Preface: Mathematics of Numerical Computation

The extensive use of computers for solving scientific and engineering problems has had a profound effect on the evolution of the mathematical techniques of numerical computation. It has given new impetus to research in numerical integration of differential equations; it has stimulated fresh approaches in the areas of approximation and optimization; and, in general, it has increased interest in numerical methods for the solution of a variety of problems. This special issue of the *Journal* reports some recent developments and innovations in the computational methods of numerical analysis.

The computer has not only been a stimulus to research, but it has also introduced certain demands upon the user. The traditional problem of finding a better and more efficient algorithm acquires new facets when the algorithm is to be executed on a computer. In addition to the usual question of how to minimize the number of arithmetic operations, another consideration is the way data are to be stored and moved. A good part of the gain derived, for example, from using sparse matrix techniques is obtained by avoiding arithmetic operations that involve zeros.

Other complexities in modern numerical mathematics are those that face the computer designer, rather than the user. A good example is the use of a hardware technique for evaluating elementary functions. The problems here are, of course, similar to those in other areas of approximation theory, but it is the strong interaction between the software and the specific hardware design that gives this set of problems its particular flavor.

These new aspects of numerical analysis are reflected in the 13 papers in this issue, which have been grouped into three topical sections, plus two communications.

In the first, four papers describe novel and efficient algorithms for the numerical integration of differential equations. Liniger and Odeh introduce a new theoretical principle for integration of stiff systems of ordinary differential equations. Alsop and Goodman describe a modified finite element method for solving partial differential equations and apply it to a study of wave propagation around sharp corners. Gourlay and Morris extend the hopscotch method to hyperbolic systems. And Canosa and Penafiel develop an improved technique for solving boundary value problems for a class of ordinary differential equations subject to roundoff instability.

In the area of approximation, Micchelli and Rivlin investigate numerical methods of the highest degree of precision for evaluating Chebyshev coefficients; Rissanen describes an algorithm for calculating matrix-valued Padé approximants; and Chen discusses the implemention of a new method, in hardware, for automatically computing logarithms, exponentials, ratios, and square roots. Brent and Miranker look at the problem of the convergence of iteration schemes. Brent shows that the Davidenko-Branin method does not necessarily converge globally, and Miranker develops a way to make matrix iteration methods converge as fast as possible. Bosarge and Smith discuss efficient numerical methods that utilize multivariate splines and apply their algorithm to typical control problems.

The papers in the third section deal with optimization. The contribution by Wolfe reveals some important limitations of a method for maximizing a function under inequality constraints, and the one by Crowder and Wolfe deals with gradient procedures for minimizing general nonlinear functions. Guignard and Spielberg give enumerative schemes for discrete optimization problems. Tomlin describes the application of sparse matrix techniques to linear programming. The paper by Hoffman and Winograd gives a new method for finding the shortest distances in a directed graph.

Even though the papers in this issue do not touch all important aspects of the broad field of numerical analysis, the results reported are representative of the kinds of research presently being conducted in this area. The continuing challenge of present and future work is the mutual interaction between computational technique and mathematical insight.

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