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Automatic Equalizers having Minimum Adjustment Time

Abstract: Two new types of automatic equalizers for telephone lines are discussed. The modular configurations shown are designed for minimum adjustment time and are ready to receive data as soon as the response of the unequalized line has been measured. Two forms of such equalizers are compared with respect to upper bounds on the residual distortion.

Introduction

The efficient use of a multipoint telephone line by a group of digital modems requires that the initial adjustment times of the modems be small, relative to the message lengths. When the modems employ line equalizers, the adjustment times of the equalizers become an important consideration.

Most automatic time-domain equalizers use a single transversal filter as the equalizing device, and the tap gains are adjusted so as to minimize the peak distortion [1] or, alternatively, the mean-square error [2,3] at the equalizer output. In either case, it is possible to compute the desired tap gains from the sampled pulse response of the line to be equalized. When fast, auxiliary computational equipment is available, the tap gains can be rapidly computed. The fast computers are not usually available for this purpose, and the need for low cost equalizers has made it necessary to compute the tap gains with minimal auxiliary hardware.

This desire for economy has led to designs that perform a sequence of feedback corrections of the tap gains, thereby causing them to converge to the correct values. While some of these designs converge the gain values more rapidly [4,5] than others, none of them is ready to receive data as soon as the pulse response of the line has beeen measured.

By employing an equalizing device that is modular, but more complex than a single transversal filter, we can achieve the minimum adjustment time. It is assumed that the receiver has means for acquiring the carrier frequency and the sample timing signal (i.e., clock) from the transmitter. Beyond this, only a short sequence of training pulses is needed to measure the response of the line. Furthermore, we measure the equalizer input rather than its output, thus minimizing the data that must be stored for control of the tap gains.

The new equalizers are suboptimal. They achieve neither the minimal peak distortion nor the minimal mean-square error attainable with a given number of filter taps. They also share a weakness of the zero-forcing type of equalizer; when the line distortion exceeds unity, it is not always reduced by the equalizers. Their advantages lie in speed of adjustment and simplicity of control.

The increased complexity of the structures of the new equalizing devices is partly ameliorated by use of the modular forms that will be presented here. A variety of modular forms is available; but the discussion below will be limited to two of them, since the others offer no significant advantages over these.

Notation

The equalizers will employ time-domain filters which may be either the sampled-data analog type or the digital type. Considerations of cost and precision will probably weigh in favor of digital implementation.

Networks of digital filters may be realized in many different ways including, of course, software-controlled arithmetic processors. We shall not be concerned with the details of implementation at this level. In order to present this material simply, we shall discuss the filters in terms of the functionally equivalent tapped delay lines, the output of each filter being derived by summing the weighted taps [6].

Let $z = \exp [s\tau]$ be the standard z-transform variable, s the complex frequency variable, and τ the sampling interval. Then, z is the unit advance operator and multiplication by z^{-1} represents a unit delay.

Simplified schematics will be used for the filters, as illustrated in Fig. 1. The nonrecursive filter in Fig. 1 (a) contains a tapped delay line (or a shift register) with N units of delay. The weighted and summed taps give the filter response, $F = \sum_{i=0}^{N} f_i z^{-i}$, where the ith tap has weight f_i and occurs after i units of delay. Thus, for any input signal whose z-transform is I, we get a filter output FI and a delayed output $z^{-N}I$, in the z domain.

The recursive filter shown in Fig. 1 (b) can be implemented when $f_0 = 0$. Its response, taken at the input to the delay line, is $(1-F)^{-1}$. This is stable whenever $1-F$ has no zeros on or outside the unit circle in the z-plane. As a practical matter, the zeros should be bounded away from the unit circle by some reasonable margin. A sufficient condition for stability is $\sum_{i=1}^{N} |f_i| < 1$. While this condition is stronger than necessary, it is expected to hold in the application at hand.

The unequalized response of the transmission medium will be written in the forms

$$H = \sum_{i=-M}^{N} h_i z^{-i} = 1 - P - T = 1 - F. \tag{1}$$

In Eq. (1) and hereafter we assume that the coefficient of the main pulse has been normalized to unity, that is, $h_0 = 1$. All other coefficients represent undesirable interference caused by dispersive transmission. Furthermore,

$$-P = \sum_{i=-M}^{-1} h_i z^{-1} \tag{2}$$

represents *precursor* interference that arrives before the desired pulse while

$$-T = \sum_{i=1}^{N} h_i z^{-i} \tag{3}$$

represents the *tail* of the interference, following the desired pulse. The negative signs are chosen for convenience in the subsequent discussion. The total interfering signal is $-F$.

If the equalizer output is $A = \sum_{i=-K}^{L} a_i z^{-i-K}$ and a_0 represents the desired pulse, we define the residual *distortion* to be

$$\delta[A] = \|A - z^{-K}\| = |a_0 - 1| + \sum_{i=-K}^{-1} |a_i| + \sum_{i=1}^{L} |a_i|. \tag{4}$$

This is the greatest possible error amplitude in a random stream of positive and negative pulses. Therefore, the norm used in Eq. (4) is a reasonable choice.

Since the equalizer input is assumed to be normalized, the initial distortion is

$$\delta[H] = \|H - 1\| = \sum_{i=-M}^{-1} |h_i| + \sum_{i=1}^{N} |h_i|.$$

Distortion reduces noise margins in the detection of digital data. Sufficiently large distortion can cause errors even in the absence of noise.

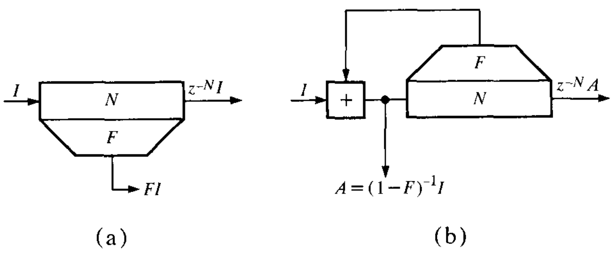

Figure 1 (a) Representation of a time-domain nonrecursive filter with N units of delay and impulse response F. (b) Representation of a recursive filter with N units of delay and impulse response $(1-F)^{-1}$.

Equalizer approximations

If the z-domain response of the equalizer is represented by G, it would be ideal if $HG = 1$, or $G = 1/H$. However, some excess delay will be necessary, and the inversion of H will be only approximated.

We begin with the algebraic identity,

$$1/H = (1-F)^{-1} = 1 + F + F^2 + \cdots + F^n + (1-F)^{-1}F^{n+1}. \tag{5}$$

If the last term is sufficiently small, then we can approximate G by $\hat{G}_n = 1 + F + F^2 + \cdots + F^n$.

This is not a realizable form because F contains precursor terms. However, $z^{-M}F$ is a causal and realizable filter response. Therefore, setting

$$D = z^{-M}, \tag{6}$$

we have the realizable approximation,

$$\hat{G}_n = D^n(1 + F + F^2 + \cdots + F^n) = D^n + D^{n-1}(DF) + \cdots + (DF)^n. \tag{7}$$

The residual interference is then $H\hat{G}_n - D^n = -D^nF^{n+1}$ and the residual distortion is

$$\delta[H\hat{G}_n] = \|D^nF^{n+1}\| = \|F^{n+1}\| \le (\|F\|)^{n+1} = (\delta[H])^{n+1}, \tag{8}$$

since D is a pure delay. The inequality becomes equality when all coefficients in F are nonnegative. Thus $(\delta[H])^{n+1}$ is the least upper bound for the residual distortion.

From the inequality (8), we see that $\delta[H\hat{G}_n] \to 0$ as n becomes large whenever $\delta[H] < 1$. We shall assume that this sufficient condition holds, in order to avoid more complex considerations.

A different equalizer approximation will now be derived from the identity,

$$1/H = (1-T-P)^{-1} = (1-T)^{-1}(1-Q)^{-1}$$
$$= (1-T)^{-1}[1+Q+Q^2+\cdots+Q^n+ (1-Q)^{-1}Q^{n+1}], \qquad (9)$$

where

$$Q = P(1-T)^{-1}. \qquad (10)$$

The quantity Q is not realizable, but $DQ = DP(1-T)^{-1}$ is realizable. Therefore, we have the realizable approximation

$$G_n = D^n(1-T)^{-1}(1+Q+Q^2+\cdots+Q^n)$$
$$= (1-T)^{-1}[D^n + D^{n-1}(DQ)+\cdots+(DQ)^n]. \quad (11)$$

The residual interference is

$$HG_n - D^n = -D^nQ^{n+1} = -D^nP^{n+1}(1-T)^{-n-1}. \qquad (12)$$

The residual distortion is

$$\delta[HG_n] = \|(P/(1-T))^{n+1}\| \le [x/(1-y)]^{n+1}, \qquad (13)$$

where $\quad x = \|P\|, \qquad y = \|T\|. \qquad (14)$

The inequality becomes equality when all coefficients of P and T are nonnegative. Therefore $[x/(1-y)]^{n+1}$ is the least upper bound (l.u.b.) for the residual distortion.

To compare the l.u.b.'s for residual distortion resulting from $\hat{G}_n$ and G_n, let us assume that the sufficient condition for convergence holds, and that

$$\delta[H] = x + y = 1 - \epsilon, \qquad 0 < \epsilon < 1. \qquad (15)$$

Then the residual distortion l.u.b.'s are, respectively,

$$\hat{\delta}_n = (1-\epsilon)^{n+1}, \qquad \delta_n = \left(\frac{1-\epsilon-y}{1-y}\right)^{n+1}. \qquad (16)$$

The ratio of the two is

$$\frac{\hat{\delta}_n}{\delta_n} = \left(1+\epsilon\frac{y}{x}\right)^{n+1}. \qquad (17)$$

This shows that δ_n is smaller whenever $y > 0$, which is always the case in practice. When $y > x$, which is usually true, then

$$\delta_n < \frac{\hat{\delta}_n}{(1+\epsilon)^{n+1}} = \left(\frac{1-\epsilon}{1+\epsilon}\right)^{n+1}. \qquad (18)$$

For example, if $x + y = 0.8$ and $y \ge x$, then $\delta_2 \le 8/27 = 0.296$.

Realizations

The polynomials G_n and $\hat{G}_n$ have modular realizations that correspond to factored forms and to a simple recursive computation. We shall illustrate the latter.

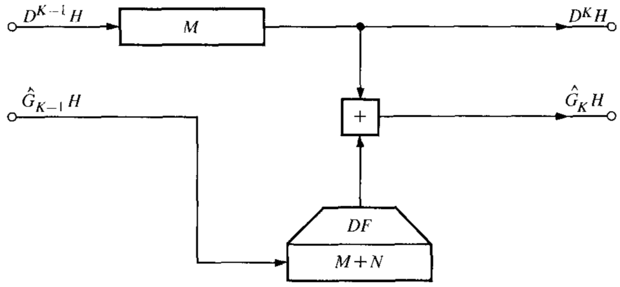

Figure 2 The K^{th} module of the equalizer, $\hat{G}_n$. Each module has two inputs and two outputs.

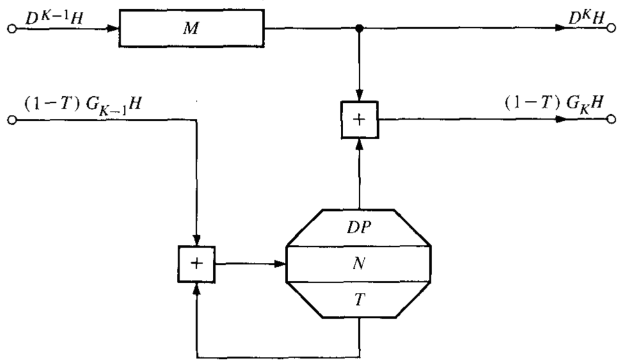

Figure 3 The K^{th} module of the equalizer G_n. The filter in the lower signal path has the response $DP(1-T)^{-1}$.

Consider polynomials of the form $U_n = A^n + A^{n-1}B + \cdots + AB^{n-1} + B^n$. It is obvious that $U_n = BU_{n-1} + A^n$.

Therefore we can compute U_n by means of n steps, each of which involves multiplication by B and addition of a power of A. The initial value is $U_0 = 1$.

Both $\hat{G}_n$ and G_n may be realized in this form, although G_n requires one final multiplication by $(1-T)^{-1}$.

A module for the realization of $\hat{G}_n$ is shown in Fig. 2. Each module has two inputs, the first of which provides a delayed signal and the second of which provides a partially equalized signal. Each module also has two outputs and the modules are connected in tandem.

A module for the realization of G_n is shown in Fig. 3. Again, each module has two inputs and two outputs. The filter in this module has the response, $DP(1-T)^{-1}$. Its recursive and nonrecursive portions share a common delay line. The number of delay sections must be the greater of M (for DP) and N (for T). In most cases, N will be greater than or equal to M.

G_n is realized by the tandem connection of n modules

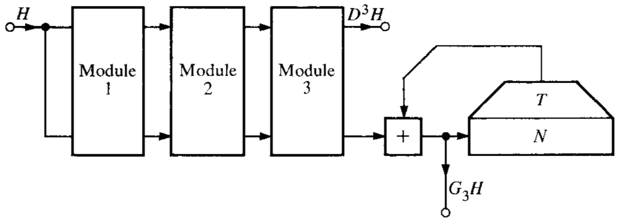

Figure 4 A tandem connection of modules to form the equalizer G_3. Each of the three modules is similar to the one shown in Fig. 3. The delayed signal, D^3H, is not required for automatic equalization.

of the type shown in Fig. 3, preceded or followed by the filter, $(1-T)^{-1}$. The latter arrangement is illustrated for G_3 in Fig. 4.

Conclusion

We have illustrated two of many possible modular equalizer configurations having the following properties:

1. They are ready to receive data when the unequalized response of the line has been measured.
2. Only the sampled values of the line response need to be stored for the purpose of controlling the tap weights.

These equalizers do not, however, achieve the minimum possible distortion for a given number of filter taps. Furthermore, they do not assure reduction of the distortion unless the initial distortion is less than unity. They may be of interest when speed of adjustment and simplicity of control are paramount as, for example, in the case of medium-speed modems on multipoint lines or on separate polled lines.

References

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