R. T. Sha
D. T. Tang

A New Class of Automatic Equalizers*

Abstract: A new class of automatic equalizers with very fast convergence is discussed in this paper. This fast convergence is accomplished by means of an iterative procedure which successively makes higher-order approximations on the desired equalizer function. The iterative procedure can be conveniently realized in the form of transversal filter stages in cascade. For a given distortion $D < 1$, the residual distortion at the equalizer output after n iterations can be reduced to no more than D^{2^n} in the noise-free cases. When this approach is applied to the feed-forward part of a recursive structure, the front distortion (due to precursors) can be reduced in a similar fashion without unduly increasing the overall distortion. The resulting distortion, mainly in the rear end, can then be cancelled out via feedback paths. Other topics treated include certain generalizations, the truncation error due to limited length in cascaded equalizer sections, and the effect of noise. Several numerical examples are also presented to illustrate the effectiveness of the approach.

1. Introduction

One of the significant developments in recent years in digital data transmission has been the use of transversal filters as time-domain equalizers. Various algorithms have been developed to automatically adjust the tap-gain settings in such an equalizer during a training period in which a train of isolated reference pulses is transmitted [1-3]. Adaptiveness can be achieved during actual data transmission by adjusting the tap-gain settings as a function of estimated channel response [4-7]. It is clear that, for high-speed data transmission systems, fast convergence in automatic equalization is important to keep the turn-around time small. In this paper, we describe a new equalization technique with very fast convergence time. The technique is based on an iterative procedure that makes successively higher-order approximations on the desired equalizer transfer function. This procedure can be conveniently implemented in the form of cascaded transversal filter stages. We also describe how this technique can be applied to the feed-forward part of a recursive structure, resulting in the suppression of the front distortion without unduly increasing the overall distortion. Other topics treated include certain generalizations of this technique, the effect of truncation (limiting the number of delay units in an equalizer stage), and the effect of noise. Several examples with numerical results obtained via an APL simulation program are also included to illustrate the effectiveness of the approach. It should be noted that base-band pulse transmission is assumed throughout. A number of related topics of practical importance, such as time jitter, digital round-off errors, and other implementation details are beyond the scope of this paper.

*Some of the results contained in this paper were presented at the Sixth Annual Princeton Conference on Information Science and Systems, March 23-24, 1972.

2. Equalizer structures

• *Nonrecursive equalizer with cascaded stages*

The first equalizer structure considered in this paper consists of n cascaded stages of transversal filters as shown in Fig. 1. The channel output feeds the first stage of the equalizer. The output of the first stage in turn feeds the second stage, and so on. Let us denote the channel impulse response by the sample sequence $\{\alpha_k^{(0)}\} = \{\alpha_{-N_i}^{(0)}, \cdots, \alpha_{-1}^{(0)}, \alpha_0^{(0)}, \alpha_1^{(0)}, \cdots, \alpha_{N_i}^{(0)}\}$. Similarly, the output of the ith stage is denoted by $\{\alpha_k^{(i)}\} = \{\alpha_{-N_i}^{(i)}, \cdots, \alpha_{-1}^{(i)}, \alpha_0^{(i)}, \alpha_1^{(i)}, \cdots, \alpha_{N_i}^{(i)}\}$. On the other hand, tap-gain settings of the ith stage form a vector $\boldsymbol{\beta}_k^{(i)} = (\beta_{-M_i}^{(i)}, \cdots, \beta_{-1}^{(i)}, \beta_0^{(i)}, \beta_1^{(i)}, \cdots, \beta_{M_i}^{(i)})$. The term $\alpha_0^{(i)}$ represents the main pulse, while other terms $\alpha_k^{(i)}$ $(k \neq 0)$ represent sidelobes. The length of the sequence $\{\alpha_k^{(i)}\}$ is $2N_i + 1$, where N_i is a function of i. There is no loss of generality in assuming N_i samples to exist on either side of the main pulse $\alpha_0^{(i)}$ since certain samples can always take zero values. Note that the tap gain $\beta_k^{(i)}$ in the ith stage is used mainly to suppress the interference term $\alpha_k^{(i-1)}$ in the input sequence of that stage; thus $M_i \leq N_{i-1}$.

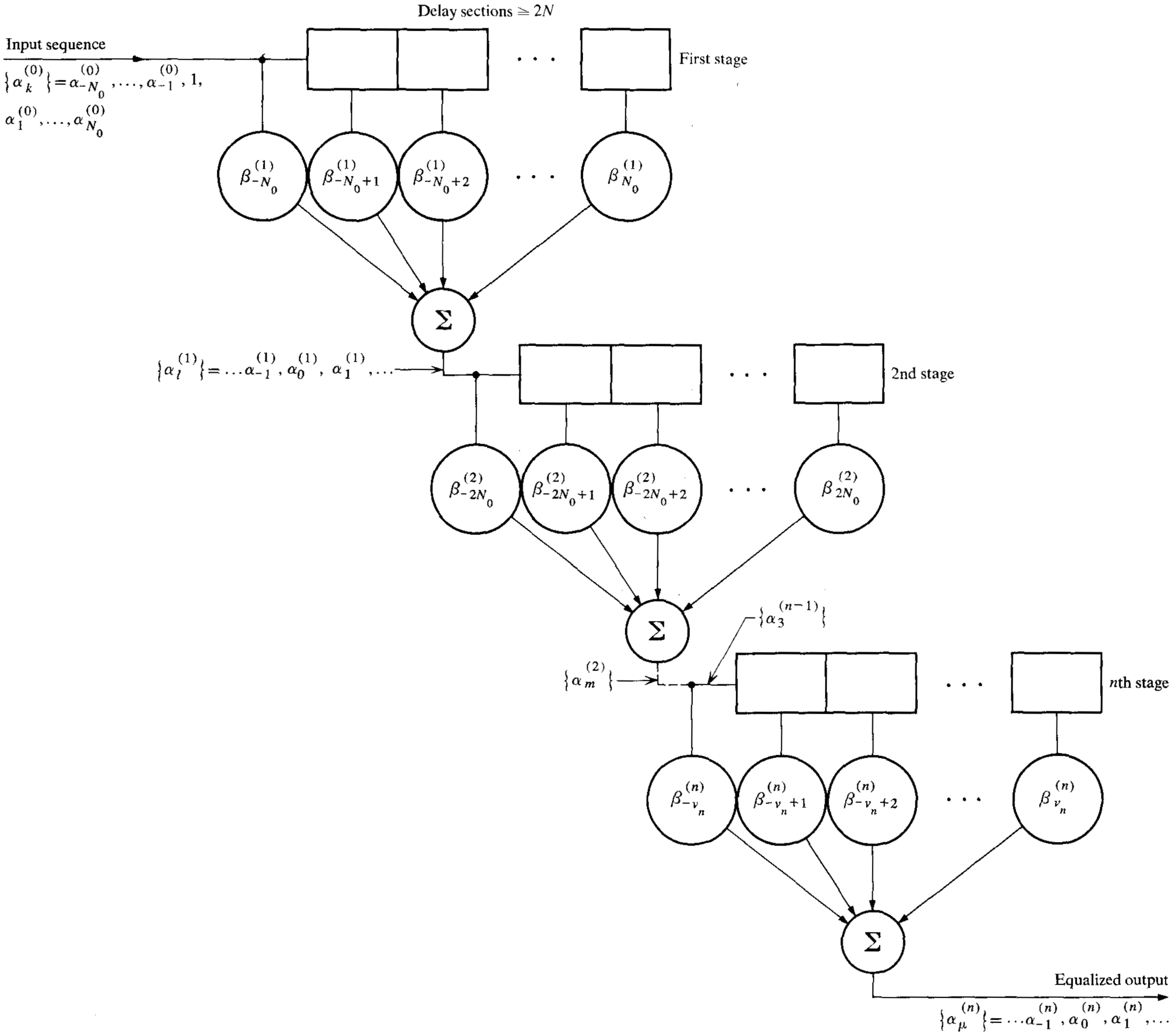


Figure 1 System diagram for the cascaded nonrecursive equalizer.

• *Recursive equalizer with cascaded stages*

Another equalizer structure considered in this paper is that of a recursive filter with n cascaded stages in the feed-forward section (Fig. 2). In the cascaded section, the tap-gain settings of the ith stage form a vector $\boldsymbol{\beta}_i^{(i)} = (\beta_{-M_i}^{(i)}, \cdots, \beta_{-1}^{(i)}, \beta_0^{(i)})$, and the output of the ith stage is $\{\alpha_k^{(i)}\} = \{\alpha_{-N_i}^{(i)}, \cdots, \alpha_{-1}^{(i)}, \alpha_0^{(i)}, \alpha_1^{(i)}, \cdots, \alpha_{N_0}^{(i)}\}$. Here again $M_i \leq N_{i-1}$. Note that no attempt is made to suppress the interference in the rear end; this results in a constant width for rear-end sidelobes in the output of all stages. These rear-end sidelobes at the output of the feed-forward section are then cancelled out in the recursive section via feedback paths weighted by tap gains $\boldsymbol{\beta}_k = (\beta_1, \beta_2, \cdots, \beta_{N_0})$ as shown in Fig. 2.

3. Gain-setting algorithms for cascaded equalizers

• *Nonrecursive type*

The following procedure is observed during the training period:

1. Preset $\beta_0^{(1)}, \beta_0^{(2)}, \cdots, \beta_0^{(n)}$ to unity and all other weights to zero.
2. In the ith iteration ($i = 1, 2, \cdots, n$), the tap gains of the ith stage are set according to the response of the ith training pulse at the input of the ith stage, as follows:

$\beta_k^{(i)} = -\alpha_k^{(i-1)}$ for $k \neq 0, -M_i \leq k \leq M_i$ and

$\beta_0^{(i)} = 2 - \alpha_0^{(i-1)}$.

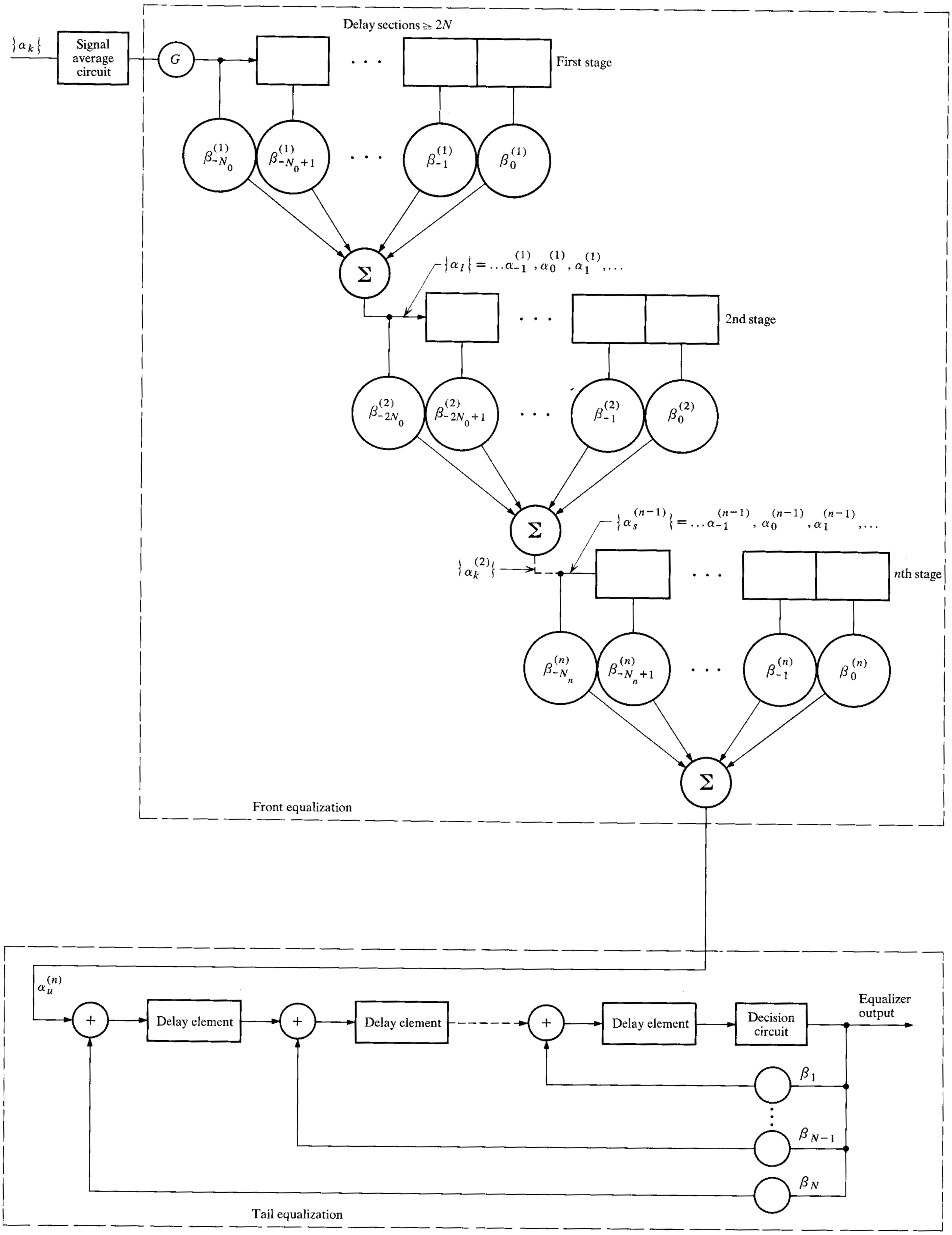


Figure 2 System diagram for cascaded recursive equalizer.

3. The procedure ends either when tap gains in all n stages are set or when the desired eye-opening is obtained.

Without truncation, we have $M_i = N_{i-1}$. In such a case, the widths of both front and rear sidelobes double after each iteration. That is, $N_i = 2^i N_0$.

• *Recursive type*

The following procedure is observed during the training period:

1. Preset $\beta_0^{(1)}, \beta_0^{(2)}, \cdots, \beta_0^{(n)}$ to unity and all other weights to zero.
2. In the ith iteration ($i = 1, 2, \cdots, n$), the tap gains of the ith stage in the feed-forward section are set according to the response of the ith training pulse at the input of the ith stage, as follows:

$$\beta_k^{(i)} = -\alpha_k^{(i-1)} \quad \text{for } M_i \leq k < 0, \quad \text{and}$$
$$\beta_0^{(i)} = 2 - \alpha_0^{(i-1)}.$$

3. The above iteration ends either when tap gains in all n stages in the feed-forward section are set or when the front distortion is sufficiently small.
4. The tap gains in feedback paths of the recursive section are set according to the rear sidelobes at the output of the cascaded feed-forward section:

$$\beta_k = -\alpha_k^{(n)} \quad \text{for } 0 \leq k \leq N_0.$$

If $M_i = N_{i-1}$, the width of the front sidelobe doubles after each iteration ($N_i = 2^i N_0$), while the rear sidelobe maintains a constant width of N_0.

4. Performance analysis

The channel response sequence $\{\alpha_k^{(0)}\}$ can be expressed in terms of the z-transform expansion:

$$A^{(0)} = \sum_{k=-N_1}^{N_0} \alpha_k^{(0)} z^{-k}. \tag{1}$$

An initial distortion sequence can be defined, giving:

$$\Lambda^{(0)} = 1 - A^{(0)}. \tag{2}$$

The initial total (peak) distortion, $D_0^{(0)}$ is defined as the sum of the magnitudes of sidelobes when the main pulse is normalized to unity. Thus

$$D^{(0)} = |A^{(0)}/\alpha_0^{(0)}| - 1, \tag{3}$$

where $|\cdot|$ denotes the sum of absolute values of coefficients in the z-transform expansion. The eye-opening of the initial channel response sequence is defined as

$$I^{(0)} = 1 - D^{(0)}. \tag{4}$$

Similarly, one can define the peak distortion $D^{(i)}$ and eye-opening $I^{(i)}$ of the ith stage output sequence. Eye-opening is a measure of the margin against noise in the presence of worst-case cumulative intersymbol interference when using simple threshold detection of the sampled signal.

Let us now consider the gain-setting algorithm for nonrecursive cascaded equalizers without truncation as described in the last section. According to the algorithm, the transfer function for the ith stage can be written as a polynomial in z^{-1}:

$$B^{(i)} = z^{-N_{i-1}}(2 - A^{(i-1)}) = z^{-N_{i-1}}(1 + \Lambda^{(i-1)}), \tag{5}$$

where $N_i = N_0 2^{i-1}$. With the input at the ith stage being $A^{(i-1)} = 1 - \Lambda^{(i-1)}$, the output at this stage is

$$A^{(i)} = A^{(i-1)} B^{(i)} = z^{-N_{i-1}} [1 - (\Lambda^{(i-1)})^2]. \tag{6}$$

It follows that, with the initial input being $A^{(0)} = 1 - \Lambda^{(0)}$, the output at the nth stage is

$$A^{(n)} = z^{-(N_0+N_1+\cdots+N_{n-1})}[1 - (\Lambda^{(0)})^{2^n}]$$
$$= z^{-N}[1 - (\Lambda^{(0)})^{2^n}], \tag{7}$$

where $N = N_0(2^n - 1)$. The overall transfer function for the n cascaded equalizer stages is:

$$J^{(n)} = \prod_{i=1}^{n} B^{(i)} = z^{-N} \prod_{i=0}^{n-1} [1 + (\Lambda^{(0)})^{2^i}]. \tag{8}$$

It is clear from the expression of $A^{(n)}$ that, as long as $|\Lambda^{(0)}| < 1$, the quantity $|\Lambda^{(n)}| \leq |\Lambda^{(0)}|^{2^n}$ converges to zero exponentially as n increases.

We shall next show that if the initial input is scaled so that $|\alpha_0^{(0)}| = 1$, then the final total distortion and eye-opening are bounded as follows:

$$D^{(n)} \leq [D^{(0)}]^{2^n} \tag{9}$$

and

$$I^{(n)} \geq 1 - [D^{(0)}]^{2^n}. \tag{10}$$

This can be done by first showing in general, $D \leq |\Lambda|$. That is, the unnormalized total distortion is always greater than or equal to the normalized one. Consider the following cases:

(a) If $|\alpha_0| = 1$, then $D = |\Lambda|$. (11)

(b) If $|\alpha_0| > 1$, let $|\alpha_0| - 1 = \epsilon$, where $\epsilon > 0$. Then

$$D = [|\Lambda| - \epsilon]/(1 + \epsilon) < |\Lambda|. \tag{12}$$

(c) If $|\alpha_0| < 1$, let $1 - |\alpha_0| = \epsilon$ where $0 < \epsilon < 1$. Then

$$D = [|\Lambda - \epsilon|]/(1 - \epsilon)$$
$$= |\Lambda| - (1 - |\Lambda|)\epsilon/(1 - \epsilon) < |\Lambda| \tag{13}$$

as long as $|\Lambda| < 1$.

Since $|\alpha_0^{(0)}| = 1$ implies $D^{(0)} = |\Lambda^{(0)}|$, we have

$$D^{(n)} \leq |\Lambda^{(n)}| \leq |\Lambda^{(0)}|^{2^n} = |D^{(0)}|^{2^n}, \text{ and} \tag{14}$$

$$I^{(n)} \geq 1 - [D^{(0)}]^{2^n} \tag{15}$$

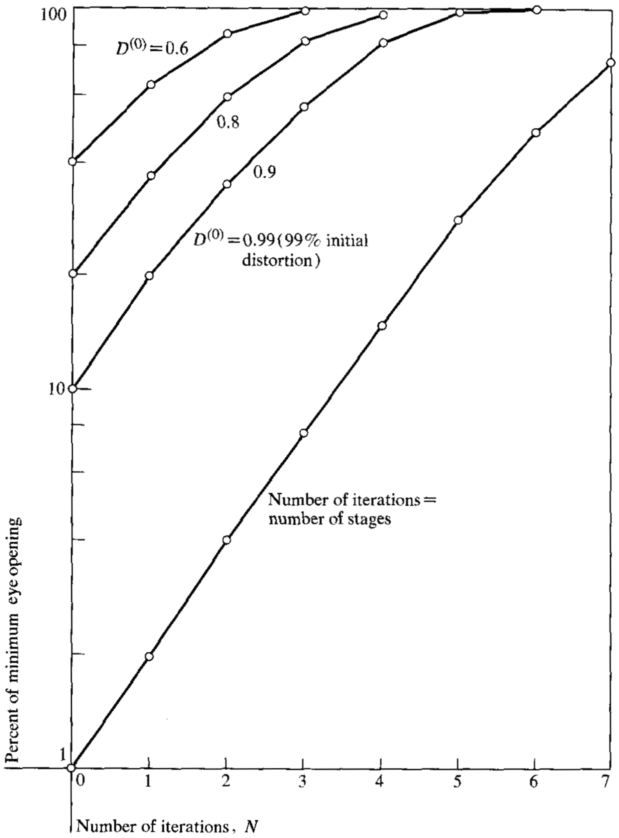


Figure 3 Minimum eye-opening of the nonrecursive equalizer vs number of iterations with a given initial distortion, $D^{(0)}$.

as long as the initial condition $D^{(0)} = |\Lambda^{(0)}| < 1$ is satisfied. It should be noted that this is a sufficient but not a necessary condition for the convergence of the total distortion. However, given $D^{(0)} \geq 1$, it is possible to have $D^{(i+1)} \geq D^{(i)}$ for all i. This situation exists, for instance, when all $\alpha_k^{(0)} (k \neq 0)$ have negative signs.

The minimum (guaranteed) eye-openings vs the number of iterations are plotted in Fig. 3 with various values of initial total distortion $D^{(0)}$. This shows what can be expected of the cascaded equalizer described above under noise-free conditions.

We next consider the gain-setting algorithm for the cascaded feed-forward section in a recursive equalizer as described in the last section. Let Λ_f denote the front distortion sequence including the distortion at the main pulse, i.e.,

$$\Lambda_f^{(i)} = 1 - \alpha_0^{(i)} - \sum_{k=-N_i}^{-1} \alpha_k^{(i)} z^{-k} . \tag{16}$$

Similarly, the rear distortion sequence gives

$$\Lambda_r^{(i)} = - \sum_{k=1}^{N_i} \alpha_k^{(i)} z^{-k} . \tag{17}$$

Thus

$$A^{(i-1)} = 1 - \Lambda^{(i-1)} = 1 - \Lambda_f^{(i-1)} - \Lambda_r^{(i-1)} . \tag{18}$$

According to the algorithm, the transfer function for the ith stage in the cascaded section is

$$B^{(i)} = z^{-N_{i-1}} (1 + \Lambda_f^{(i-1)}) . \tag{19}$$

The output at the ith stage is therefore

$$\begin{aligned} A^{(i)} = A^{(i-1)} B^{(i)} &= z^{-N_{i-1}} (1 - \Lambda_f^{(i-1)} - \Lambda_r^{(i-1)}) (1 + \Lambda_f^{(i-1)}) \\ &= z^{-N_{i-1}} [1 - \Lambda_f^{(i-1)} (\Lambda_f^{(i-1)} + \Lambda_r^{(i-1)}) \\ &\quad - \Lambda_r^{(i-1)}] . \end{aligned} \tag{20}$$

Thus

$$\begin{aligned} |\Lambda^{(i)}| = |1 - A^{(i)}| &\leq |\Lambda_f^{(i-1)}| |\Lambda^{(i-1)}| + |\Lambda_r^{(i+1)}| \\ &< |\Lambda_f^{(i-1)}| + |\Lambda_r^{(i-1)}| = |\Lambda|^{(i-1)}| \text{ for } |\Lambda^{(i-1)}| < 1 . \end{aligned} \tag{21}$$

Therefore if the initial input is normalized with $|\alpha_0^{(0)}| = 1$, then

$$D^{(n)} \leq |\Lambda^{(n)}| < |\Lambda^{(0)}| = D^{(0)} \text{ for } D^{(0)} < 1 .$$

That is, the total distortion decreases. On the other hand,

$$|\Lambda_f^{(i)}| < |\Lambda_f^{(i-1)} (\Lambda_f^{(i-1)} + \Lambda_r^{(i-1)})| \leq |\Lambda_f^{(i-1)}| |\Lambda^{(i-1)}| . \tag{22}$$

If we define D_f and D_r, respectively, as the total front and rear distortions, i.e., contributions of, respectively, front and rear sidelobes to the total distortion, and if $|\alpha_0^{(0)}| = 1$, then

$$D_f^{(n)} < D_f^{(0)} \prod_{i=0}^{n-1} D^{(i)} < D_f^{(0)} [D^{(0)}]^n . \tag{23}$$

It should be pointed out that the above inequality gives a very loose bound for $D_f^{(n)}$. In practice $D_f^{(i)}$ converges much faster than $D_f^{(0)} [D^{(0)}]^i$ since a) $D^{(i)}$ itself converges, b) only a portion of $\Lambda_f^{(i-1)} \Lambda_r^{(i-1)}$ in Eq. (22) actually becomes part of the front distortion, and c) many product terms with different signs tend to partly cancel one another.

After n stages of front equalization, feedback paths are used to cancel out the final rear distortion. Since components in $D_f^{(n)}$ are multiplied by appropriate components in $D_r^{(n)}$ and fed back recursively, the worst-case condition of an eye-opening can be seen to be

$$I = 1 - [D_f^{(n)} / (1 - D_r^{(n)})] . \tag{24}$$

Since $D_f^{(n)} \to 0$ as n increases, and since $D_r^{(n)} \leq D^{(n)} \leq D^{(0)}$, the eye-opening approaches unity as n increases. Note that there can be no guarantee for eye-opening if $D_r^{(n)} \geq 1$, and it is therefore vital for the gain-setting algorithm to keep the rear distortion below unity.

The effectiveness of these gain-setting algorithms will be illustrated later by numerical examples.

5. Effect of truncation

We shall define the "truncation error" to be the residual distortion resulting from our iterative equalization procedure, in which the maximum number of delay units per stage is limited to a constant K while the number of stages is unlimited. For simplicity, let

$$K = 2^p M \,, \tag{25}$$

where p is a positive integer and M is the number of sidelobe samples in the original channel response. Thus,

$$p = \log_2 (K/M) \tag{26}$$

is the number of stages without truncation. We shall denote the corresponding truncation error by $E^{(p)}$.

The upper bound of the truncation error $E^{(p)}$, with a given initial distortion, can be readily derived from the worst-case input sequence, which has only one sidelobe in addition to the main pulse:

$$E^{(p)} \leq [D^{(0)}]^{2^{p+1}} . \tag{27}$$

Although bounds of truncation error, as indicated in the above equation, can actually be attained under worst-case conditions, truncation error in practical cases may fall well below these bounds. A reasonable approximation of the truncation error can be written in the following linear form:

$$E^{(p)} \approx b_{p+1} [D^{(0)}]^{2^{p+1}} + \cdots + b_n [D^{(0)}]^{2^n}$$
$$= \sum_{k=p+1}^{n} b_k [D^{(0)}]^{2^k} . \tag{28}$$

This is based on the observation that the output at the $(p+1)$th stage consists of two parts; the uncontrollable sidelobes which lie outside the span of K delay units, and the controllable sidelobes which lie inside the span. It is then assumed that the contribution of the first $(p+1)$ stages in $E^{(p)}$ is $b_{p+1}[D^{(0)}]^{2^{p+1}}$. The contribution of the $(p+2)$th stage to $E^{(p)}$ is a fraction of its output when its input is the actual output of the $(p+1)$th stage with uncontrollable sidelobes removed. This is approximated by $b_{p+2}[D^{(0)}]^{2^{p+2}}$. The same reasoning is applied to the following stages. The coefficients are determined by the estimated fraction of the total distortion corresponding to uncontrollable sidelobes. When the total distortion becomes "uncontrollable" after $(p+1)$ stages, we have $b_{p+1} = 1$ and $b_k = 0$ for $k > p+1$, in which case Eq. (28) reduces to Eq. (27).

We have found the following case to yield a conservative but realistic estimate of $E^{(p)}$: The input sequence to the equalizer (original channel response) is assumed to have M flatly distributed sidelobes with identical magnitudes and the worst-case sign pattern. We also assume $p \geq 1$, i.e., the maximum number of delay sections allowed in each stage is at least $2M + 1$. The outputs of the first two iterations with $p = 1$ are shown in Fig. 4.

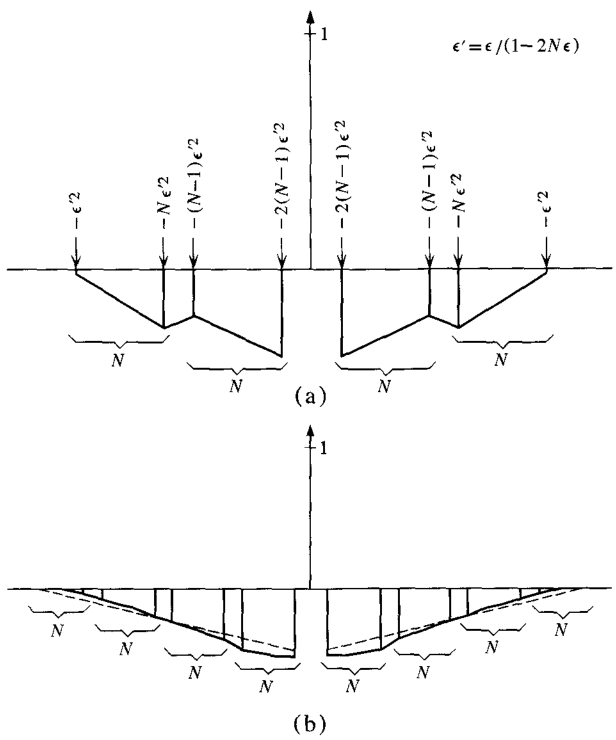


Figure 4 First and second iterations of the nonrecursive equalizer with flatly distributed sidelobes as input; (a) output sequence after first iteration, (b) output sequence after second iteration.

Clearly, if $D^{(0)} < 1$, only a few terms in (28) are necessary to give a good approximation. For small b_2, we may further simplify the approximation of (28) to give

$$E^{(1)} \approx b_2 [(D^{(0)})^{2^2} + (D^{(0)})^{2^3} + (D^{(0)})^{2^4}] . \tag{29}$$

By inspecting the output of Fig. 4(b), one can estimate b_2, the fraction of uncontrollable distortion to be about one fifth, hence

$$\hat{E}^{(1)} \approx 0.2 \sum_{k=2}^{4} [D^{(0)}]^{2^k} . \tag{30}$$

The estimated truncation error $\hat{E}^{(1)}$ and the actual $E^{(1)}$ after five iterations are tabulated in Table 1 under the assumption that the inputs to the equalizers all have worst-case flatly distributed sidelobes with initial distortions of 0.9, 0.8, 0.7, and 0.6, respectively,

Example 1:
Given $\{\alpha_k^{(0)}\} = (0.005, -0.064, -0.138, 1, 0.315, -0.131, -0.059)$, which has an initial distortion, $D^{(0)} = 0.712$, we wish to find 1) the number of iterations, and 2) the number of delay sections for the nonrecursive type, truncated nonrecursive type, and recursive type equalizers in order to obtain an eye-opening of 95% or better. The results are shown in Table 2.

Table 1 Comparison of estimated and simulated truncation error.

Initial distortion $D^{(0)}$	Percent of truncation error $E^{(1)}$, estimated by Eq. (30)	Percent of truncation error $E^{(1)}$ determined by simulation	
		$N=4$	$N=5$
0.9	26.1	24.7	23.3
0.8	12.1	9.7	9.2
0.7	6.9	4.4	4.2
0.6	2.9	2.1	1.9

Table 2 Results of example 1.

Type of equalization	No. of iterations required	Max. no. of delay units per stage	Eye-opening in percent
Nonrecursive	3	24	98.9
Nonrecursive with truncation	3	12	98.1
Recursive	2	6	97.7

6. Noise consideration

So far we have discussed gain-setting algorithms for cascaded equalizer sections and their performance under the noise-free assumption. We shall now investigate the effect of noise, making the common assumption that it is additive and Gaussian.

Let us introduce a signal-averaging circuit just before the equalizer. The output signal sequences from the averaging circuit represent the estimates of the channel response. These estimates of the channel response are used successively to tune (set the tap gains) the cascaded stages. Although the training pulses at the input of the averaging circuit can be separated at a regular minimum of $2N+1$ samples, the estimated channel responses at the output of the averaging circuit are separated at appropriate (and varying) distances to satisfy: a) minimum delays required for the signal to pass through the equalizer stages without overlapping, and b) the allowable noise level at the stage being tuned. If the ith estimated channel response is used to tune the ith cascaded stage, then consideration a) above gives a minimum separation of $3N_0+1$ sample periods between the first and the second, and $(3\times 2^{i-1})N_0+1$ sample periods between the ith and the $(i+1)$th estimated channel responses. This requirement can be satisfied by the following scheme: When the second test pulse is being received, the averaging circuit produces, with negligible delay, the first estimated channel response by averaging, sample-by-sample, the first two test pulses. In a similar fashion, when the 2^ith test pulse is being received, the averaging circuit produces, almost simultaneously, the ith estimated channel response. The separation between the ith and the $(i+1)$th estimated channel responses is thus $2^i(2N_0+1)$ sample periods [8]. The fact that the first 2^i test pulses are averaged for stage i simplifies greatly the division procedure involved, since only shifts on the accumulated sample values in binary representation are necessary.

Another interesting property of the above averaging scheme is that more test pulses are averaged for larger i, and thus more accurate estimates of channel responses are used for finer tuning of later stages. More specifically, since we double the number of test pulses averaged for each additional stage, a 3dB gain in signal-to-noise ratio is obtained automatically per stage. If we denote the initial signal-to-noise ratio in dB by $(\mathrm{S/N})_0$ and that of the ith averaging circuit output by $(\mathrm{S/N})_i$, then we have

$$(\mathrm{S/N})_i=(\mathrm{S/N})_0+3i\,. \tag{31}$$

Clearly, one should always limit the noise to a level considerably below the residual distortion at all stages, i.e.,

$$(\mathrm{S/N})_i>-20\log_{10}D^{(i)}$$

$$\text{or } (\mathrm{S/N})_0+3i>-20\times 2^i\log_{10}D^{(0)}\,. \tag{32}$$

If the above condition is satisfied for all $i\le n$ with a certain prespecified margin, then the averaging scheme suggested above should be satisfactory. Note that since $D^{(i)}$ in dB changes exponentially with i, the above inequality needs to be checked only for $i=n$ in most cases. If, on the other hand, Eq. (32) is not satisfied for some i, then it is necessary to average over more test pulses for these i. For instance, the separation between the ith and the $(i+1)$th estimated channel response may now be 2^{j_i}, where j_i satisfies the inequality

$$(\mathrm{S/N})_0+3j_i>-20\times 2^i\log_{10}D^{(0)} \tag{33}$$

for all i with a certain specified margin.

Example 2:

Given a binary transmission system with a transmission rate of 4800 bits/sec and an initial channel signal-to-noise ratio, $(\mathrm{S/N})_0=25$ dB, where the exact channel response is $\{\alpha_k^{(0)}\}=(-0.012,\ 0.023,\ -0.081,\ -0.314,\ 1.0,\ -0.189,\ -0.115,\ 0.093,\ -0.048,\ 0.014)$, it is desired to achieve a final eye-opening of 95% or better.

The initial distortion in this case is $D^{(0)}=0.89$. From Fig. 3, one finds $n=5$ to yield $D^{(5)}=2.5\%$. If 2^i test pulses are averaged for stage i, the final estimated channel response $(i=n=5)$ has a signal-to-noise ratio

$$(\mathrm{S/N})_5=25+(3\times 5)=40\text{ dB}\,.$$

Since this implies an expected noise perturbation of 1%, well below the guaranteed noise-free final distortion of

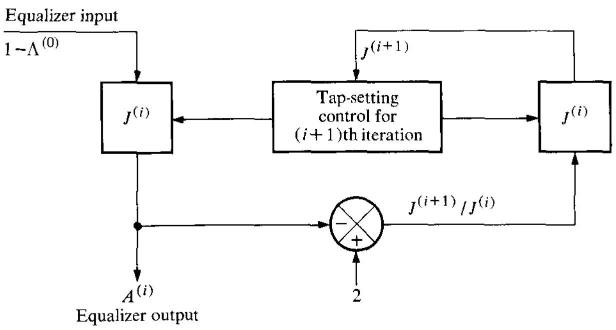


Figure 5 Software implementation of the automatic nonrecursive equalizer.

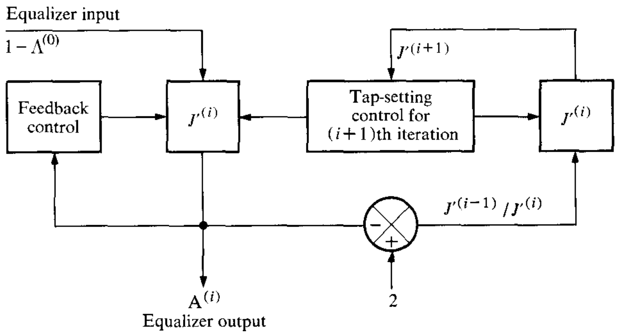


Figure 6 Software implementation of the automatic recursive equalizer.

2.5%, no additional averaging is necessary. The settling time T_s in this case is

$$T_s = (1/4800)\ 2^i\ (M_0 + 1) = 32 \times 10/4800 \approx 67 \text{ ms}.$$

We assumed earlier that the averaging circuit is capable of producing, sample-by-sample, an estimated channel response with negligible delay. This is true when the high-speed logic circuit performs the additions and shifts needed for averaging in less than one sample period—a feat easily accomplished with the LSI technology over a wide range of transmission speeds. In fact, one can take further advantage of the logic speed by driving the clock of the equalizer stages at the limit of the circuitry after an estimated channel response is obtained, thus minimizing the delay in obtaining the signal at the input of the equalizer stage to be tuned. In cases where $(S/N)_0$ is high, the settling time can be reduced to approximately the time it takes to receive a sufficient number (say 2^j) of test pulses such that

$$(S/N)_n = (S/N)_0 + 3j > -20 \log_{10} D^{(n)} \tag{34}$$

is satisfied with a specified margin.

7. Some generalizations

• *Single stage implementation*

The gain setting algorithms described in this paper can be viewed as iterative procedures to approximate desired equalizer responses. The cascaded structure is mainly a hardware implementation in which each stage in cascade corresponds to an iteration. Various other forms of implementation are possible with different software and hardware combinations. For instance, a completely general equalizer transfer function for the ith iteration can be written as

$$J^{(i)} = z^{-(N_0+N_1+\cdots+N_{i-1})} \prod_{j=1}^{I} B^{(i)}, \tag{35}$$

where $B^{(i)}$ is the transfer function of the equivalent ith cascaded equalizer stage, which depends on the $(i-1)$th iteration output

$$A^{(i-1)} = A^{(0)} J^{(i-1)} \tag{36}$$

and the algorithm used. In the case of the nonrecursive equalizer algorithm described in Section 3, we have

$$\begin{aligned} J^{(i)} &= z^{-N_{i-1}}\ (2 - A^{(i-1)})\ J^{(i-1)} \\ &= z^{-N_{i-1}}\ (2 - A^{(0)} J^{(i-1)})\ J^{(i-1)}. \end{aligned} \tag{37}$$

The above iterative expression can be implemented in a form shown in Fig. 5. Here all calculations may be done in software, or one may retain the left $J^{(i)}$ box in the conventional equalizer form while realizing the function of the right $J^{(i)}$ box by either software or special purpose hardware. The equalizer in this case no longer consists of cascaded stages as shown in Fig. 1.

When the noise level is not negligible, an averaging circuit can be used in the same way as discussed in Section 6. Furthermore, only slight modifications are needed for the equalizer structure of Fig. 5 to operate in a recursive mode, i.e., to operate as the equivalent of the recursive equalizer with cascaded feed-forward section described earlier in this paper. This is shown in Fig. 6.

In Figs. 5 and 6, each $J^{(i)}$ or $J'^{(i)}$ box represents a conventional single stage equalizer. Duplicate $J^{(i)}$ boxes are required for software implementations. The first four iterations of the nonrecursive equalizer (see Fig. 5) are illustrated in Table 3.

• *Partial adjustment of tap gains*

We shall next consider the effect of a proportionality constant $c (c > 0)$ which is now introduced in our gain-setting adjustment algorithm as follows: In the $(i+1)$th iteration, the input is first scaled so that $a_0^{(i)} = 1$. The tap-gain settings are: $\beta_k^{(i+1)} = -c\alpha_k^{(i)}$ for $k \neq 0$ and $\beta_0^{(i+1)} = 1$. For $0 < c < 1$, this modification amounts to a partial adjustment of tap gains in comparison with the original algorithm. The output of the $(i+1)$th iteration with the

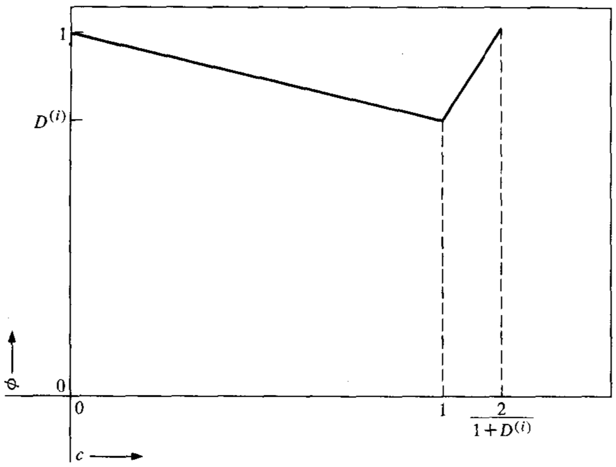


Figure 7 ϕ vs proportionality constant, c.

proportionality constant c is denoted by $A'^{(i+1)}$, where

$$A'^{(i+1)} = 1 - \Lambda'^{(i+1)} = A^{(i)}[(1-c) + c(2 - A^{(i)})]$$
$$= (1-c)(1-\Lambda^{(i)}) + c(1-\Lambda^{(i)})(1+\Lambda^{(i)}), \quad (38)$$

$$\Lambda'^{(i+1)} = (1-c)\Lambda^{(i)} - c|[\Lambda^{(i)}]^2|, \quad (39)$$

and

$$|\Lambda'^{(i+1)}| \leq |1+c|\ |\Lambda^{(i)}| + c|[\Lambda^{(i)}]^2| \quad (40)$$

with the equality satisfied for $\{\alpha_k^{(i)}\}$ having worst-case sign patterns. From Eqs. (11) - (13), we have

$$D'^{(i+1)} \leq |\Lambda'^{(i+1)}| \leq |1-c|\ |\Lambda^{(i)}| + c|[\Lambda^{(i)}]^2|$$
$$= D^{(i)}[|1-c| + cD^{(i)}] = \phi D^{(i)}, \quad (41)$$

where

$$\phi = |1-c| - cD^{(i)}. \quad (42)$$

The eye-opening is therefore

$$I'^{(i+1)} = 1 - \phi D^{(i)}. \quad (43)$$

The plot of ϕ as a function of c is shown in Fig. 7. It can be seen that the rate of convergence in the distortion $D^{(i)}$ is scaled down almost linearly for $\{\alpha_k^{(i)}\}$ with worst-case sign patterns. However, it has been found that for many other sign patterns, the use of a proportionality constant $c < 1$ may well improve the convergence rate. This is shown in Figs. 8(a) and 8(b) for various sign patterns of a particular numerical example. Figs. 9(a) and 9(b) show the improvement of eye-opening as functions of c, again with various sign patterns for a different input sequence. It is interesting to note that, although here the initial eye-opening is closed, the use of a proportionality constant c in the range of 0.6 to 0.8 yields good eye-openings in most cases.

• *Adaptive operation*

While an automatic equalizer is capable of reducing the intersymbol interference in the channel response by automatically adjusting its tap gains based on a set of isolated training pulses received prior to actual data transmission, an adaptive equalizer is capable of equalizing the channel and tracking the time variations of the channel response during data transmission. Such adaptive operation can be realized in the cascaded structure.

Figure 10 shows the system diagram of a cascaded adaptive equalizer. We note that the input to the first stage of the equalizer is no longer a set of isolated impulse responses of the channel during the data transmission. However, the desired impulse response of the channel can be estimated by cross-correlating the input to the equalizer and the decision output [9]. This information is used to adjust the tap gains of the first stage according to the same algorithm for automatic equalization. Similarly, the combined impulse response of the channel and the first stage can be estimated by cross-correlating the

Table 3 Tap-settings and output response for nonrecursive equalizer.*

No. of iterations, i	*Equalizer tap-setting for ith iteration, $J^{(i)}$*	*Output response, $\Lambda^{(i)}$, of the equalizer*	*Equalizer tap-settings for (i+1)th iteration, $J^{(i+1)}$*
0 (initial)	1	$1-\Lambda$	$1+\Lambda$
1	$(1+\Lambda)$	$1-\Lambda^2$	$\prod_{i=1}^{2}(1+\Lambda^{2^{(i-1)}})$
2	$\prod_{i=1}^{2}(1+\Lambda^{2^{(i-1)}})$	$1-\Lambda^4$	$\prod_{i=1}^{3}(1+\Lambda^{2^{(i-1)}})$
3	$\prod_{i=1}^{3}(1+\Lambda^{2^{(i-1)}})$	$1-\Lambda^8$	$\prod_{i=1}^{4}(1+\Lambda^{2^{(i-1)}})$
4	$\prod_{i=1}^{4}(1+\Lambda^{2^{(i-1)}})$	$1-\Lambda^{16}$	$\prod_{i=1}^{5}(1+\Lambda^{2^{(i-1)}})$

$^*\Lambda^{(0)} = \Lambda$

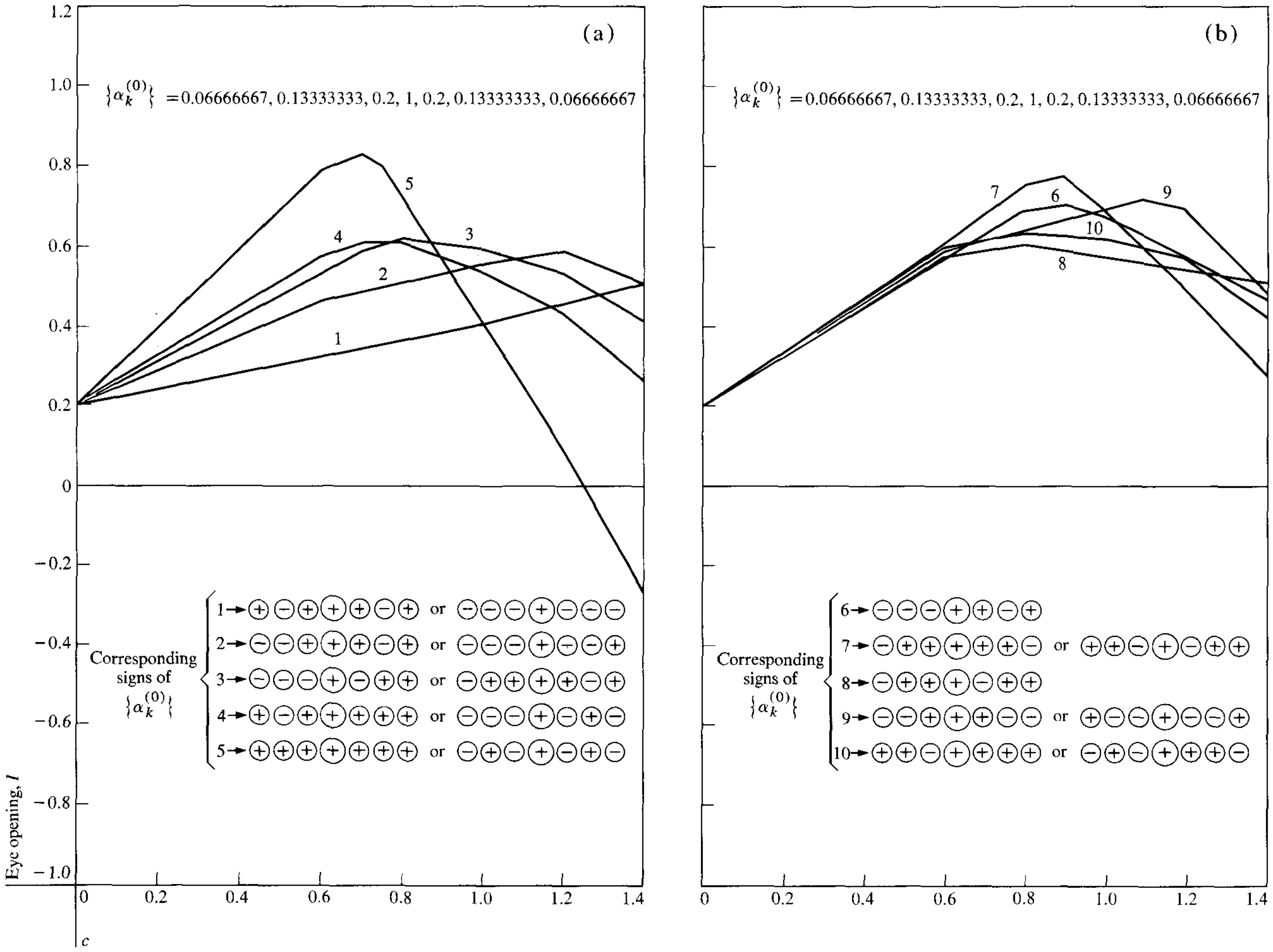


Figure 8 Eye opening vs proportionality constant c with given sign patterns $\{\alpha_k^{(0)}\}$.

input to the second stage and the decision output, which, in turn, is used to adjust the tap-gain setting of the second stage. Analogous procedures are employed for all n stages. It should be noted that the adaptive operation discussed above works only if the decision output contains relatively few errors such that their effect is averaged out during the cross-correlation.

Once the initial equalization is completed, some minor tuning may still be necessary to track the possible channel variations. It is likely here that only the last stage will need adjustment in most cases, since the channel is not expected to change too much during a normal period of transmission. In such a case, the inpulse estimator circuit cross-correlates the last stage input and the decision circuit output using no more than a certain number of past data bits.

We point out that an efficient equalization can often be achieved by combining automatic and adaptive modes of operation. The automatic pre-equalization is carried out only to an extent sufficient for the adaptive mode to take over. The adaptive mode will then refine the initial equalization and track the time variations of the channel.

8. Concluding remarks

A new type of automatic equalizer algorithm has been developed and analyzed. Its main advantage is the fast convergence of gain settings, especially in cases with high signal-to-noise ratios. The main idea is applicable to recursive as well as nonrecursive equalizers. The general iterative procedure can be implemented in various forms. The cascaded implementation, having certain potential advantages enhanced by the LSI technology, is studied in greater detail. The effects of truncation error and additive noise are also analyzed.

Finally, we note that fast convergent equalizers are not only necessary in high-speed data transmission, but may also find applications in lower-speed transmission systems in which time-sharing of a fast equalizer offers an attractive cost-performance advantage.

Acknowledgments

The authors express their appreciation to D. Jessep for his help in writing the APL simulation program. They are particularly indebted to M. Karnaugh for his stimulating discussions during the course of this study.

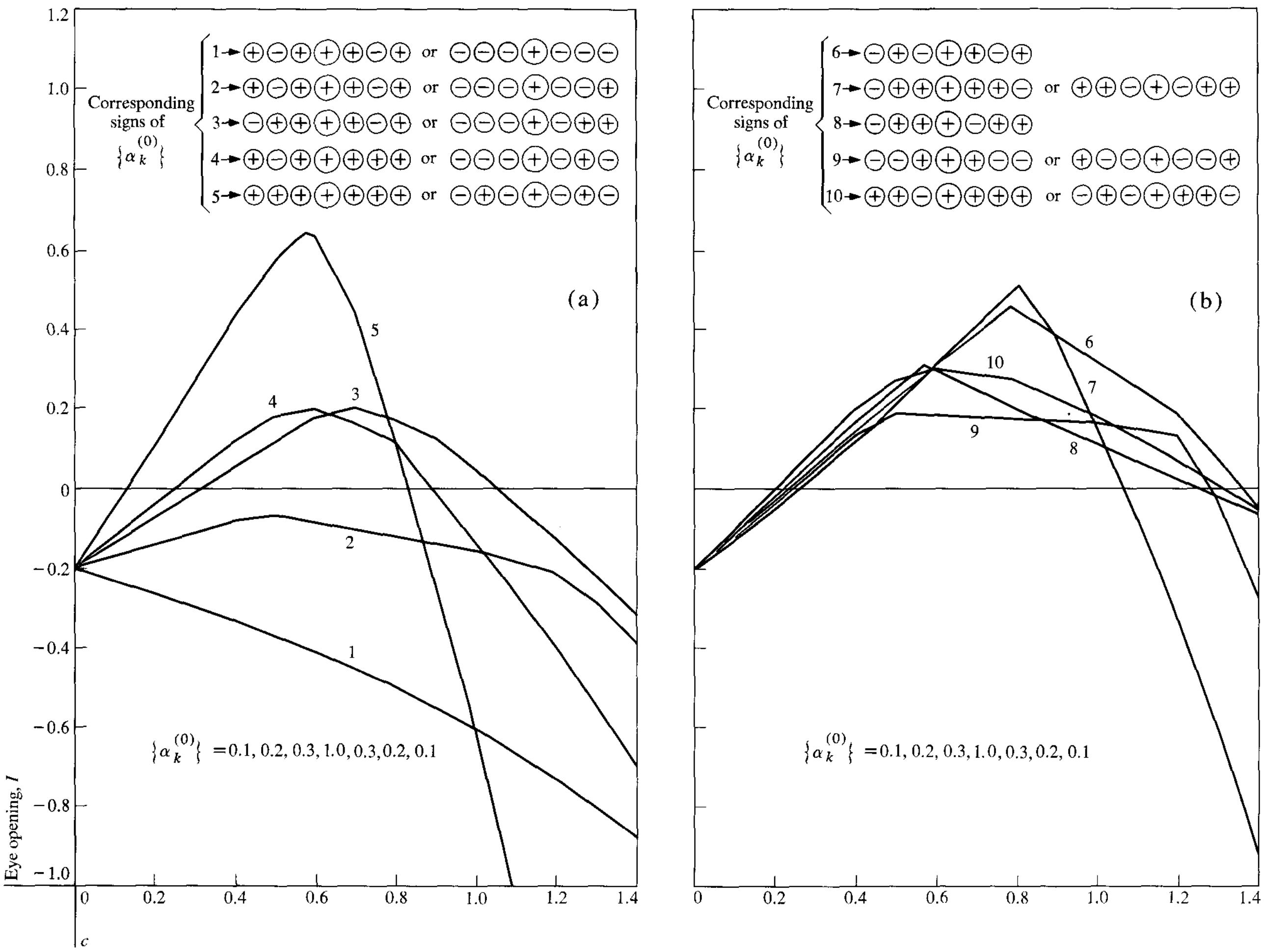
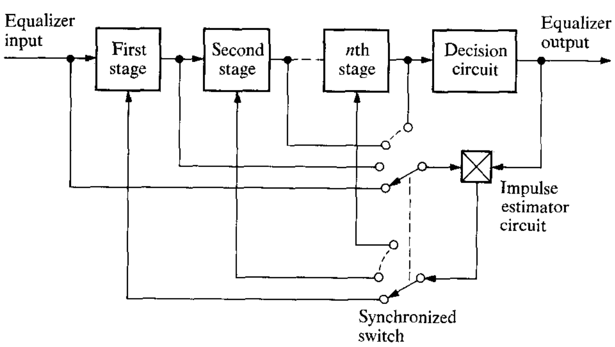


Figure 9 Eye opening vs proportionality constant c with given sign patterns for $\{\alpha_k^{(0)}\}$.

Figure 10 Implementation of a cascaded adaptive equalizer.



References and notes

1. K. E. Schreiner, H. L. Funk and E. Hopner, "Automatic Distortion Correction for Efficient Pulse Transmission," *IBM J. Res. Develop.* **9**, 20 (1965).
2. R. W. Lucky, "Automatic Equalization for Digital Communication," *Bell System Tech. J.* **44**, 547 (1965).
3. A. Gersho, "Automatic Time-Domain Equalization with Transversal Filters," National Electronic Conference, Chicago, Oct. 3–5, 1966.
4. M. J. Di Toro, "A New Method of High-Speed Adaptive Serial Communication Through Any Time-Variable and Dispersive Transmission Medium," *Conference Record*, IEEE Annual Communication Convention (1965), pp. 763.
5. R. W. Lucky, "Techniques for Adaptive Equalization of Digital Communication Systems," *Bell System Tech. J.* **45**, 255 (1966).
6. A. Gersho, "Adaptive Equalization of Highly Dispersive Channels for Data Transmission," *Bell System Tech. J.* **48**, 55 (1969).
7. D. Hirsch and W. J. Wolf, "A Simple Adaptive Equalizer for Efficient Data Transmission," 1969 WESCON Section II.
8. Another form is $2^i(M_0 + 1)$ sample periods where M_0 is the total number of sidelobe samples. This form is applicable when front and rear sidelobes are not of the same width.
9. The same technique has been used by Lucky in his adaptive equalizer study. See Ref. 5 or Lucky, Salz and Weldon, *Principles of Data Communication*, McGraw-Hill Book Co., Inc., New York, 1968, pp. 160–1.

Received July 12, 1971; revised July 19, 1972

The authors are located at the IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598.