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Viscoelastic Behavior of Computer Tape Subjected to Periodic Motion

Abstract: The purpose of this study was to develop a theoretical means for predicting the longitudinal motion of computer tape in a high-performance tape drive. In particular, this paper treats the motion that is governed by traveling velocity-stress wave reflections, attenuations, and interactions in the length of tape between the tangency point at the capstan and the tangency point at the stubby column in the drive.

The motion of the tape was determined by solving the classical, damped, one-dimensional wave equation subject to the appropriate boundary conditions. J. C. Snowdon's low-damping constitutive model was used to describe the viscoelastic behavior of the tape. The solutions for simple boundary conditions were experimentally verified by mechanical impedance techniques. More complex boundary conditions, such as those for vacuum columns, were experimentally studied to determine the true mathematical boundary conditions.

This paper also discusses simple unreflected harmonic waves, simple reflected harmonic waves, and general periodic reflected waves as examples. The significance of the wave interactions in the design of tape drives is considered.

Introduction

In typical high-performance computer tape drives such as the IBM 2420-7 or the more recent 3420-7, the magnetic tape moves at a steady speed of a few hundred inches per second. It must start and stop in one or two milliseconds, and complete a full reversal in three to five milliseconds. The control unit frequently requires these tape drives to start and stop (shoeshine) about once every 20 to 60 milliseconds. Controlling the motion of tape running at a steady speed is well understood. Not so well understood is the process of bringing the tape to operating speed and holding it there under start-stop and shoeshine motion.

In the above-mentioned tape drives, the read/write head is located at a point along roughly a foot of tape between a capstan and a stubby vacuum column. Excursions and velocities of the tangency point at the stubby column are small as compared, respectively, with the relevant length of tape and the tape velocity. The sonic velocity of IBM Series/500 tape is about 6500 ft/s. The time required for motion to propagate from the capstan to the stubby column and back is of the order of 0.3 ms (about 1/5 of the specified start time), or from 60 to 200 times longer than for the start-stop and shoeshine motions already mentioned. If there is little or no damping in the stubby column, at the head, or in the boundary layer of air adjacent to the tape, then the internal damping in the tape determines the length of the startup or reversal transients, and thus determines the motion of tape at the read/write head.

Wave and constitutive equations

The one-dimensional wave equation is obtained from the momentum equation for a continuum,

$$\rho \frac{d\mathbf{u}}{dt} = \nabla \cdot \mathbf{T} + \rho \mathbf{f}, \tag{1}$$

where ρ is the mass density, $\mathbf{u}$ is the velocity in an inertial Eulerian frame, $\mathbf{T}$ is the stress tensor, and $\mathbf{f}$ is the summation of all body forces (per unit mass). If transverse velocities and body forces are ignored, and only pure tension is allowed in the stress tensor, the momentum equation in one dimension becomes

$$\rho \frac{du}{dt} = \frac{\partial \sigma}{\partial x}, \tag{2}$$

where σ is the tensile stress, u is the longitudinal velocity component, and x is the longitudinal coordinate. In general, a linearized relation between uniaxial stress and the strain component along that axis can be written as $\sigma = E^*\epsilon$, where ϵ is the strain and E^* is a real (or complex) elastic modulus. Also, du/dt is given by $\partial u/\partial t + u(\partial u/\partial x)$. If the ratio between u and the sonic velocity (or the propagation velocity of a small disturbance) is small, then the convective term can be ignored. Therefore, to the first order, $du/dt = \partial u/\partial t$. Finally, if the strain is small everywhere, then the relationship between velocity and strain, $\partial\epsilon/\partial t = \partial u/\partial x$, is correct to first order.

If we use the above relations in the momentum equation, the one-dimensional wave equation is obtained:

$$\rho \frac{\partial^2 u}{\partial t^2} = E^* \frac{\partial^2 u}{\partial x^2}. \tag{3}$$

In its most general linear form, E^* can be rewritten as

$$E^* = E/(n^*)^2, \tag{4}$$

where E is the (real) elastic modulus and n^* is a complex number, both possibly functions of a harmonic wave frequency. For convenience, the time and spatial coordinates and the velocity can be converted to dimensionless parameters as follows:

$$T = \frac{t}{\ell/\sqrt{E/\rho}};$$

$$X = x/\ell;$$

$$U = u/u_0, \tag{5}$$

where ℓ is a characteristic length of tape and u_0 is a reference velocity in the tape. In dimensionless form, the wave equation becomes

$$(n^*)^2 \frac{\partial^2 U}{\partial T^2} = \frac{\partial^2 U}{\partial X^2}. \tag{6}$$

The modulus E^* can also be written as $E(1 + j\delta)$, where δ is a real number and $j = \sqrt{-1}$. Now E and δ may be functions of frequency. Snowdon [1] observed that for many materials, neither E nor δ varies with frequency below some critical frequency (and above some minimum frequency). Snowdon's low-damping constitutive model was used in this project, and was experimentally verified for computer tape. (One might note here that computer tape is not simply a polymer web, but rather a sandwich of 25 percent iron oxide particles in a polymer binder system. The damping may therefore not be the same as that in the polymer substrate.) From the above, $(n^*)^2 = 1/(1 + j\delta)$. Two new variables, P and q, were defined by Snowdon, such that $n^* = P + jq$. From algebra, we can obtain

$$q = \left[\frac{\sqrt{1 + \delta^2} - 1}{2(1 + \delta^2)}\right]^{1/2} \tag{7}$$

and

$$P = \frac{-\delta}{[2(1 + \delta^2)\ (\sqrt{1 + \delta^2} - 1)]^{1/2}}. \tag{8}$$

A small-δ approximation for P and q can help us see the effect of attentuation and phase velocity on wave propagation. Experimentally, the value of δ was always found to be between 0.05 and 0.01. When $\delta \ll 1$, we can use the limiting values of P and q as $\delta \to 0$. These values are given by

$$\lim_{\delta \to 0} P = -1, \tag{9}$$

and

$$\lim_{\delta \to 0} q = \frac{\delta}{2}, \tag{10}$$

and are used throughout this paper.

Reflected and unreflected simple waves

The simplest one-dimensional wave in a viscoelastic material is the propagation of a harmonic train in a semi-infinite member, such as a semi-infinite piece of computer tape. Reflections do not occur in this member. An understanding of reflected waves can be obtained from the study of this case.

The velocity of the tape, determined by solving the wave equation, is written in exponential form as

$$U(X,T) = Be^{j\omega(n^*X+T)}, \tag{11}$$

where the imposed end velocity is simply

$$U(0,T) = Be^{j\omega T}. \tag{12}$$

Substituting $n^* = P + jq$ gives

$$U(X,T) = Be^{-q\omega X}e^{j\omega(PX+T)}. \tag{13}$$

The wave velocity is generally considered the velocity at which an observer moves such that a traveling disturbance appears unchanged in time. In undamped materials (i.e., $q = 0$) the wave solution is

$$U(X,T) = Be^{j\omega(PX+T)}. \tag{14}$$

The dimensionless wave velocity is $-1/P$. However, the harmonic wave train attenuates as it propagates. To avoid ambiguity, the velocity $-1/P$ will be called the phase velocity, for both damped and undamped waves.

If materials are considered in which $\delta \ll 1$, then the values

$$P \approx -1 \text{ and } q \approx \frac{\delta}{2} \tag{15}$$

can be substituted to give

$$U(X,T) \approx Be^{-\delta\omega X/2}e^{j\omega(T-X)}. \tag{16}$$

In this case, the dimensionless phase velocity is simply $+1$, and the wave amplitude envelope is given by

$$\frac{U(X,T)}{Be^{j\omega(PX+T)}} = e^{-q\omega X}. \tag{17}$$

For $\delta \ll 1$, this envelope becomes $\exp(-\frac{1}{2}\delta\omega X)$. The ratio of the amplitudes over two successive spatial periods (i.e., when ωX has increased by 2π) is then $\exp(-\pi\delta)$, which is approximated by $1 - \pi\delta$ for small values of δ. For δ constant in frequency, the spatial attenuation per wavelength is independent of the spatial frequency. Also, if δ does not exceed 0.05, the maximum attenuation per wave length is about 15 percent.

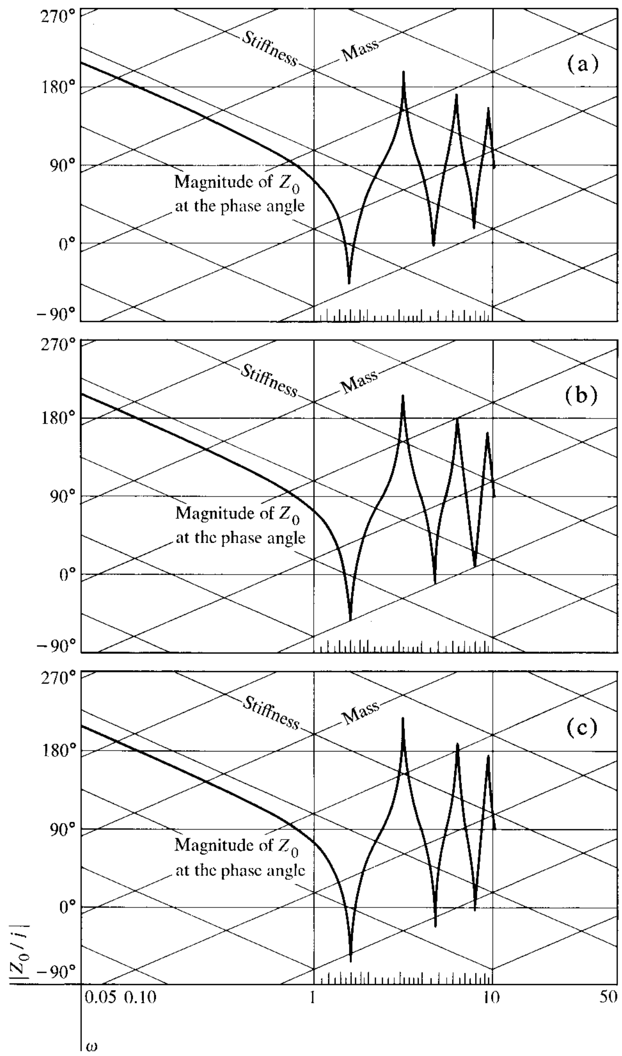
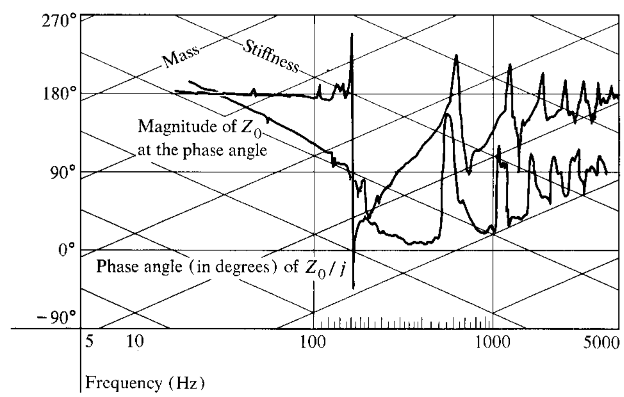


Figure 1 Theoretical plots of mechanical impedance (magnitude) vs frequency for a constant complex elastic modulus. End of tape rigidly clamped. δ = (a) 0.040; (b) 0.032; (c) 0.024.

Figure 2 Experimental plot of mechanical impedance vs frequency for IBM Series/500 tape, 64 in. long. End of tape rigidly clamped.



In general, the segments of tape under study are less than a few wavelengths long. In cases like these, a traveling wave reaches the terminating point and is reflected in some manner. Thus, waves generally propagate in both directions along the tape. Snowdon's solution to the wave equation that describes this propagation is

$$U(X,T) = B'[e^{-jn^*\omega X} + (Re^{j\phi})\, e^{jn^*\omega X}]\, e^{j\omega T}. \tag{18}$$

Re is a reflection coefficient that describes how the wave is reflected. If the driven end of the tape is defined by $X = 1$ and the other end by $X = 0$, a rigidly clamped end (i.e., $U = 0$ at $X = 0$) corresponds to $Re^{j\phi} = -1$. If the velocity of the driven end is given by $U(1,T) = Be^{j\omega T}$, then the velocity of the tape is

$$U(X,T) = Be^{j\omega T}\left(\frac{e^{-jn^*\omega X} - e^{jn^*\omega X}}{e^{-jn^*\omega} - e^{jn^*\omega}}\right). \tag{19}$$

In analyzing the vibration of continuous systems, the concept of mechanical impedance is frequently used to describe the dynamic response of a system, and is applied as a laboratory method of experimentally determining the dynamic properties of materials. This impedance is defined for our purposes as the ratio between the stress and the velocity at the driven end of the tape. If the tape has a constant preload, this impedance can be defined as the ratio between the time-varying stress and the velocity. The time-varying component of stress, σ', can be shown to be

$$-\sqrt{E\rho}\, u_0 B'\, e \quad (e^{-jn^*\omega X} + e^{jn^*\omega X})/n^*. \tag{20}$$

The impedance, Z, is then found to be

$$Z = \frac{\sqrt{E\rho}}{n^*}\left(\frac{-e^{-j\omega n^*} - e^{j\omega n^*}}{e^{-j\omega n^*} - e^{j\omega n^*}}\right). \tag{21}$$

The dimensionless impedance Z_0 is defined as $Z/\sqrt{E\rho}$ and is given by

$$\begin{aligned} Z_0/j = {} & [1/(P^2+q^2)] \\ & \times \{[(q \sinh \omega q \cosh \omega q - P \sin \omega P \cos \omega P) \\ & + j(P \sinh \omega q \cosh \omega q + q \sin \omega P \cos \omega P)] \\ & \div (\sinh^2 \omega q + \sin^2 \omega P)\}. \end{aligned} \tag{22}$$

The following magnitude and phase angle can be obtained by algebraic manipulation:

$$\left|\frac{Z_0}{j}\right| = \frac{1}{\sinh^2 \omega q + \sin^2 \omega P} \times \left[\frac{\sinh^2 \omega q \cosh^2 \omega q + \sin^2 \omega P \cos^2 \omega P}{P^2 + q^2}\right]^{1/2}; \tag{23}$$

$$\theta_{Z_0} = \tan^{-1}\left\{\left[\frac{\sin \omega P \cos \omega P}{\sinh \omega q \cosh \omega q} - \left(\frac{q}{P}\right)\right] \div \left[1 + \left(\frac{\sin \omega P \cos \omega P}{\sinh \omega q \cosh \omega q}\right)\left(\frac{q}{P}\right)\right]\right\}. \tag{24}$$

Figure 1 shows the first function plotted for a few values of δ. (Note: The stiffness lines are impedance plots, one decade apart in magnitude, for pure springs. The mass lines are the same for pure masses, and the horizontal lines are the same for pure, linear dampers. The scales are logarithmic, except for the phase angle, for which the vertical scale is linear. These comments apply to all plots of impedance and transfer functions.) Figure 2 shows an experimental plot of both the magnitude and the phase angle. The theoretical and the experimental plots are similar, but the antiresonance frequencies and the magnitudes of the resonance and the antiresonance impedance are in considerable disagreement. The antiresonance magnitudes diminish along a spring line, whereas the resonance magnitudes increase along a mass line. At antiresonance, the inertial and the spring forces are in equilibrium at the driven end, so that only the dissipative stresses remain. In this test the tape is rigidly clamped at one end, and the other end is fastened to a shaker-driven impedance head. The impedance head is merely a force transducer and an accelerometer. The total mass of the instrument driven through the force transducer is two to three times the total mass of the tape. This mass can be partially canceled out electronically. However, it cannot be exactly canceled out at all frequencies, and small errors in cancellation cause gross errors in resonance magnitudes and frequencies, as well as significant errors in the antiresonance magnitudes. Consequently, mechanical impedance was not used to determine E and δ for computer tape.

The concept of a transfer function is also used to study the vibration of continuous systems, both to describe system response and to experimentally determine the properties of materials. This transfer function, which is similar to the impedance, is defined as the ratio of the time-varying stress at the rigidly clamped end of the tape to the velocity at the driven end. The ratio between the varying stress at the clamped end and that at the driven end, denoted by Γ, is simply

$$\Gamma = 2(e^{-jn*\omega} + e^{jn*\omega})^{-1}. \tag{25}$$

Note that the dimensionless transfer function, denoted by Λ, can thus be written as $\Lambda = \Gamma Z_0$. If we recall the expression calculated for the impedance Z_0, we can perform the necessary manipulations to show that

$$\begin{aligned}\Lambda = -[&(P \sinh \omega q \cos \omega P + q \sin \omega P \cosh \omega q) \\ &+ j(P \sin \omega P \cosh \omega q - q \sinh \omega q \cos \omega P)] \\ &\div [(\sinh^2 \omega q + \sin^2 \omega P)(P^2 + q^2)].\end{aligned} \tag{26}$$

After more manipulation, the magnitude and the phase angle are found to be the following:

$$|\Lambda| = [\sin^2 \omega P + \sinh^2 \omega q)(P^2 + q^2)]^{-1/2} \tag{27}$$

and

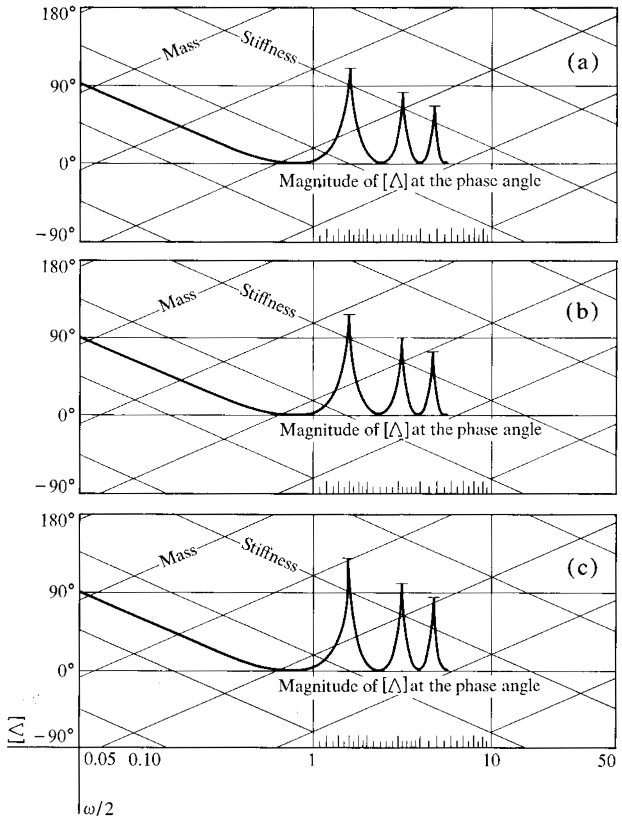


Figure 3 Theoretical plots of transfer function (magnitude) vs frequency for constant complex elastic modulus. End of tape rigidly clamped. δ = (a) 0.040; (b) 0.032; (c) 0.024.

$$\theta_\Lambda = \tan^{-1} \frac{q \sinh \omega q \cos \omega P - P \sin \omega P \cosh \omega q}{P \sinh \omega q \cos \omega P + q \sin \omega P \cosh \omega q}. \tag{28}$$

The first function is plotted for values of δ of 0.024, 0.032, and 0.040 (see Fig. 3). A few actual experimental plots of $|\Lambda|$ and θ are shown in Fig. 4. Although the impedance measurements had errors in canceling the instrument mass, the transfer-function measurements are free of these inherent measurement errors. Over the frequency range of 100 Hz to 5 kHz, the sonic velocity was found to vary less than ± 2.5 percent (corresponding to ± 5 percent variation in E_0), and δ was found to vary less than ± 0.0025. The excellent agreement between the theoretical and the experimental plots shows the validity of Snowdon's constitutive model with constant E and δ. This method was used to determine δ and E for the computer tape and the substrate material only.

A final experiment was designed to verify the constitutive model. The ultimate objective of this project was to determine the response of the computer tape to a general periodic excitation. Snowdon's constitutive equation leads to a linear wave equation. The next step is to construct the periodic excitation by a Fourier series and

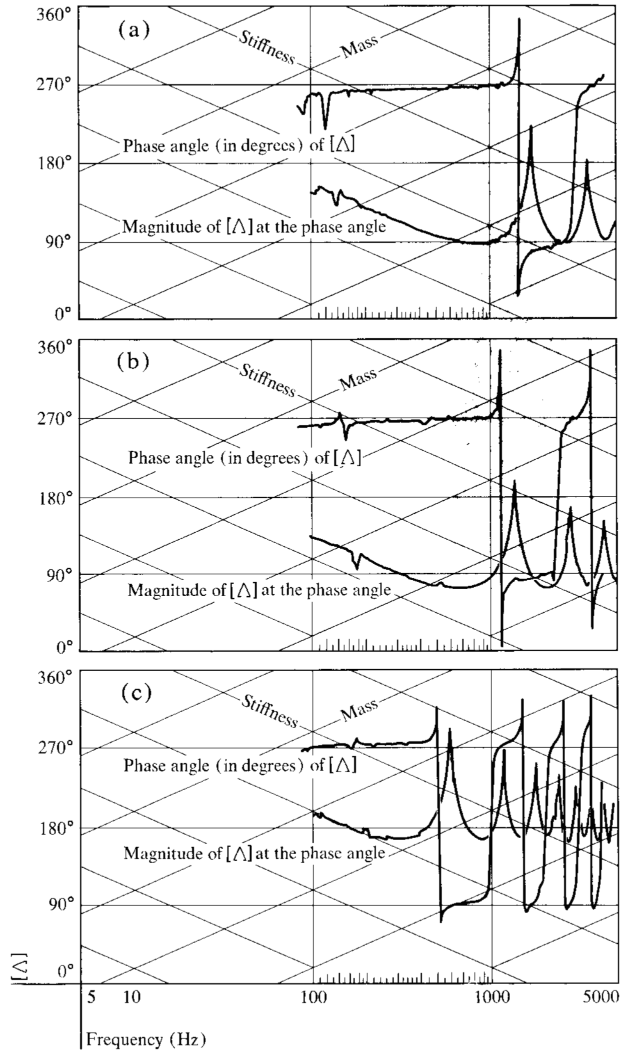


Figure 4 Experimental plots of transfer function vs frequency for tape and substrate; end of tape rigidly clamped. (a) IBM Series/500 tape, 23 in. long; (b) Mylar (® E. I. DuPont de Nemours Co.) substrate; (c) IBM Series/500 tape, 66.5 in. long.

sum the responses of the tape to the several Fourier components.

The constitutive equation, being linear, implies independence between two or more wave trains simultaneously traveling in the same or in opposite directions. To verify this independence, the driven end of the tape was simultaneously excited at two frequencies. One frequency was held constant while the other was swept from 100 Hz to 5 kHz. The force response was viewed through a narrow-band tracking filter that followed the sweep frequency. These plots of transfer functions were made without a constant signal, as well as with contstant signals of 1, 2, 3, 4, and 5 kHz. The lack of a detectable difference between any two plots verified the linear constitutive model, and this supports the use of the linear, damped wave equation.

Series of reflected waves

The natural route to a solution of this problem is through Fourier analysis. If the periodic excitation velocity is restricted to satisfy the Dirichlet conditions, it can be expanded in a Fourier series. The velocity at the driven end of the tape can be represented by

$$U(1,T) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 T + b_n \sin n\omega_0 T), \qquad (29)$$

where

$$a_n = \frac{\omega_0}{\pi} \int_{-\pi/\omega_0}^{\pi/\omega_0} f(T') \cos n\omega_0\, T'\, dT', \qquad (30)$$

and

$$b_n = \frac{\omega_0}{\pi} \int_{-\pi/\omega_0}^{\pi/\omega_0} f(T') \sin n\omega_0\, T'\, dT'. \qquad (31)$$

Here, ω_0 is defined to be 2π/period.

The wave equation in complex notation is

$(1 + \delta_j)\ \partial^2 U/\partial X^2 = \partial^2 U/\partial T^2$.

If U is understood to be of the form $e^{jn\omega_0 t}$, this complex form is equivalent to

$$\frac{\partial^2 U}{\partial X^2} + \frac{\delta}{n\omega_0} \frac{\partial^3 U}{\partial X^2 \partial T} = \frac{\partial^2 U}{\partial T^2}. \qquad (32)$$

Recalling the relationship $\partial u/\partial x = \partial\epsilon/\partial t$, we can solve the wave equation only if two combinations of U and ϵ are known at $X = 0$ or 1. Since we want to study the tape response to an imposed velocity, $U(1,T)$ will be one boundary condition. In a modern tape drive, one end of the tape terminates in a stubby vacuum column, and the read/write head is at some intermediate point. The tape velocity at the head is the primary concern. If longitudinal forces at the head are ignored, and if the stiffness, the damping, and the inertial effects of the stubby column are negligible, the second boundary condition will be a time-invariant stress at the end of the vacuum column. Most generally, of course, the stress at the end would be described in terms of the end velocity by some function of frequency. This will be discussed later in this paper.

The total solution to the wave equation can be written in trigonometric and exponential functions as

$$U(X,T) = \sum_{n=1}^{\infty} U_n(X,T) + A_5, \qquad (33)$$

where

$$\begin{aligned} U_n(X,T) = e^{-n\omega_0 qX}[{}_nA_1 \sin n\omega_0(T - PX) \\ + {}_nA_2 \cos n\omega_0(T - PX)] \\ + e^{n\omega_0 qX}[{}_nA_3 \sin n\omega_0(T - PX) \\ + {}_nA_4 \cos n\omega_0(T + PX)]. \end{aligned} \qquad (34)$$

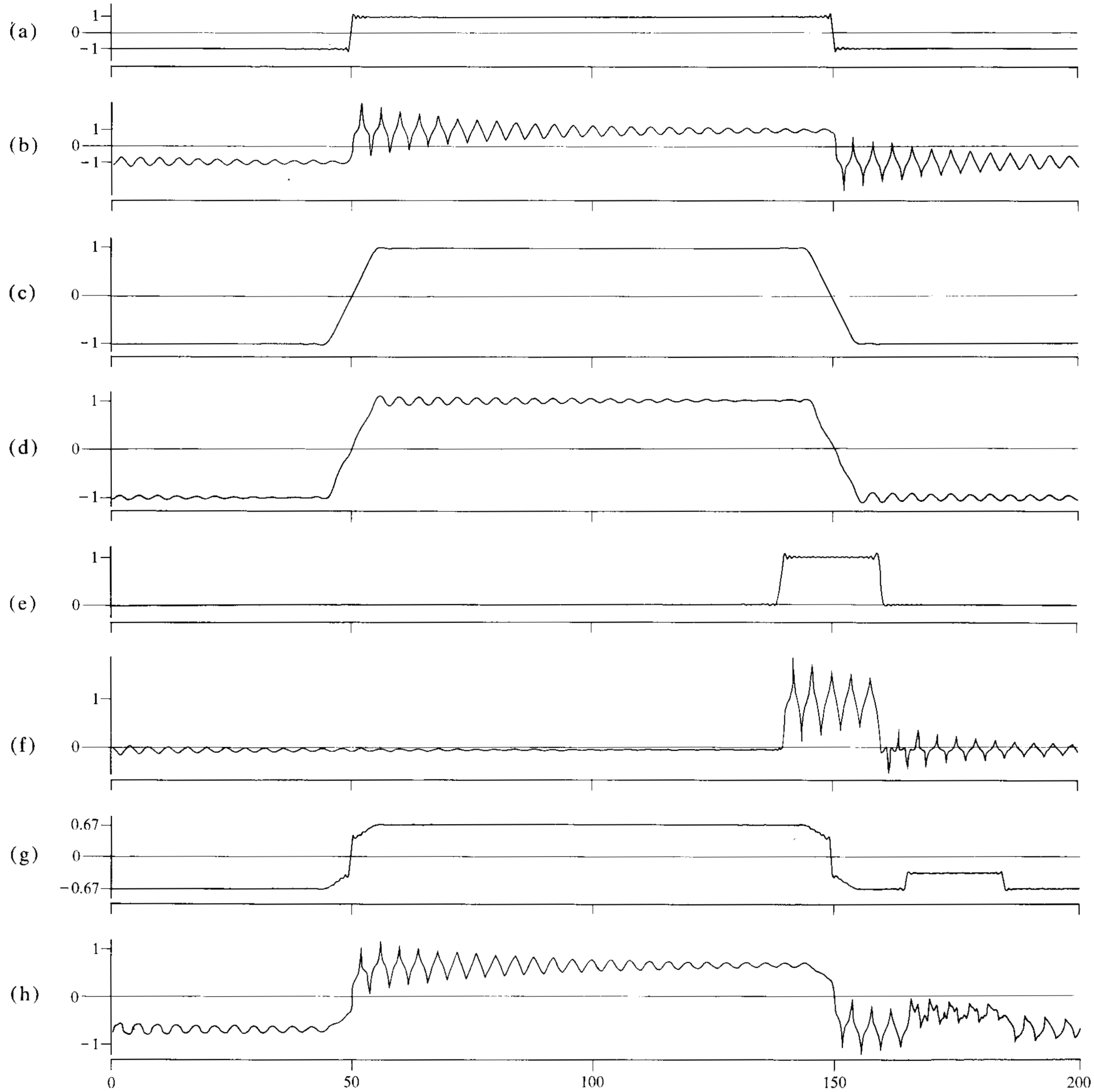


Figure 5 Motion plots with truncated Fourier representations of $U(X,T)$. (a) $U(1,T)$ vs T for a rectangular wave; (b) $U(0.7,T)$ vs T for a rectangular wave; (c) $U(1,T)$ vs T for a trapezoidal wave; (d) $U(0.7,T)$ vs T for a trapezoidal wave; (e) $U(1,T)$ vs T for an impulsive wave; (f) $U(0.7,T)$ vs T for an impulsive wave; (g) $U(1,T)$ vs T for the sum of the three waves; (h) $U(0.7,T)$ vs T for the sum of the three waves.

The boundary condition $\sigma' = 0$ at $X = 0$ determines that

$$ {}_nA_3 = {}_nA_1, \qquad {}_nA_4 = {}_nA_2. \tag{35} $$

Then from the condition

$$ U(1,T) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 T + b_n \sin n\omega_0 T), \tag{36} $$

we get $A_5 = a_0/2$, (37)

$$ {}_nA_1 = \frac{b_n \cos n\omega_0 P \cosh n\omega_0 q + a_n \sin n\omega_0 P \sinh n\omega_0 q}{2(1 + \sinh_2 n\omega_0 q - \sin^2 n\omega_0 P)}, \tag{38} $$

and

$$ {}_nA_2 = \frac{a_n \cos n\omega_0 P \cosh n\omega_0 q - b_n \sin n\omega_0 P \sinh n\omega_0 q}{2(1 + \sinh^2 n\omega_0 q - \sin^2 n\omega_0 P)}. \tag{39} $$

Therefore, we need only to sum the solutions on n to obtain the total response of tape at any point X in time T. Figure 5 shows plots of these responses for even rectangular, trapezoidal and uneven rectangular waves, and for a combination of these.

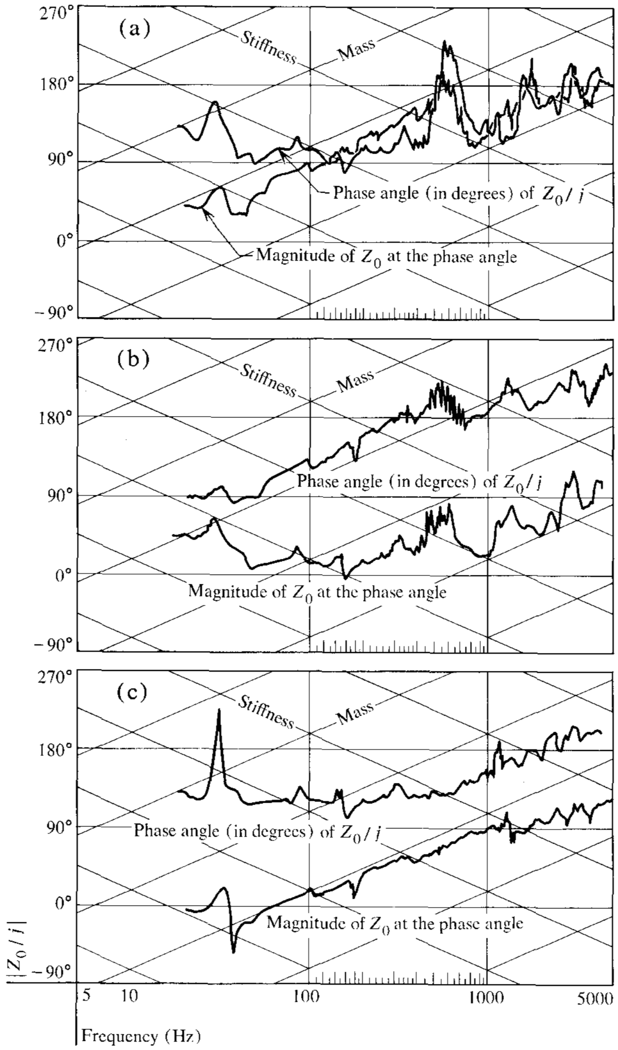


Figure 6 Experimental plots of mechanical impedance vs frequency for tape in a vacuum column. (a) Glass-beaded column, 20.5 in. of water vacuum, 34 in. of tape; (b) machined column, 10.8 in. of water vacuum, 34 in. of tape; (c) machined column, 11.2 in. of water vacuum, 9 in. of tape.

At this point, the solution can be rewritten as follows:

$$\begin{aligned} U_n(X,T) = {}_nA_1 [&e^{-n\omega_0 qX} \sin n\omega_0 (T - PX) \\ &+ e^{n\omega_0 qX} \sin n\omega_0 (T + PX)] \\ + {}_nA_2 [&e^{-n\omega_0 qX} \cos n\omega_0 (T - PX) \\ &+ e^{n\omega_0 qX} \cos n\omega_0 (T + PX)]. \end{aligned} \quad (40)$$

The solution contains waves traveling in the $+X$ and $-X$ directions, attenuating as they travel. Note that the attenuation coefficients are exponential and the attenuation of a wave over the length of tape is $\exp(-n\omega_0 q)$. If the ratio of wavelength to tape length for the lowest harmonic is 10 or 100 (i.e., ω_0 is very small), the lowest harmonic then travels and is reflected in an essentially undamped manner. If the lowest harmonic has a wavelength equal to the tape length, all harmonics of order $n>10$ are diminished to less than eight percent of their original amplitude after traveling the length of tape and back twice (for $\delta = 0.02$). In the tape drives described in the introduction, motion periods of 30 ms occur for corresponding propagation times of 0.15 ms. In this case, the 100th harmonic is attenuated only about two percent after traveling the length of tape and back five times, or in other words, after traveling for 1.5 ms (again for $\delta = 0.02$).

Thus, in the cases for which the lowest harmonic frequency is of order one, the wave attenuation and phase lag (PX) dominate the motion of the tape. On the other hand, for cases in which the lowest harmonic is of order 0.01, the wave interactions greatly dominate the tape motion in the absence of external damping forces. In this problem, we should also account for the slight increase with frequency in the phase velocities of waves shown by the measurements of the transfer functions. This effect can cause substantial changes in the waveform during propagation.

Experimental measurements of vacuum-column forces on tape

In the analytical prediction of tape response in a tape drive, the boundary condition at the undriven end was assumed to be zero stress. If a totally unstiff vacuum column with an infinite frequency response were used, no dynamic spring forces, damping forces, or inertial forces could exist at that end of the tape. The possibility of acoustical resonance in the column leads one to consider spring and inertial forces. The concept of volumetric displacement and corresponding flow into and out of the column leads one to consider viscous pressure forces. Actual measurements of these effects are necessary to verify the foregoing analysis.

A test setup was constructed in which one end of a piece of tape was fastened rigidly to a vacuum-column wall; the other end was fastened to a shaker-driven impedance head. A loop of tape was then formed in the column, just as in a tape drive, and the impedance was measured at the driven end of the tape for two different vacuum levels, and for both a smooth, machined-column wall and a glass-beaded wall. Plots of impedance vs frequency are shown in Fig. 6.

Unfortunately, the length of the tape and the length of the vacuum column corresponded to approximately the same first antiresonance frequency in each. However, the antiresonant frequency ratios for the free-ended tape (the assumed model) are given by the odd integers 1, 3, 5, 7, · · ·, whereas the antiresonant frequency ratios for a closed-end column are simply all integers 1, 2, 3, 4, · · ·. In the plot for tape with high vacuum and beaded-column walls, the first antiresonance is rather broad and occurs at nearly 600 Hz (see Fig. 6). The second and third are extremely sharp and occur at about 1800 and 3000 Hz, showing tape antiresonance and tape damping. In the plots with high or low vacuum and machined-aluminum column walls, the first antiresonance frequency is about 600 Hz. However, the next two antiresonant frequencies are about 1300 and 3000 Hz. The 1300-Hz disturbance therefore seems to be acoustical, whereas the 3000-Hz antiresonance is probably caused by combined acoustical and mechanical effects.

One other effect is shown in the plots. Adding the glass beads to the wall brought about a frequency-independent drop in impedance. If we consider a viscous laminar air film between the tape and the column wall, the impedance of such a film will be constant with frequency if the tape geometry remains constant. However, large protrusions, such as glass beads, could force the film to be thick enough that the viscous shear would be reduced or eliminated. Such a film could therefore have caused this difference in the plots.

The results of these experiments clearly point out the need for experimentally obtained reflection coefficients (as a function of frequency) to be used in any theoretical predictions of motion when end conditions are unknown. In addition, these impedance plots can help the designer choose modifications of the tape path to change the damping of the tape at critical locations in the drive. Ultimately, these experiments should enable the mechanical designer to understand the nature of waves in tape and the propagation of tape motion in a drive, and to understand how the parts of the drive interact with these waves.

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