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Computation of Incident Solar Energy

Abstract: Computations of the daily and annual totals of the solar energy incident upon south-facing tilted flat surfaces were carried out for several cloudfree atmospheric models after taking into account, somewhat arbitrarily, the contribution due to sky radiation and that due to radiation reflected by the ground. Representative variations of these quantities are discussed as a function of several parameters such as geographical latitude of the location, tilt angle of the surface, atmospheric transmission characteristics, sky-radiation contribution, and ground reflectivity.

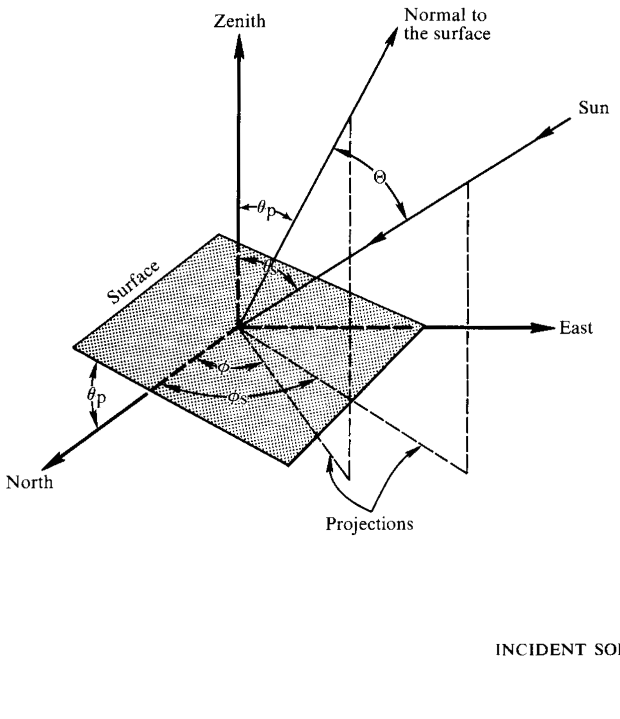
Introduction

Estimates of the daily, monthly, and annual totals of the solar energy incident on tilted flat surfaces are required for various locations and atmospheric conditions in a number of applications such as determination of heat load on buildings and selection of optimal types and positions of flat-plate collectors. Such information is also of value in several disciplines, such as illumination engineering and meteorology. Information available in the open literature on this subject is of two types: 1) hourly pyranometer measurements of the total (direct plus diffuse) radiation received by a horizontal surface located at selected Weather Bureau stations and their daily as well as monthly averages [1], and 2) computations of the solar energy received by tilted flat surfaces for a very limited number of circumstances [1–3]. This readily available information is insufficient for obtaining estimates of the incident energy as a function of various parameters, e.g., latitude of the location, tilt angle of the surface, and atmospheric transmission.

Sophisticated numerical simulations of the solar energy received by a tilted surface can be performed by adoption of one of the several techniques developed for atmospheric radiation transfer studies [4–6]. One could then investigate in detail the effect of changes in several factors such as geographical location of the site, tilt angle of the surface, amount and vertical distribution of absorbing gases and aerosols, optical characteristics of aerosols, and reflecting properties of the ground underlying the atmospheric model. Such a detailed investigation would provide a great deal of information about the spectral and directional characteristics of the total sky brightness needed in photo-voltaic harvesting of solar energy.

However, the amount of computer time required for such an investigation is rather considerable and constitutes a barrier to its undertaking. Consequently, in order to obtain some of this information and to demonstrate the potential of atmospheric radiation modeling for solar energy studies, while at the same time incurring only modest cost, we have undertaken a crude approximation to the above-mentioned study. Some implications of the simplifying approximations made by us are brought out at appropriate places in the text. Expressions used

Figure 1 Definition of the angles used in computing incident solar energy.



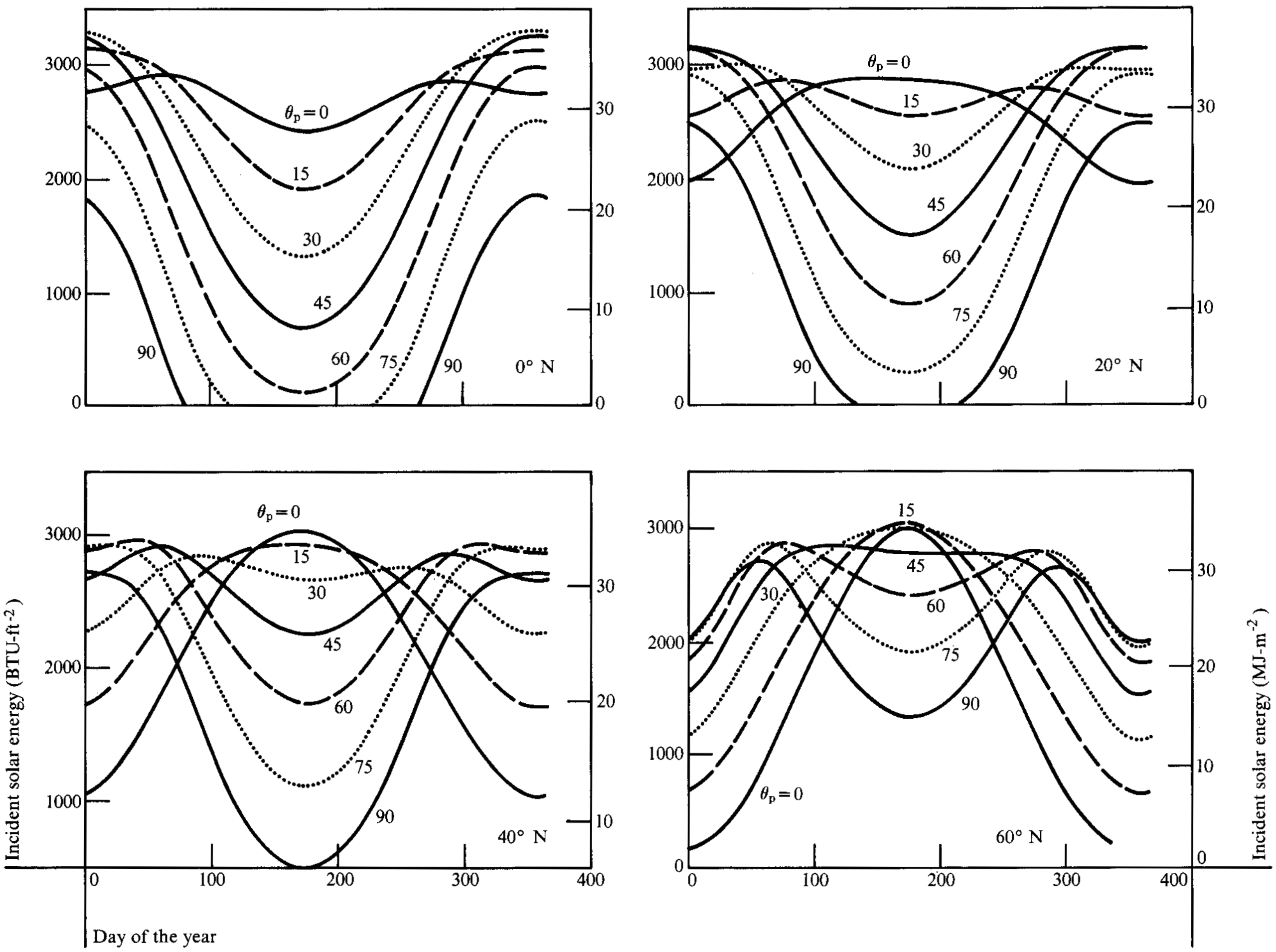


Figure 2 Variation of the daily total solar energy incident per unit area on a south-facing flat surface as a function of the tile angle θ_p of the surface at the indicated latitude. Parameters: Normal optical thickness $\tau = 0$; sky radiation $k = 0$; ground reflectivity $R = 0$.

for computations of the solar energy received by a tilted flat surface are given in the next section. The remaining parts of this paper are related to discussion of the solar energy received by south-facing tilted flat surfaces located at various latitudes in the northern hemisphere. In particular, we have addressed ourselves to the problem of dependence of this quantity on the day of a calendar year, on atmospheric transmission, the sky-radiation contribution, and ground reflectivity.

Necessary expressions

Let the symbols θ_s and ϕ_s represent angles which the incident direct solar radiation and its projection make with the local vertical and the local true north, respectively (Fig. 1). These two quantities are functions of three parameters, viz., latitude (L in degrees) of the location, local apparent time (t in hr) of a day, and the serial number (n) of the day in a calendar year. In fact, the n dependence of θ_s and ϕ_s comes through the quantity δ, which is the declination of the sun. For our work, it is sufficient and convenient to compute a value of δ for a given value of n from the following series [7]:

$$\delta = 0.302 - 22.93 \cos x - 0.229 \cos 2x - 0.243 \cos 3x + 3.851 \sin x + 0.002 \sin 2x - 0.055 \sin 3x, \quad (1)$$

where $x = 2\pi n/365$. Values of θ_s and ϕ_s are then computed by making use of the following astronomical expressions:

$$\cos \theta_s = \sin \delta \sin L + \cos \delta \cos L \cos h, \quad (2)$$

and

$$\sin \phi_s = -\cos \delta \sin h / \sin \theta_s. \quad (3)$$

The quantity h appearing in Eqs. (2) and (3) is the hour angle. Its value in degrees is given by the equation

$$h = 15(t - 12), \quad (4)$$

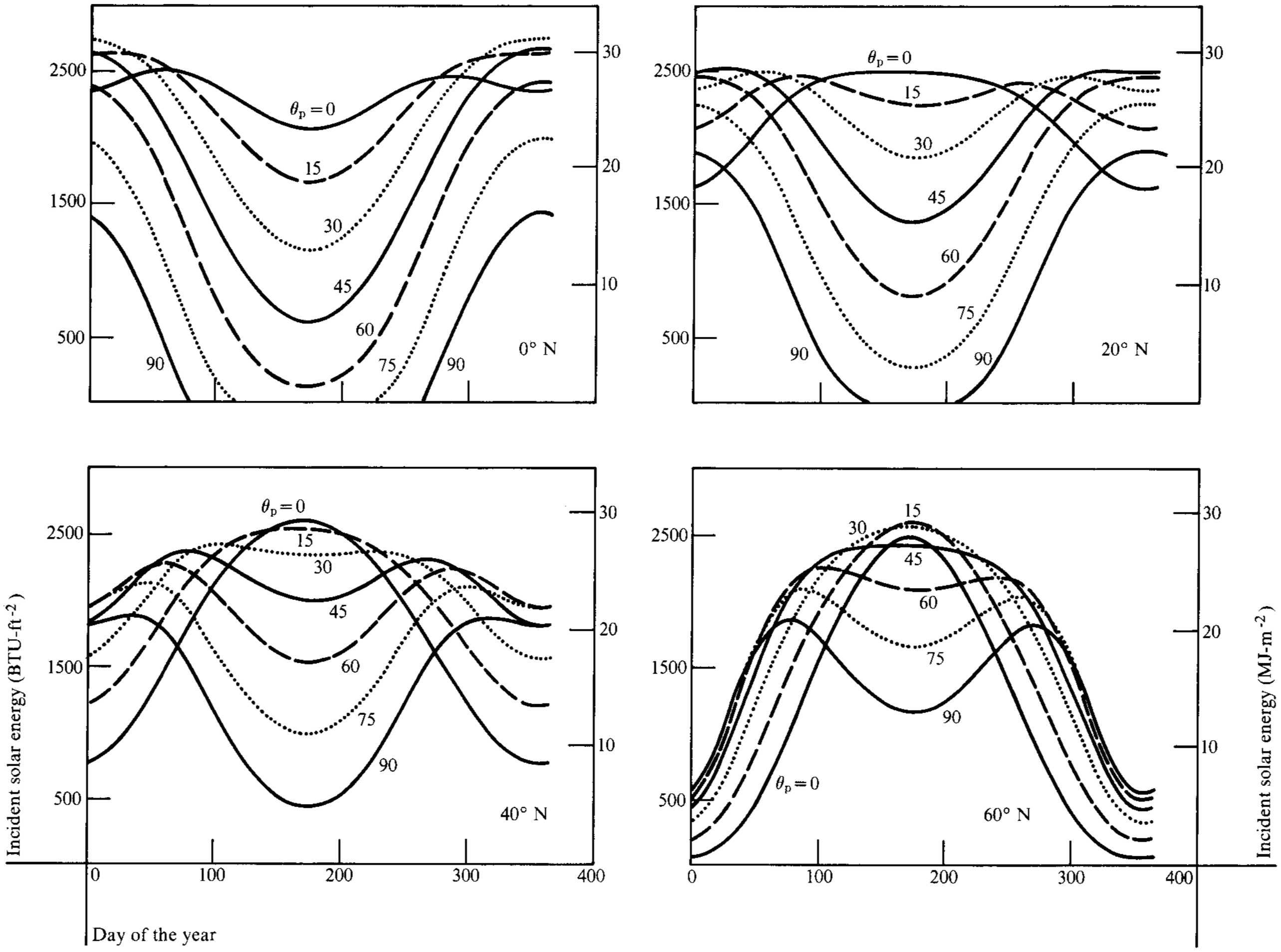


Figure 3 Incident solar energy as in Fig. 2 with $\tau = 0.1$, $k = 0$, $R = 0$.

where $t = 0$ at midnight and $t > 12$ for the afternoon hours.

The quantity ϕ_s computed from Eq. (3) can vary in the range $\pm 90°$, only. The following criterion is used for obtaining a value of ϕ_s referred to the true north: If $\cos h \geq \tan \delta / \tan L$, required $\phi_s = 180° -$ computed ϕ_s. If $\cos h < \tan \delta / \tan L$, required $\phi_s = 360° +$ computed ϕ_s.

Let θ_p be the tilt angle of the flat surface, i.e., the angle which the surface makes with the local horizontal plane. Then, a normal to the surface also makes an angle θ_p with the local vertical. Furthermore, let ϕ_p be the azimuth angle which the projection of the outward normal to the surface makes with the local true north (Fig. 1). The angle Θ between the direct solar radiation incident on the surface and the outward normal to the surface is given by

$$\cos \Theta = \cos \theta_p \cos \theta_s + \sin \theta_p \sin \theta_s \cos (\phi_p - \phi_s). \quad (5)$$

Evidently, the angle Θ depends upon the parameters θ_p, ϕ_p, L, t, and n. The sun is behind the surface for negative values of $\cos \Theta$.

The energy incident upon a tilted flat surface receives contributions from the direct solar radiation, diffuse sky radiation, and sun as well as sky radiation reflected by the ground underlying the atmosphere. Accordingly,

$$E(n, t) = E_{sun}(n, t) + E_{sky}(n, t) + E_{ground}(n, t). \quad (6)$$

The quantity $E(n, t)$ also depends upon several additional parameters such as θ_p, ϕ_p, L, and optical properties of the atmosphere and the ground.

The transmission characteristics of the atmosphere for a pencil of radiation are best represented by a parameter τ called the normal optical thickness of the atmosphere. The direct solar radiation traveling at an angle θ_s with respect to a local vertical in a plane-parallel atmosphere of infinite extent in the horizontal directions

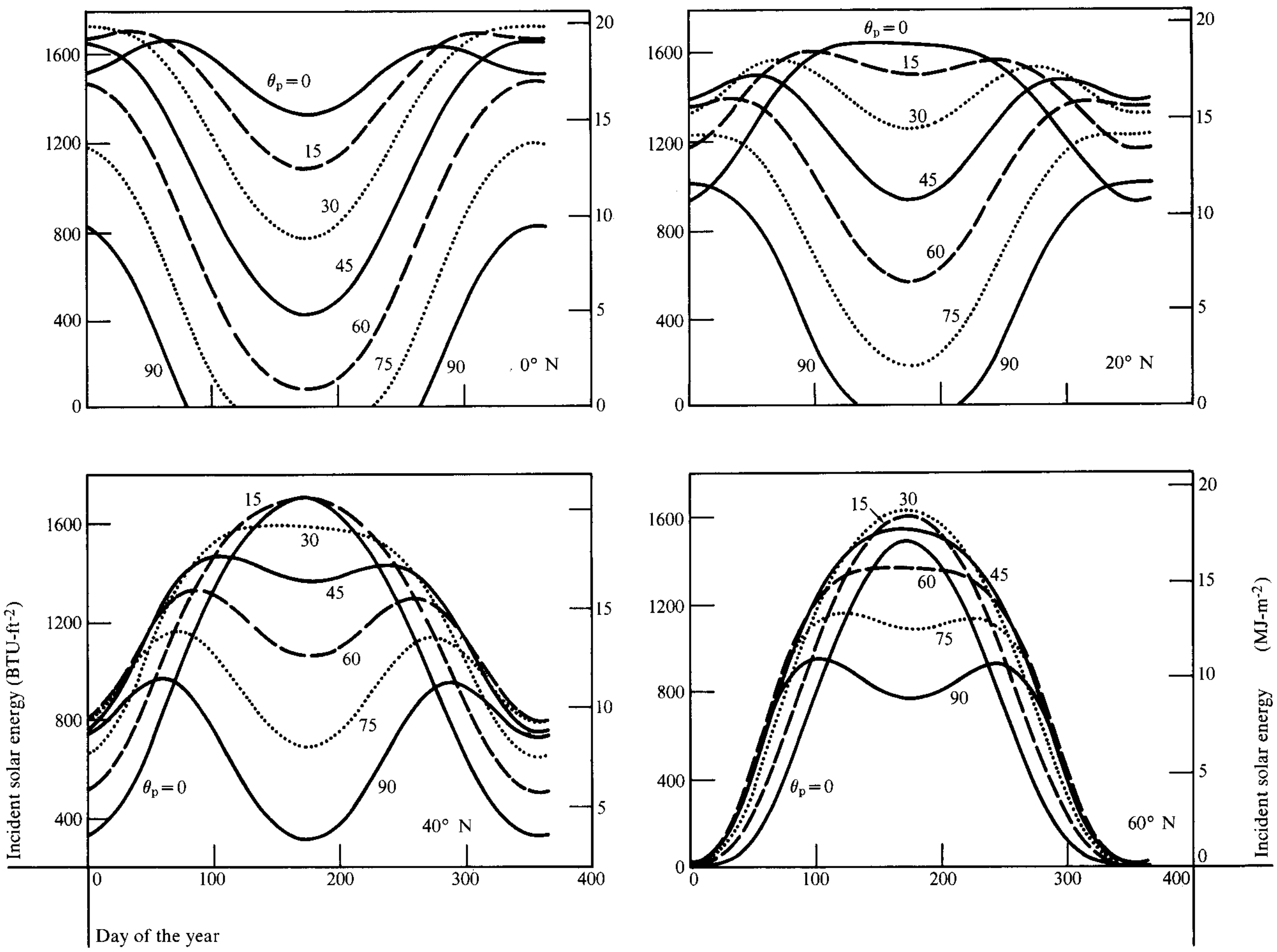


Figure 4 Incident solar energy as in Fig. 2 with $\tau = 0.4$, $k = 0$, $R = 0$.

undergoes an attenuation of $\exp(-\tau \sec \theta_s)$ before reaching the ground. Thus,

$$E_{sun}(n, t) = I_0(n) \cos \Theta \exp(-\tau \sec \theta_s). \tag{7}$$

Values of $E_{sun}(n, t)$ obtained by making the assumption of a plane-parallel atmosphere can be in significant error when the sun is close to the horizon, i.e., when θ_s is of the order of 85° or more. However, we are primarily interested in the daily total $E(n)$ given by the equation

$$E(n) = \int_{t_{sunrise}}^{t_{sunset}} E(n, t)\, dt, \tag{8}$$

and magnitudes of $E(n, t)$ are very small indeed for the sun near the horizon. Thus, the assumption of a plane-parallel atmosphere cannot be expected to affect our results significantly.

The quantity $I_0(n)$ is the intensity of the solar radiation incident at the top of the atmosphere. It is expressed in the terms of BTU-ft^{-2}-hr^{-1} because the results of this investigation are expected to be of great interest to building engineers and designers (SI metric values are given in parentheses). Its value for a given value of n was computed by making use of the following series [7]:

$$I_0(n) = 368.44 + 24.52 \cos x - 1.14 \cos 2x - 1.09 \cos 3x + 0.58 \sin x - 0.18 \sin 2x + 0.28 \sin 3x. \tag{9}$$

Because of the atmospheric attenuation, an amount $I_0(n) \cos \theta_s[1 - \exp(-\tau \sec \theta_s)]$ of energy is removed from the direct solar beam. A fraction k of this amount (to be henceforth referred to as the *sky parameter*) can return in the form of diffuse sky radiation due to scattering by air molecules and dust particles in the atmosphere. The quantity $E_{sky}(n, t)$ is given by

$$E_{sky}(n, t) = k\, I_0(n) \cos \theta_s[1 - \exp(-\tau \sec \theta_s)]\tfrac{1}{2} \times (1 + \cos \theta_p) \tag{10}$$

if we assume that the diffuse sky radiation is isotropically

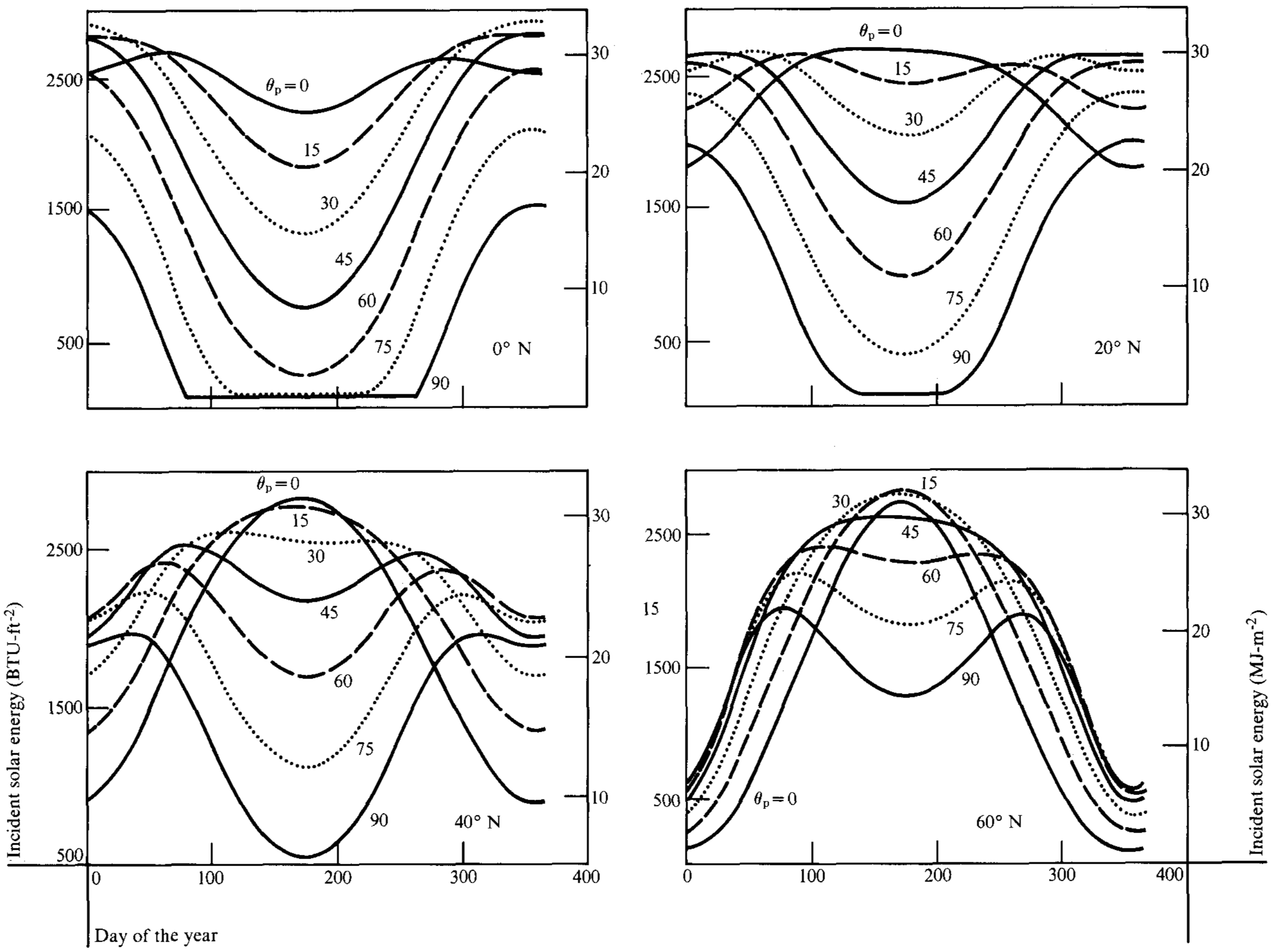


Figure 5 Incident solar energy as in Fig. 2 with $\tau = 0.1$, $k = 0.5$, $R = 0$.

distributed. It should be pointed out that this assumption about the sky-radiation distribution is rarely valid. It generally exhibits a strong maximum near the horizon and/or near the sun, and/or a weak broad maximum near the local zenith, depending upon the atmospheric composition, position of the sun, azimuth angle of the direction of observation, and the wavelength of radiation. There is no simple procedure for estimating the effect of this approximation on the accuracy of the ultimate results. Thus, a value of 0 to 1 of the parameter k can be expected to provide lower and upper bounds, respectively, of the sky-radiation contribution.

The horizontal ground underlying the atmosphere receives direct solar radiation in the amount $I_0(n) \cos \theta_s \exp(-\tau \sec \theta_s)$, and sky radiation in the amount $k\, I_0(n) \cos \theta_s[1 - \exp(-\tau \sec \theta_s)]$. This sky radiation is due to illumination of the atmosphere from above by the direct solar radiation. There is a secondary component of the sky radiation that results from illumination of the atmosphere from below by the radiation reflected by the ground. We have neglected this secondary sky-radiation component in our investigation because it is expected to be small for most cases and, furthermore, the magnitude of $E_{\text{ground}}(n, t)$ is subject to greater error because of other assumptions, e.g., flat horizontal terrain. If a fraction R (Lambert reflectivity of the ground) of the energy incident on the ground is isotropically reflected by it, we have

$$E_{\text{ground}}(n, t) = R\, I_0(n) \cos \theta_s \times [k + (1-k) \exp(-\tau \sec \theta_s)] \times \tfrac{1}{2}(1 - \cos \theta_p). \quad (11)$$

Remarks on $I_0(n)$

The solar radiation incident on the top of the atmosphere [quantity $I_0(n)$ given by Eq. (9)] shows a maximum in January and a minimum in July. These variations, of the order of a few percent, are due to the change in the distance between the Earth and the sun. The value of $I_0(n)$ as obtained from Eq. (9) varies in the range 345 to 391

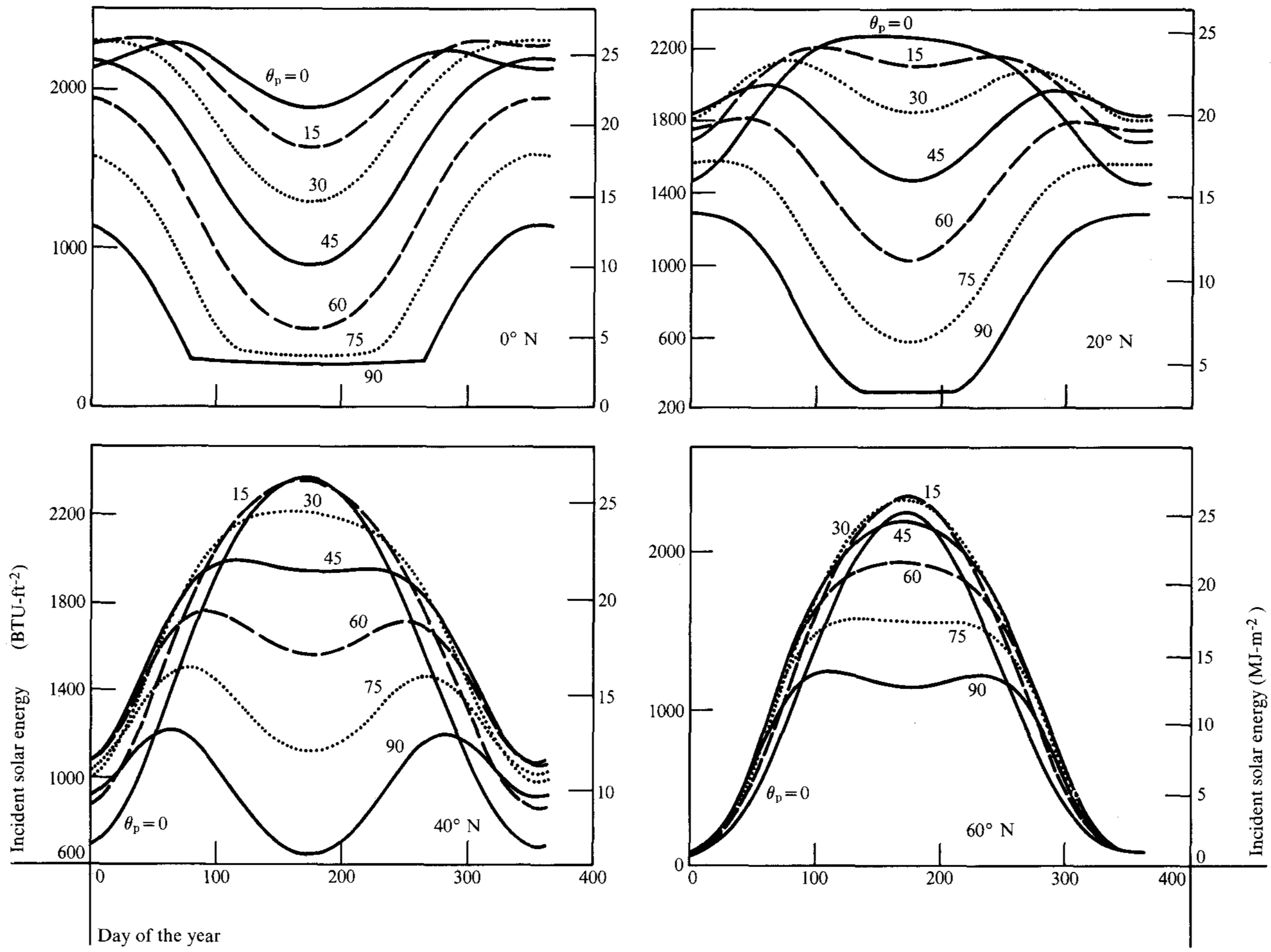


Figure 6 Incident solar energy as in Fig. 2 with $\tau = 0.4$, $k = 0.5$, $R = 0$.

BTU-ft^{-2}-hr^{-1} (1.09 to 1.23 kw-m^{-2}). The authors of Ref. 1 quote a value of 442 BTU-ft^{-2}-hr^{-1} (1.39 kw-m^{2}). This later value, corresponding to 2.00 cal-cm^{-2}-min^{-1} in other units commonly used in the scientific literature, is considered to be unreasonably high [8]. The mean value of $I_0(n)$ as used by us in this investigation can be judged to be somewhat lower when compared to the same most recent standard [8]. However, it is used in order to maintain compatibility with other work in building-design fields [7, 9].

Computations

Values of $E(n, t)$ were computed for a south-facing ($\phi_p = 180°$), tilted flat surface for the following values of various parameters:

$L = 0°\ (20°)\ 60°$ in the northern hemisphere;
$\theta_p = 0°\ (15°)\ 90°$;
$n = 1\ (1)\ 365$;
t = 15-minute intervals from the time of local noon to local sunset with a smaller time step near the sunset time, if necessary;
$k = 0.0, 0.25, 0.4$, and 0.5;
$R = 0.0, 0.1, 0.3, 0.5, 0.8$; and
$\tau = 0.0, 0.1, 0.2$, and 0.4.

The reasons for selecting these particular values of the parameters k, R, and τ are given in the sections discussing their effects on $E(n)$.

The quantity $E(n)$, the daily total of the solar energy received by a tilted flat surface, was then obtained by applying the trapezoidal rule of integration to Eq. (8). The annual total of the solar energy received by the tilted flat surface for a given set of parameters L, θ_p, τ, k, and R was then computed by performing sums of corresponding values of $E(n)$ for all days of the year.

- *Variations of* $E_{sun}(n)$

The daily total of the direct solar energy received by a tilted flat surface [$E(n)$ for k and $R = 0$ or the quantity $E_{sun}(n)$] is plotted, for various tilt angles θ_p, as a function of the serial number n of the day in a calendar year in Figs. 2 through 4. Results for $\tau = 0$, corresponding to the case of positioning of surfaces outside the atmosphere, are presented in Fig. 2. Figure 3 contains results for $\tau = 0.1$, which is the normal optical thickness of a pure molecular earth atmosphere (no absorption by gases, no aerosols) at a median wavelength of about 0.55 μm. In Fig. 4 we have shown results for $\tau = 0.4$, simulating conditions encountered in very hazy situations. (A pure molecular atmosphere with average gaseous absorption due to ozone, carbon dioxide, and water vapor has a normal optical thickness of about 0.2.)

Each of these three figures (and also Figs. 5 through 8) consists of four distinct diagrams, one for each geographic location of the surface (indicated thereon). For example, the top right diagram represents results for a surface located at 20° N. Variations of E_{sun} as a function of the parameter n are shown in each diagram for seven tilt angles of the surface and for values of the parameters τ, k, R, and L listed in the diagram.

The daily total of the solar energy received by a horizontal ($\theta_p = 0$) flat surface located outside the atmosphere but at the equator (left top diagram of Fig. 2) shows two maxima, one near the vernal equinox [2 898 BTU-ft^{-2} (32.91 MJ-m^{-2}) at $n = 61$] and the other near the autumnal equinox [2838 BTU-ft^{-2} (32.23 MJ-m^{-2}) at $n = 289$]. The displacements of the maxima from their true equinox positions and the differences in their magnitudes are due to the variations of $I_0(n)$ vs n discussed in a previous section. A minimum in the $E_{sun}(n)$-vs-n curve near the summer solstice is due to the northernmost position of the sun, combined with a minimum in the $I_0(n)$-vs-n curve near that point. This minimum in the $E_{sun}(n)$-vs-n curve becomes more pronounced with increase in the tilt angle of the south-facing flat surface. In fact, a south-facing flat surface tilted at 75° from the horizontal receives no direct sunlight during a period of 100 days (n in the range 123 to 223) because the sun is behind the surface at all times during that period. It should be noted that for the 0° latitude, the highest amount of direct solar energy is received by a horizontal flat surface in summer but by a flat surface tilted at 30° from the horizontal position in winter.

The conspicuous summer minimum in the $E_{sun}(n)$-vs-n curve for the $\tau = 0$, $\theta_p = 0$ case turns into a broad maximum as the location of the south-facing surface is moved from the equator to 20° N. A further increase in latitude results in a very significant sharpening of this summer maximum. The rate of decrease of $E_{sun}(n)$ with increase in θ_p decreases with northward movement of the south-facing flat surface. For 60° N location (right bottom diagram of Fig. 2), a surface with 15° tilt angle receives the strongest dose of direct solar radiation at the summer solstice.

As for the winter months, the tilt angle for which a south-facing flat surface can receive the highest amount of direct solar radiation increases from about 30°, to 45°, to 60°, to 80° for the latitude regions 0° N, 20° N, 40° N, and 60° N, respectively.

The discussion thus far has been restricted to surfaces located outside the atmosphere. The $E_{sun}(n)$-vs-n curves for seven values of θ_p and four values of L are given for the south-facing surfaces located at the bottom of a very clear ($\tau = 0.1$) and a very hazy ($\tau = 0.4$) atmosphere in Figs. 3 and 4, respectively. In a general sense, trends exhibited by these curves are similar to those of the corresponding curves for the $\tau = 0$ case. However, there are some significant differences also. As for example, for the 60° N location, values of $E_{sun}(1)$ for a vertical ($\theta_p = 90°$) flat surface are 2 043, 583, 212, and 32 BTU-ft^{-2} (23.20, 6.62, 2.41, and 0.36 MJ-m^{-2}) for $\tau = 0$, 0.1, 0.2, and 0.4, respectively. Thus, these sets of curves can be used to estimate values of $E_{sun}(n)$ for any reasonable combination of the parameters n, θ_p, L, and τ.

• *Contribution due to sky radiation*

An approximate method for obtaining bounds for the sky-radiation contribution to the daily total of the solar energy received by a tilted flat surface was outlined during the development of Eq. (10). This method makes use of the parameter k which we refer to as the sky-radiation parameter. It represents the fraction of the energy that is removed from the direct solar beam and returned to the ground in the form of sky radiation. This parameter can assume any value between 0 and 1 depending upon the composition of the atmosphere, wavelength of radiation, and zenith angle of the sun. A value of zero for the parameter k implies that there is no skylight. Such a condition can occur if all the energy removed from the direct beam is absorbed within the atmosphere and/or the molecules and particulate constituents exhibit very strong back-reflecting characteristics. At the other extreme, a value of one for the parameter k implies that atmospheric constitutents prefer to direct all energy removed by them in the directions close to that of the original beam.

Experience has shown that such extreme conditions can rarely prevail in the terrestrial atmosphere. This sky-radiation parameter assumes a value of 0.5 for a pure molecular atmosphere obeying Rayleigh's law of scattering and containing no gaseous absorbers, provided the sun is not very far from the local zenith [10]. From the results presented in Tables 1 and 2 of Ref. 11, it can be seen that this sky-radiation parameter varies in the range 0.2 to 0.6 when typical amounts of dust and absorbing gases are added to the model. Thus, a value of 0.5 for k can be considered to be a reasonable upper limit for typical atmospheric conditions.

The daily total solar energy received by a tilted flat surface [$E_{sun}(n) + E_{sky}(n)$, or $E(n)$ for $R = 0$] is plotted

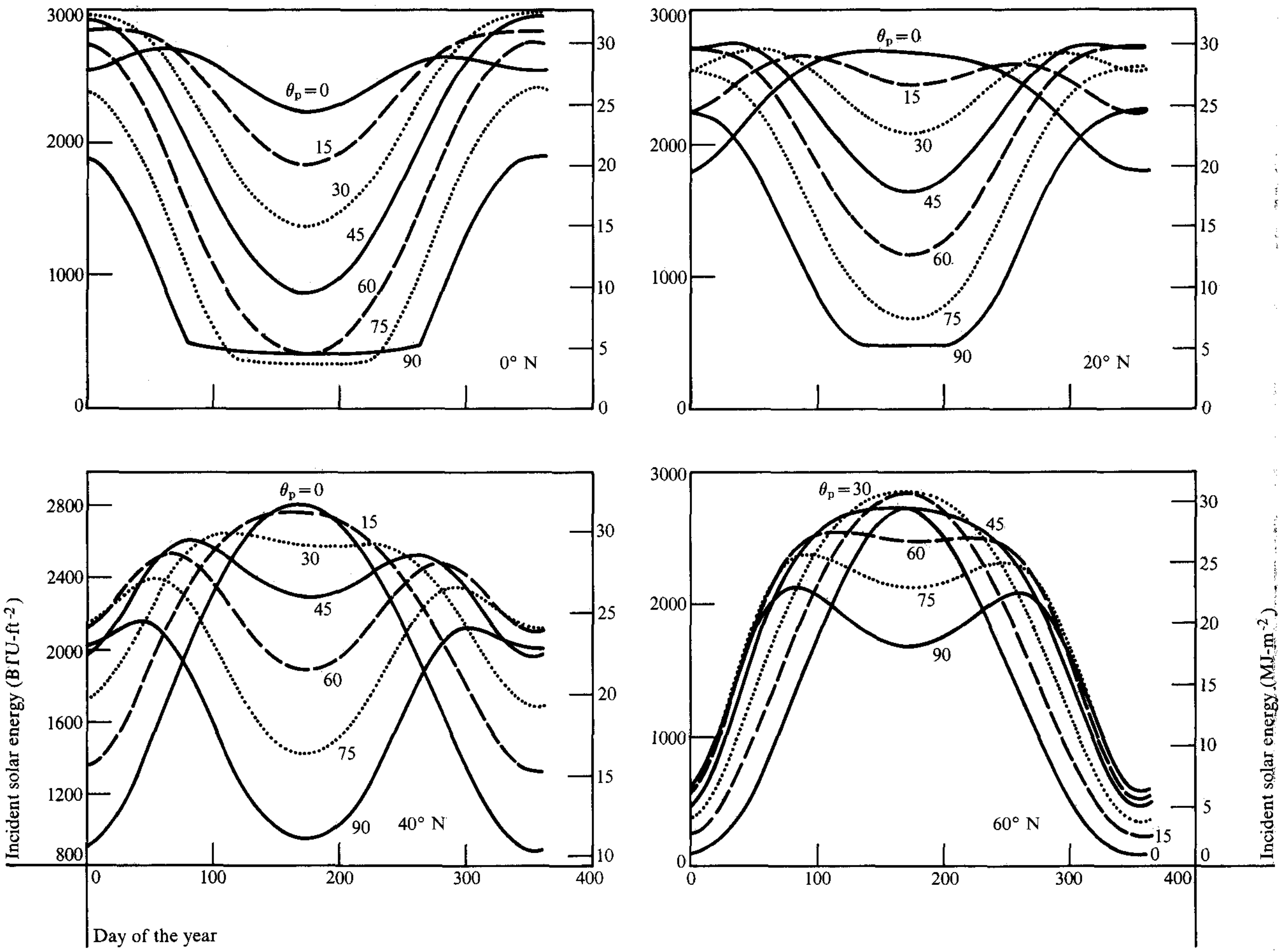


Figure 7 Incident solar energy as in Fig. 2 with $\tau = 0.1$, $k = 0.5$, $R = 0.3$.

as a function of n in Figs. 5 and 6 for the cases $\tau = 0.1$, $k = 0.5$ and $\tau = 0.4$, $k = 0.5$, respectively. No sky radiation contribution can exist for the $\tau = 0$ case. Results presented in these two diagrams can be studied in conjunction with those shown in Figs. 3 and 4 for estimating the effect of the sky-radiation contribution on the daily total of the solar energy received by a south-facing, tilted flat surface. In general, the ratio $E_{sky}(n)/E_{sun}(n)$ lies in the range 0.05 to 0.25 for the $\tau = 0.1$ case and in the range 0.3 to 1.0 for the $\tau = 0.4$ case, provided $E_{sun}(n) \approx 1\,000$ BTU-ft^{-2} (11.36 MJ-m^{-2}) or more. The exact value of this ratio depends upon the parameters θ_p and L and assumes very large values especially when $E_{sun}(n)$ is very small. For example, during the periods when the south-facing surfaces receive no direct solar radiation (e.g., curves for $\theta_p = 75°$ and 90° for 0° N in Fig. 4), the skylight contribution amounts to about 250 BTU-ft^{-2} (2.84 MJ-m^{-2}) on an average summer day.

• *Effect of ground reflection*

To demonstrate the relative importance of the ground-reflected radiation in the daily total of the solar energy received by a south-facing flat surface, we consider a ground surface isotropically reflecting 30 percent of the total energy incident upon it. Alternately, this ground is said to have a Lambert reflectivity of 0.3. Bare ground exhibits such a reflectivity. Values of $E(n)$ for $k = 0.5$ and $R = 0.3$ are plotted as a function of the serial number of the day in a calendar year in Figs. 7 and 8 for the $\tau = 0.1$ and $\tau = 0.4$ cases, respectively. As mentioned previously, each figure consists of four diagrams, one for each of the four latitude values indicated. Furthermore, each diagram contains seven curves corresponding to seven tilt positions of a south-facing flat surface. Ground-reflected radiation can contribute significantly to the daily total of the solar energy received by a tilted flat surface located at the top of the atmosphere ($\tau = 0$ case).

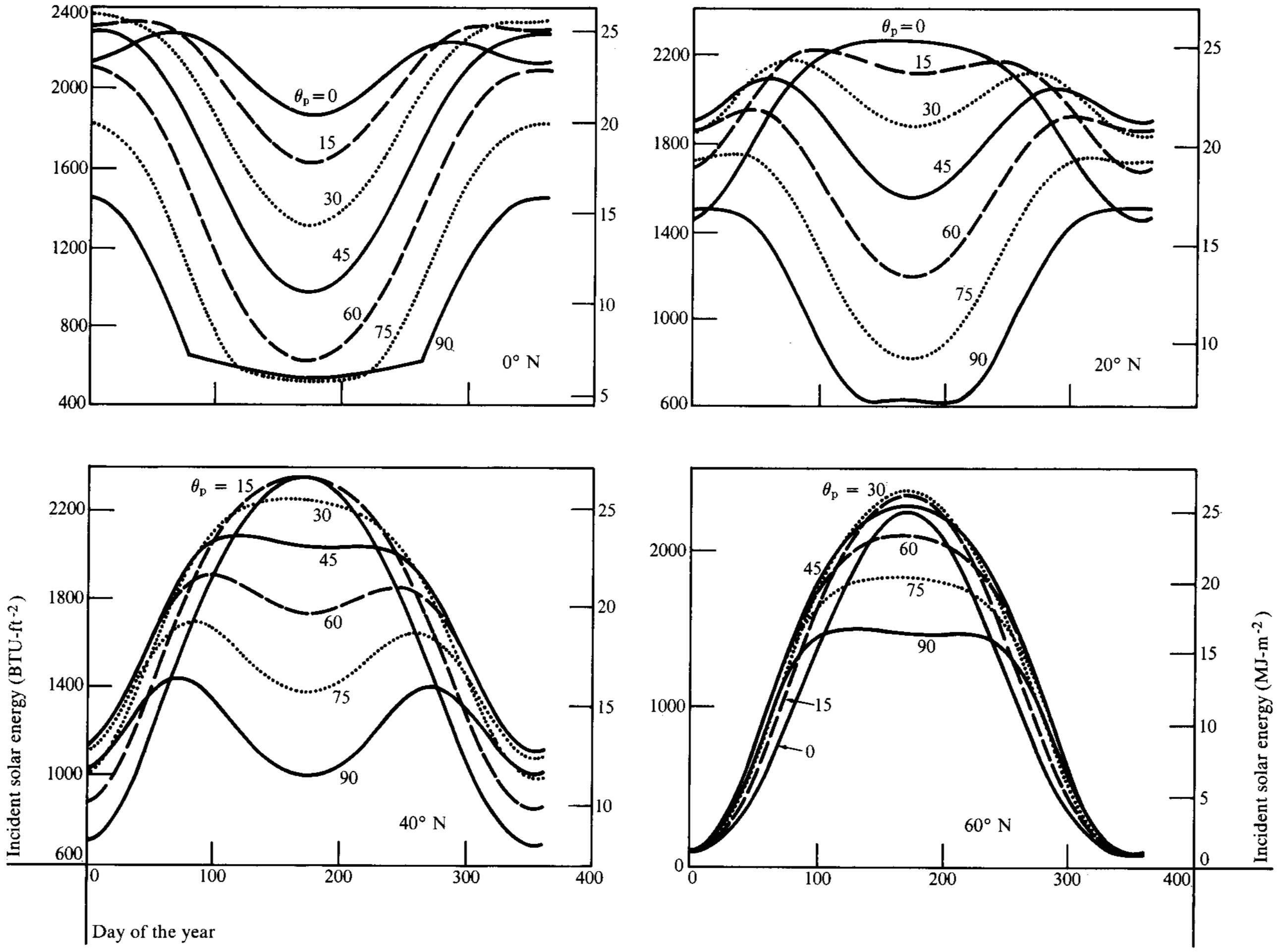


Figure 8 Incident solar energy as in Fig. 2 with $\tau = 0.4$, $k = 0.5$, $R = 0.3$.

This is especially so since we then have to take into account energy reflected by clouds, which generally exhibit reflectivity in the range 50 to 100 percent. However, numerical results for this special case are of very little interest to engineers primarily concerned with ground-based projects.

In general, the presence of a partly reflecting ground increases the daily total of the solar energy received by a tilted flat surface provided the surface is not horizontal, i.e., $\theta_p \neq 0$. (This is because we have neglected sky radiation originating as a result of illumination of the atmosphere from below.) The relative importance of the ground-reflected contribution to a tilted surface increases with an increase in θ_p. A vertical south-facing surface receives more energy than a surface tilted at an angle of 75° from the horizontal position during the summer period in the equatorial region (see $\theta_p = 75°$ and 90° curves for 0° latitudes in Figs. 3 through 8).

• *Annual total*

In Fig. 9 we have plotted the solar energy received per unit area (MBTU-ft^{-2}, GJ-m^{-2}) by a south-facing flat surface over the entire calendar year (annual total) as a function of the tilt angle (θ_p) of the surface. The solid curves represent results when contributions due to the sky radiation and/or ground-reflected radiation are not taken into account; i.e., the parameters k and R are zero for these curves. Results obtained after inclusion of the sky-radiation contribution ($k = 0.5$, $R = 0$) and the sky as well as ground-radiation contributions ($k = 0.5$, $R = 0.8$) are shown by dashed and dotted curves, respectively.

The curves of annual total solar energy received by the flat surface vs its tilt angle exhibit rather broad maxima whose broadness increases with increase in the ground reflectivity. To a good first approximation, the positions of these broad maxima coincide with the latitudes of the

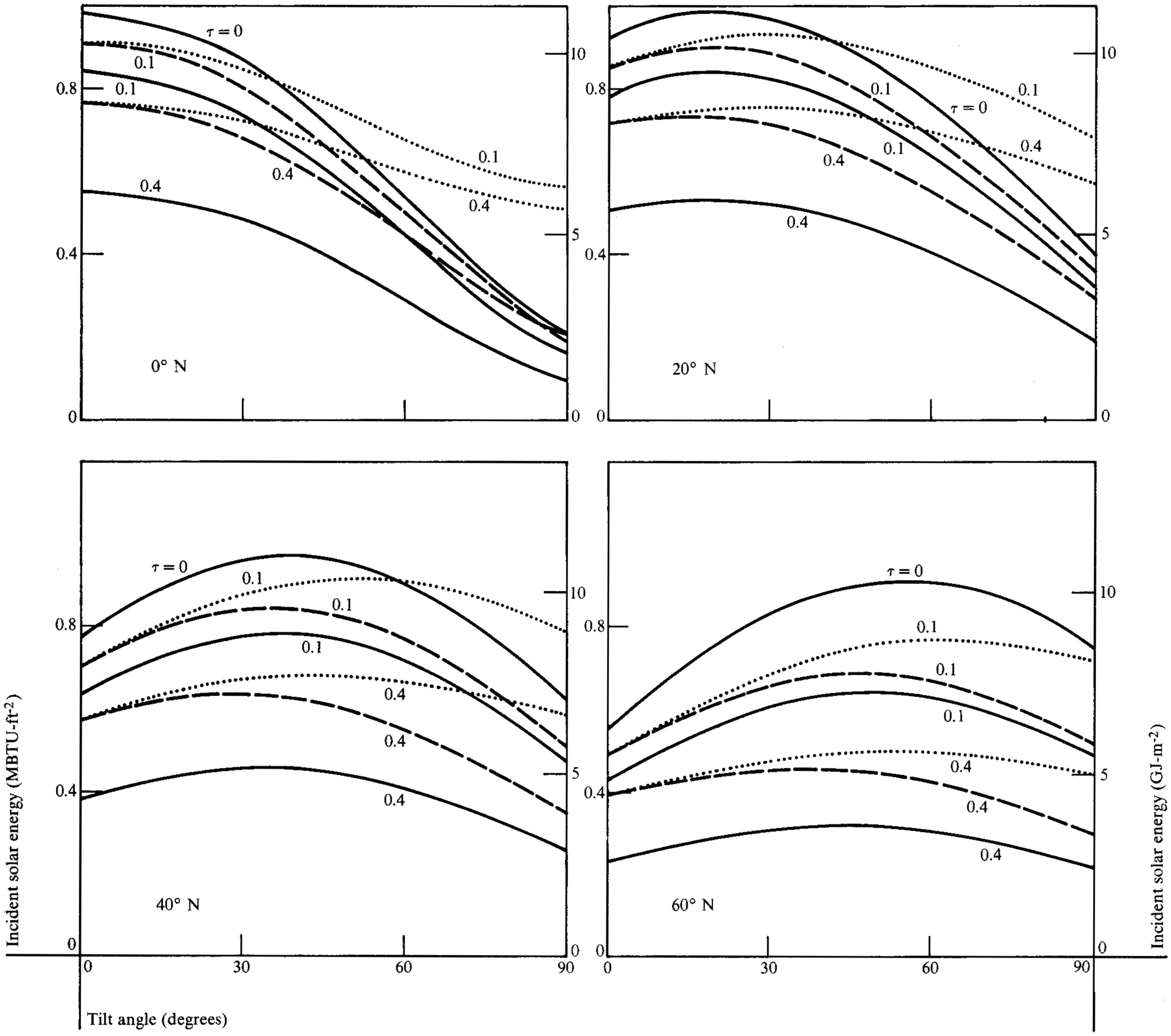


Figure 9 Variation of the annual total solar energy incident per unit area on a south-facing flat surface as a function of the normal optical thickness τ at the indicated latitude. Solid line: $k = 0$; $R = 0$; dashed line: $k = 0.5$, $R = 0$; dotted line: $k = 0.5$, $R = 0.8$.

geographical locations of the surfaces. It is interesting to note that highly tilted surfaces can receive more energy over a year when they are located at the bottom of the atmosphere with high ground reflectivity than they could outside the atmosphere without any ground reflection. This is not the case when the tilted surfaces are located far north of the equator.

Summary

In the foregoing sections, we have presented selected results of numerical simulation of the daily totals and annual totals of the solar energy received by south-facing tilted flat surfaces located at four geographical latitudes in the northern hemisphere, viz., 0° N. 20° N, 40° N, and 60° N. These results are for cloud-free models and were obtained after assuming wavelength independence of the solar spectrum and of the optical characteristics of the earth-atmosphere system. Furthermore, sky-radiation and ground-radiation contributions are accounted for in a crude fashion only. Thus, the results presented in this paper are expected to be helpful primarily for inferring general trends and for providing numerical values that can assist in preliminary planning of a variety of solar energy projects.

One of the by-products of this investigation is the diurnal variation of the solar energy received by a tilted surface for a multitude of parameters, such as latitude, season, tilt angle, atmospheric conditions, and ground reflectivity. We have not discussed this aspect.

Further investigations with tilted flat surfaces oriented in any arbitrary direction can be of considerable help in several applications. They can be used in estimation of heat loads on buildings and of tradeoffs when flat-plate solar-energy collectors have to be mounted away from the true south [12]. Still another extension of such investigation would include estimation of the reduction in daily total of the solar energy received by tilted flat surfaces due to the presence of other buildings, trees, and hills. Such a study could then be extended to the selection of optimal angles for fixed-type, flat-plate, solar energy collectors when the location under consideration has some prominent climatic feature, e.g., the early-morning, low-level cloudy and/or foggy conditions observed in California.

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