of Walsh coefficients needed to get F_1 and F_2 depends on the amount of harmonic and non-harmonic content of $g(t)$. The significance of this content has not yet been settled by field experience, and is currently being debated. Phadke et al. [12] say that the typical, observed, faulted waveform (analog filtered) is about as smooth as those exhibited in Figs. 3(a) and 4(a). This observation supports the formulas given. Phadke et al. caution, however, that the raw data should be corrected before use in the impedance algorithms discussed above for an exponentially decaying "dc offset" which is to be expected in faulted waveforms. They have devised an algorithm to do this, and have used it with the 12-point Fourier method to more accurately obtain the impedance modulus from laboratory data.

Accurate calculation of phase using only the Add, Subtract, and Shift operations has proved to be the more difficult problem. The second amplitude and phase theorem permits an extremely fast determination of phase by table look-up to an accuracy of $\pm 11.25°$; however, this operation must be preceded by a Compare operation using one multiplication by $(\alpha-\gamma)/\beta$, a constant. Hence, the Walsh method does yield a good approximate value of phase in essentially the time for one fixed-constant multiply. Based upon this approximate value, several algorithms look feasible for getting an accurate value in a time less than the customary, slow, arctangent operation. However, rather than develop these algorithms, which do not appear to be an order of magnitude faster than the arctangent, we note that the current practice [12] seems to be moving away from the use of phase as a decision parameter. Use tends to be made of the real and imaginary components of fault impedance, R_f and I_f, respectively, since these define a quadrilateral relay characteristic in the complex Z-plane. In the notation of this paper, the current and voltage phasors and the impedance are

$$I = \sqrt{2}\,(F_1 + j\,F_2),$$

$$V = \sqrt{2}\,(F_1' + j\,F_2').$$

Hence,

$$\begin{aligned} Z_f &= R_f + j\,I_f, \\ &= \frac{F_1'F_1 + F_2'F_2}{F_1^2 + F_2^2} + j\frac{F_1'F_2 - F_2'F_1}{F_1^2 + F_2^2}. \end{aligned} \tag{21}$$

While the Walsh method has been shown to be considerably faster than even the fastest Fourier method under the assumptions made in this paper, the 12-point Fourier method is certainly competitive from the standpoint of speed, and "naturally" recommends itself to the problem because the end result has to be the Fourier coefficients F_1 and F_2. Furthermore, it has been suggested that, by analog scaling of the input signals, the 12-point Fourier algorithm (see also [12]) can be implemented so much more quickly that it is comparable to the Walsh algorithms. Whether the introduction of an analog device to do the scaling would seriously degrade the high reliability expected of the digital algorithms is still a question to be answered.

An area not touched by this paper is the use of multiple processors for protective relaying; rather, we have chosen to explore the possibilities in fully utilizing a single minicomputer. The computer architecture most appropriate for substation control, monitoring, and protection is also a subject of much current investigation.

Summary

A new method for computing impedance has been elaborated. This method appears to be a suitable candidate for impedance relaying applications. Its advantage over methods based on Fourier analysis lies in its superior speed within acceptable limits of accuracy. The saving in time for this function can be well utilized by a substation computer either to perform other substation functions or to protect additional lines. Tests of one impedance algorithm using 17 samples per cycle exhibit a response to fault conditions that is at least as satisfactory as an equivalent algorithm based on standard Fourier analysis. The method leads to an extremely fast (50 μs) "early-warning" impedance-algorithm which offers an alternative approach to the multiple-line problem.

Acknowledgment

This paper was presented at the IEEE Power Engineering Society Summer Meeting, San Francisco, CA., July 20-25, 1975.

Appendix A: 17-point Walsh coefficients

Let $g(t)$ be sampled at 17 equally spaced points and denote these samples by $g_1, \cdots, g_{17}$. Denote the W_k for this interval as $W_k(1)$, and let $W_k(2)$ be the Walsh coefficients for the interval defined by samples $g_2, \cdots, g_{18}$, etc. The following formulas for $W_k(I)$ in terms of the 17 points in the sampling "window" are obtained from Eq. (5) using the trapezoidal rule for numerical integration. Note that W_1 and W_2 are done by recursion to reduce the number of terms.

$$W_0(1) = 0.5g_1 + g_2 + \cdots + g_{16} + 0.5g_{17},$$

$$\begin{aligned} W_1(1) = 0.5g_1 &+ g_2 + g_3 + g_4 + g_5 + g_6 + g_7 + g_8 \\ &- g_{10} - g_{11} - g_{12} - g_{13} - g_{14} - g_{15} - g_{16} \\ &- 0.5g_{17}, \end{aligned}$$

$$\begin{aligned} W_2(1) = 0.5g_1 &+ g_2 + g_3 + g_4 \\ &- g_6 - g_7 - g_8 - g_9 - g_{10} - g_{11} - g_{12} \\ &+ g_{14} + g_{15} + g_{16} + 0.5g_{17}. \end{aligned}$$

In the following, $I = 2, 3, \cdots$

$$\begin{aligned} W_0(I) = W_0(I-1) &- 0.5g_{I-1} - 0.5g_I \\ &+ 0.5g_{I+15} + 0.5g_{I+16}, \end{aligned}$$

$$\begin{aligned} W_1(I) = W(I-1) &- 0.5g_{I-1} - 0.5g_I + g_{I+7} + g_{I+8} \\ &- 0.5g_{I+15} - 0.5g_{I+16}, \end{aligned}$$

$$\begin{aligned} W_2(I) = W(I-1) &- 0.5g_{I-1} - 0.5g_I + g_{I+3} + g_{I+4} \\ &- g_{I+11} - g_{I+12} + 0.5g_{I+15} + 0.5g_{I+16}. \end{aligned}$$

In the following, $I = 1, 2, \cdots$

$$\begin{aligned} W_5(I) = 0.5g_I &+ g_{I+1} - g_{I+3} - g_{I+4} - g_{I+5} \\ &+ g_{I+7} - g_{I+9} + g_{I+11} + g_{I+12} + g_{I+13} \\ &- g_{I+15} - g_{I+16}, \end{aligned}$$

$$\begin{aligned} W_6(I) = 0.5g_I &+ g_{I+1} - g_{I+3} + g_{I+5} - g_{I+7} - g_{I+8} - g_{I+9} \\ &+ g_{I+11} - g_{I+13} + g_{I+15} + 0.5g_{I+16}, \end{aligned}$$

$$\begin{aligned} W_9(I) = 0.5g_I &- g_{I+2} + g_{I+4} - g_{I+6} + g_{I+10} - g_{I+12} \\ &+ g_{I+14} - 0.5g_{I+16}, \end{aligned}$$

$$\begin{aligned} W_{10}(I) = 0.5g_I &- g_{I+2} + g_{I+6} - g_{I+8} + g_{I+10} - g_{I+14} \\ &+ 0.5g_{I+16}, \end{aligned}$$

$$W_{13}(I) = 0.5g_I - g_{I+4} + g_{I+12} - 0.5g_{I+16},$$

$$W_{14}(I) = 0.5g_I - g_{I+8} + 0.5g_{I+16},$$

$$W_{15}(I) = 0.5g_I - 0.5g_{I+16}.$$

Appendix B: impedance algorithm using formulas (16)

Depending upon which of the two formulas for max V or max I are applicable (cf., Eq. (16)) there are then four possible formats for the impedance calculation. These can be shown for $\max_1/\max_1$, $\max_1/\max_2$, $\max_2/\max_2$, and $\max_2/\max_1$, respectively,

$$|Z| = \frac{|\Delta| + 5\Sigma}{|\Delta| + 5\Sigma} \quad \text{or} \quad \frac{3|\Delta| + (1 + \frac{1}{2})\Sigma}{(0.848)|\Delta| + 1.27\Sigma} \quad \text{or}$$

$$= \frac{2|\Delta| + 3\Sigma}{2|\Delta| + 3\Sigma} \quad \text{or} \quad \frac{2|\Delta| + 3\Sigma}{7|\Delta| + 3.54\Sigma}.$$

The approximations are better than one percent. With the cruder approximations of $0.848 \doteq 1 - (1/8)$, $1.27 \doteq 1 + (1/4)$, and $3.54 \doteq 3 + (1/2)$, Shift operations can be employed but, in the worst case, accuracy falls to 3 percent. Other formulas are possible, of course, but these are quite fast.

References and notes

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Received February 27, 1976; revised April 28, 1976

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