# Estimate of Subsidence in Venice Using a One-dimensional Model of the Subsoil

Abstract: This work applies a one-dimensional model, based on K. Terzaghi's theory of vertical consolidation, to the calculation of land subsidence in the Venetian Lagoon, caused by extraction of water from artificial wells. Certain assumptions are made in order to compensate for the scarcity of data currently available, and these are discussed with regard to both the geophysical characteristics of the subsoil and the decline in piezometric level in the various strata. The interpretation of the results shows a direct correlation between water extraction and surface subsidence.

#### Introduction

The compaction of soil following extraction of fluid from the subsoil is a phenomenon that has long been the object of scientific analysis. Fuller[1] was first to scientifically analyze its causes. Minor[2] and Pratt and Johnson[3] gave the first descriptions of oil-field areas and their progressive sinking. Many examples testify to the close connection between fluid withdrawal and subsidence of the surface. Considerable subsidence has been noticed at Goose Creek in Texas and on the shores of Lake Maracaibo in Venezuela (mineral oils), in the Po River delta in Italy (methane gas), and in the Santa Clara valley in California (water).

Venice (Fig. 1) is a recent case of particular interest. There the quantities of water extracted have been less than those that caused considerable sinking in other parts of the world. The lowering, though modest, is nevertheless a cause of concern because it compounds the effect of the exceptionally high tides in the city during winter. Geodetic surveys, though not made on a regular basis, indicate that a progressive sinking of the soil of the lagoon began in the period 1925 - 30. The total subsidence is not known, but from 1952 to 1969 (Fig. 2) it was about 14 cm at Marghera and 11 cm in Venice. In the lagoon the water is extracted from the subsoil for domestic and industrial uses. The major withdrawals are for industrial purposes and are carried out in Marghera, an area in which numerous factories are located. Here the wells have a maximum depth of about 300 m.

We have considered the problem of estimating the subsidence in Venice by relating it to the registered decline

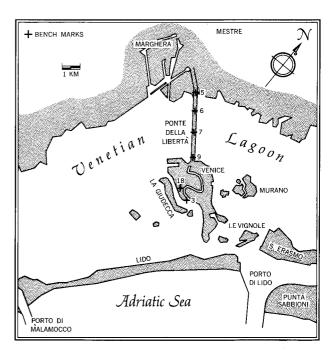


Figure 1 Map of the Venetian Lagoon.

of the piezometric levels in the aquifers [4]. Our purpose is to show that one can produce a reasonable estimate of the extent of the sinking of the land surface caused by extraction of liquids from the subterranean strata. Application of our one-dimensional model of the subsoil actually requires data only part of which is currently available for the Venetian Lagoon. Certain assumptions

have therefore been made with regard to physical quantities; these assumptions are discussed in a later section. Meanwhile, to provide the additional data, drilling operations are currently under way in the Tronchetto area.

### Subsoil model

Many authors have studied the phenomenon of compaction due to the removal of fluids from the subsoil. In 1940 Jacob[5] postulated that the irreversible compaction of the clay beds in or adjacent to exploited aquifers is a major cause of subsidence. He defined a coefficient of storage which takes this into account and which Lohman[6] used successively in a formula that he proposed for estimating the compression of an artesian aquifer, knowing the decline of its piezometric level and its hydrologic properties.

As early as 1923, however, Terzaghi[7] had developed his theory of vertical consolidation, introducing the principle of "effective stress" that Meinzer and Hard[8] shortly thereafter recognized to be acting in the compaction of water-bearing strata. Consolidation theory is now widely accepted and the details are described in the texts by Terzaghi and Peck[9], Taylor[10], and Leonards[11]. The latter text also discusses the discrepancies between theoretical results and those observed experimentally, both in the field and during laboratory tests.

A one-dimensional model is especially well suited to soil of a nonconsolidated nature, such as that of the Venetian Lagoon area, in which vertical displacements are much greater than horizontal ones. Furthermore, it makes practical a more rigorous treatment of heterogeneous ground in the one dimension than would be possible under the simplifying and constraining assumptions necessary to make a three-dimensional calculation at the present time. The limitations of the model are that it applies to a noncemented soil, composed of horizontal strata of large extent, and that it does not take into consideration the presence of gas in unsaturated soil. Although a hydrologic model would be desirable to describe the variation in time of the piezometric level of the Venetian aquifers as a function of the quantity of water extracted, none currently exists, so we substitute data taken locally in controlled wells over a period of several years of the decline of the hydraulic head in the surrounding sandy strata.

How Terzaghi's theory can be adapted to this purpose has been indicated recently in studies by American authors [12,13] and has been described fully by the author in a recent technical report [4(b)]. Here we review only the general equations and principles on which the model is based.

The fundamental relation is

$$\sigma_{c} = \sigma_{z} + p, \tag{1}$$

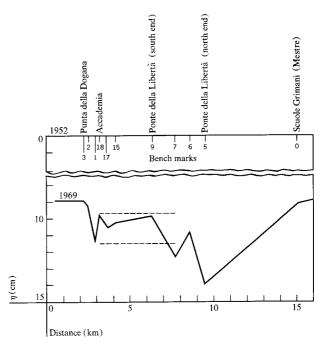


Figure 2 Subsidence in the vicinity of the Venetian Lagoon (Ref. 19).

the formulation of which dates back to Terzaghi[7]. Formula (1) is valid for any depth and expresses quantitatively the condition that the total overburden load or geostatic pressure  $\sigma_c$  of a vertical column of unit cross section is sustained partly by the intergranular stress  $\sigma_z$  (compaction is attributed to variation in  $\sigma_z$ ) and partly by the hydrostatic pressure p of the water in the pores of the subsoil. The latter can be shown to act on the entire cross section because the actual contact area among the grains is very small [4(b)].

Denoting  $c_b$  as the compressibility of a permeable stratum of thickness  $s_0$  and  $\Delta z$  as the reduction in piezometric level due to water extraction, we obtain the following expressions for the soil compaction  $\eta$ :

$$\eta_1 = c_{\text{b1}} \gamma \Delta z_1 \left( 1 - \phi_f - \phi_w \right) \left( s_{\text{01}} + \frac{1}{2} \Delta z_1 \right)$$
(2)

for a phreatic stratum (unconfined aquifer), and

$$\eta_2 = c_{\rm b2} \, \gamma \, \left[ \Delta z_2 - (\phi_{\rm f} - \phi_{\rm w}) \Delta z_1 \right] \, s_{02}$$
(3)

for an artesian stratum (confined aquifer). Here  $\gamma$  is the specific weight of water;  $\phi_{\rm f}$  is the porosity of the phreatic stratum;  $\phi_{\rm w}$  is the moisture content, as a percentage of the total volume, of the sediment lying above the phreatic level; and  $s_{\rm ol}$  is the thickness of phreatic stratum below the lowered water-table level. The vertical compressibility values may be derived from experimental data, which show the void ratio e as a function of the effective intergranular stress  $\sigma_{e}$ , using the relation

$$c_{\rm b} = - (de/d\sigma_{\star})/(1+e)$$
. (4)

131

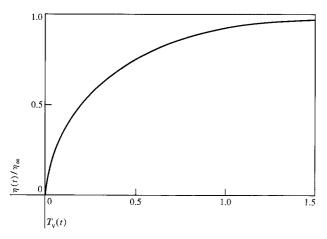
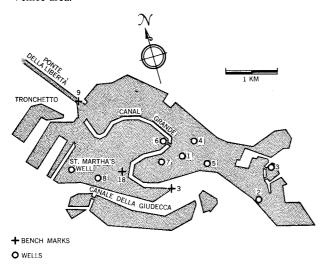


Figure 3 Terzaghi's consolidation-theory curve of compaction as a function of a dimensionless time factor.

Figure 4 Location of wells  $(\bigcirc)$  and bench marks (+) in the Venice area.



In the cases of layers and lenses of clay which separate one water-bearing stratum from another or which lie within a single aquifer, formula (3)—where  $\Delta z_2$  now means the average of the decreases in piezometric level recorded in the sandy strata in contact with the clay layer—yields only the total compaction occurring in a theoretically infinite time. The time-dependent development of the compaction as a function of the actual occurrence of the piezometric-level changes depends directly on the compressibility of the clay and the square of the layer thickness and inversely on the vertical permeability.

The influence of these three parameters is expressed by the dimensionless term

$$T_{v} \equiv 4c_{v} t/s_{0}^{2}, \tag{5}$$

called the time factor, in which in addition to the time t and the thickness  $s_0$  of the stratum, Terzaghi's coefficient of consolidation,  $c_v$ , also appears:

$$c_{\rm v} = k/\gamma c_{\rm b},\tag{6}$$

where k is the coefficient of vertical permeability. The consolidation coefficient  $c_{\rm v}$  can be determined in the laboratory using a sample of material from the soil stratum.

Definition (5) is of particular importance in that the actual compaction of a stratum (at the boundaries of which instantaneous reductions of the piezometric levels in the adjacent sands take place) relative to the final compaction (3) is a function solely of the time factor  $T_v$ ; this dependence [4(b)] is represented in Fig. 3, which shows the ratio between the compaction that took place at time t and the final compaction expressed by formula (3). Figure 3 can be used to predict how a clay stratum will compact in the course of time, including cases in which changes in hydrostatic pressure at its boundaries may also vary in time. An example of this use is given in Ref. 4(b).

#### **Data for Venice**

The data needed for the model are of two kinds: The first type comprises the stratigraphy of the subsoil, to the depth to which the effects of the extraction penetrate, and the mechanical and hydraulic characteristics of each stratum (which may be considered homogeneous); the second type describes the history of the effect of the extractions, i.e., the relation according to which, in the given time period, a decline in piezometric level is revealed in the sandy strata from which the wells draw. Currently these data, which are being supplied by the geodynamics group of the National Research Council (C.N.R.), Laboratory for the Study of the Dynamics of Large Masses[14,], are not yet complete. In the first case, although it has been possible to reconstruct the stratigraphy in its essential outlines to a depth of about 300 m, there are few measurements determining the compressibility and permeability of the strata, or their porosity and specific weight; in the second case, the data on piezometric levels, obtained from the locally registered wells are far from complete and often inconsistent and not in accord with the apparent stratigraphy. The discrepancies sometimes result from uncertainty on the part of the well driller as to the depth of the well intake, and sometimes from the nature of the survey, which includes wells that are distant from each other, and which reflect the heterogeneity of horizontal configurations in the Venetian subsoil.

Table 1, for the Tronchetto area (Fig. 4), has been prepared from the stratigraphic model developed for that area [14(a), (b)] by placing within the same stratum soils which are, as far as possible, lithologically and physically similar. The compactable thickness, which differs in some

Table 1 Simplified stratigraphy and lithology in the Tronchetto area of Venice.

Zone	Mid-zone depth (m)	Stratum thickness (m)	Compactable thickness (m)	Summary litho- logical description
1	28	56	45	Clay, mud, sand with clayey intercalations, sandy clay, peaty clay
2	71.5	31	31	Sand, clayey sand, alter- nating sand and clay
3	92	10	10	Prevalently clay
4	102	10	10	Sand with thin layers of clay
5	113	12	6	Alternate layers of clayey sand and clay
6	137	36	36	Sand, sand with clayey intercalations
7	159	8	8	Prevalently clay
8	168	10	10	Sand, clayey sand
9	194	22	22	Clayey sand, sand intercalated by clay
10	210	10	10	Prevalently clay
11	222.5	15	15	Prevalently sand
12	242.5	25	25	Prevalently clay
13	267.5	25	25	Prevalently sand, fossil- bearing strata, light sandstone, thin layers of light gravel
14	285	10	5	Alternate layers of sand and clay

cases from the total thickness, takes account of layers of sand, which are subject to very little deformation, between layers of clay. Table 1 includes a number of aquifer levels that appear frequently in the stratigraphic analyses of various locations in Venice. We refer particularly to zones 2, 4, 6, 8, 11, and 13, which are composed prevalently of sands and in which are located the vast majority of the intakes to the extraction wells. Naturally, a more accurate subdivision of the stratigraphy will be possible when the subsoil has been mapped exactly, permitting a better classification of the strata of mixed nature.

Table 2[14(c)] shows the piezometric levels measured at the beginning of the century, at various depths, in St. Martha's Well, which is near Tronchetto. Other historic data are virtually nonexistent. The correlation between the depths recorded in Table 2 and the aquifer levels of Table 1 is far from perfect. This could be due to errors made in making the measurements that figure in Table 2, but is also likely due to different geologic characteristics and the distance which separates Tronchetto from St. Martha's Well.

In more recent times, the available data[14(c)] relating to several wells in Venice, permit the compilation of Table 3, in which the piezometric levels of the various aquifers are, where possible, average values based on a

Table 2 Piezometric levels observed at St. Martha's Well in 1905.

Measurement depth (m)	Piezometric level (m above sea level)
56	2.95
91	3.80
112	4.55
135	4.90
155	5.60
170	6.25
198	6.70

Table 3 Combined data from several wells in the Venice area.

Zone	Piezometr (m above se	
	ca. 1940	1966
2	1.5ª	
4	1 a	-3
6	$2.5^{\rm b}$	
11	2	-6
13	4	-4.5

<sup>в</sup>1944 <sup>в</sup>1939

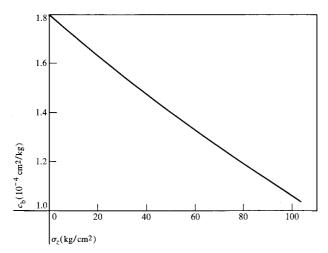
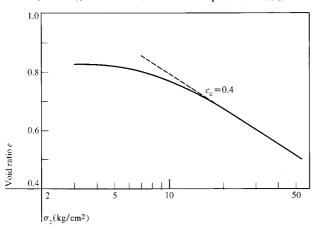


Figure 5 Extrapolated curve of the compressibility of sand from the Po River valley.

**Figure 6** Dependence of void ratio for clay on intergranular stress (Ref. 12), which determines the compression index.



number of wells of the same depth. The scattered distribution of the wells in which the measurements were made may be seen in Fig. 4 (numbered circles). Tables 2 and 3, although they enable us to define the order of magnitude of the decline in piezometric levels, do not contain sufficient information for an accurate evaluation of the decline in levels in relation to either time or depth. This deficiency had to be compensated for by the assumptions adopted in the following section.

## Assumptions and calculations

An analysis by Lofgren and Klausing[12] on nonconsolidated soil in California (like that of the Venetian Lagoon) to a depth of 230 m yielded porosity values from 0.35 to 0.55, with values near 0.4 occurring most frequently. The average specific weights in the same study varied from 2.68 to 2.73 g/cm<sup>3</sup>. We assume an average

value for the porosity  $\phi$  of 0.4 and for the specific weight  $\gamma'$  of dry soil, 2.7 g/cm<sup>3</sup>.

More difficult is the choice of average values for the compressibilities of the sands and clays, due to the wide range of values of these quantities. For the sands we refer to the research of Brighenti[15], who has experimented on sands of methane gas deposits in the plain of the Po River. The compressibility values obtained as a function of the effective intergranular stress have been interpolated with a parabola by Evangelisti and Poggi[16]. Figure 5 shows a nearly linear segment of this parabola, extrapolated to the low values of  $\sigma_z$  that are obtained in the first 300 m of depth.

For the clays the shortage of data is again limiting. Colombo and others [17], for the clays in the more superficial strata of the lagoon area, give values of the compression index  $c_{\rm c}$  that vary between 0.1 and 0.3, with major frequency around 0.2. The American authors [12] report, for the clays on which they experimented, a  $c_{\rm c}$  value of about 0.4.

The compression index is the slope of the straight-line segment in a semilogarithmic plot (Fig. 6) of the void ratio e vs the effective intergranular stress  $\sigma_z$ . If  $c_c$  is constant (straight-line segment) and we impose the condition that  $e = e_0$  for  $\sigma_z = \sigma_{z0}$ , we obtain

$$e = e_0 + c_c \log \left( \sigma_{z0} / \sigma_z \right). \tag{7}$$

Finally, recalling formula (4), we derive the relation between  $c_b$  and  $c_c$ :

$$c_b = 0.434c_c/\sigma_z \left[ 1 + e_0 + c_c \log \left( \sigma_{z0}/\sigma_z \right) \right].$$
 (8)

Formula (8) can provide values of  $c_{\rm b}$ , starting from a certain  $\sigma_z$ , on condition that we know a few pairs of values  $e_0,\sigma_{z0}$  through which a formula (7) curve passes. Lofgren and Klausing[12] have found, for the first 250 m of depth, values of e that oscillate between 0.55 and 1.2 (corresponding to the degrees of porosity 0.35 and 0.55). The maximum value that  $\sigma_{z0}$  takes on, at 300 m with the assumed values of  $\phi$  and  $\gamma'$ , is  $\sigma_{z0}^{\rm max} \approx 30.6 \, {\rm kg/cm^2}$ .

The minimum value of  $\sigma_{z0}$ , starting from which  $c_c$  may be considered constant, is not known. Apart from the uncertainty with which it can be obtained from the semilogarithmic graph, one finds that it also depends on the stress in the clay at the particular location and depth of interest. In Fig. 6, which refers to clays at a depth of about 100 m, the corresponding minimum value of  $\sigma_{z0}$  is approximately  $10 \text{ kg/cm}^2$ . Formula (8) is not valid for small values of  $\sigma_z$  because in that case  $c_b$  would take on extremely high, inadmissible values. If we accept the minimum limit provided by the American authors[12] and use  $c_c = 0.2$ , the constant part of the denominator of formula (8), i.e.,

$$k_0 = e_0 + c_0 \log \sigma_{z_0}, (9)$$

varies between 0.75 and 1.5. Correspondingly, the band of  $c_{\rm b}$  curves (Fig. 7) proves to be quite narrow, thus justifying the adoption of an "average" curve. This rationale for the choice of a stress-dependent  $c_{\rm b}$  of the clays is believed to be more realistic than a criterion of constancy of  $c_{\rm b}$  throughout the depth examined, which would be an oversimplification because the subsoil compressibility certainly decreases with depth.

Table 4 records, for each depth zone, the initial stress  $\sigma_{z0}$ , assuming unit saturation throughout, and the compressibility deduced from either Fig. 5 for sands or Fig. 7 for clays. Numerical values for the midplane of each stratum were used in the calculations [18].

The data in Tables 2 and 3 are not sufficient to support an unequivocal deduction of the nature of the decline in piezometric levels in the aguifers. However, one may note that the two deepest aquifers have had approximately equal decreases in piezometric level between 1940 and 1966, approximately twice the drop recorded in the stratum at 100 m. (No information is available before 1940 below a depth of 200 m and none after 1944 for strata 2, 6, and 8.) One is led to postulate the existence of two distinct families of "noninteracting" aquifers, separated by the 210-m clay layer. This hypothesis is reinforced by the fact that this layer is rather consistent in nature and is found extensively in the subsoil of the lagoon area, as indicated by the drilling of numerous deep wells in both Venice and Marghera[14(a)]. The other clay strata indicated in Table 1 do not have the same extensive distribution, at least not with the same well-founded certainty.

We have summarized our analysis and evaluation of the piezometric-level data currently available in the pattern represented by the "effective" data in Table 5. These particular values, in some cases different from the observations recorded in Tables 2 and 3, retain the order of magnitude involved and individually have only a slight influence on the final calculation of subsidence. The subsidence is, however, substantially affected by the differences between the piezometric levels of successive periods. The difference values have remained practically unchanged in Table 5. Between 1900 and 1944 and between 1944 and 1966 and beyond, we consider the time dependence of the piezometric level to be linear.

Finally, to make use of these data, we must have a value for the coefficient of consolidation  $c_{\rm v}$  of the clays. Referring again to the experience of the American authors, we adopt the value  $c_{\rm v}=0.45~{\rm m}^2/{\rm yr}$ , found for clay strata at different depths. For the first stratum we ignore the existence of delay in the compaction (for the first 80 m of depth,  $c_{\rm v}=29~{\rm m}^2/{\rm yr}[12]$ , which is very high), and we extend this condition to the strata with thin alternations of sand and clay, because of the thickness of the clay layers.

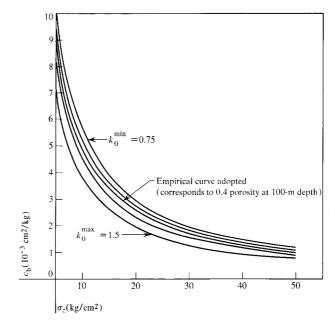


Figure 7 Stress-dependent compressibility of clay on the basis of Colombo's compression index ( $c_c = 0.2$ , Ref. 17); choice of an "average" curve.

**Table 4** Initial stress and subsoil compressibilities for various stratigraphic layers.

Zone	$\sigma_{z0} (kg/cm^2)$	$c_{\rm b}~(10^{-4}~{\rm cm}^2/{\rm kg})$
1	3	50.0
2	7	0.96
3	9	50.0
4	10	0.94
5	11	45.0
6	13.5	0.93
7	15.5	33.0
8	16.5	0.91
9	19.5	0.90
10	21.0	25.0
11	22.5	0.89
12	24.0	22.0
13	26.5	0.87
14	29.0	18.0

## Result and conclusion

Only for the period 1952 through 1969 are there recent geodetic surveys with which to compare values determined by our model. Figure 2 is a record of the observations made by the Institute of Geodesy of the University of Bologna[19]. Their measurements, relative to the period 1952-69, have as the point of reference the town of Treviso, which is generally regarded as stable. Unfortunately, the survey does not pass through the Tronchetto area, which has supplied most of the available piezometric-level and stratigraphic data. Bench mark

Table 5 Derived data for the piezometric-level decline.

Zone	Piezometric level (m above sea level		
(aquifers only)	1900	1944	1966
2	3	1.5	-3
4,6,8	5	1.5	-3
11,13	5	3.0	-5

Table 6 Theoretical estimate of subsoil compaction and total subsidence in the Tronchetto area of Venice.

Zone	$c_{\rm v} (m^2/yr)$	Piezometric-level change 1952–1969 (m) 19	Compaction 952 – 1969 (cm)
1			3.95
2		-3.5	0.1
2 3	0.45		1.2
4		-3.5	0.03
4 5		•	0.9
6		-3.5	0.1
7	0.45		0.75
8		-3.5	0.03
9			0.07
10	0.45		0.87
11		6.4	0.08
12	0.45		1.1
13		-6.4	0.13
14			0.28
		Total subsidence 1952 – 19	69 9.6 cm

9, however, which is located at the terminus of the Ponte della Libertà, is near enough to Tronchetto (Fig. 4) to permit comparison between the measured and the calculated datum. The values of Fig. 2, being actual observations, include the possible effects of regional sinking of the land surface (bradyseism) to which the eastern Po River valley is subject. The contribution of this type of sinking is not known, but it is given an average value by various authors of about 1 mm/yr.

Using our model we have calculated, for each stratum of sand and clay, the compaction from 1952 through 1969, with data from Tables 1, 4, and 5. For strata 3, 7, 10, and 12 it was necessary to calculate, with the data of Table 5, the compaction that occurred between 1900 and 1952 and between 1900 and 1969. This was done using Fig. 3 and the procedure described in Ref. 4(b), distributing the reduction in piezometric level proportionally over the periods 1900–10, 1910–20, 1920–30, 1930–40, 1040–44, 1944–52, 1952–60, and 1960–69. It was assumed that the effect of the extraction had not penetrated below 290 m, the depth of the deepest well in the lagoon area, below which we do not yet know the stratigraphic composition. The results of our calculations are given in Table 6.

We have calculated the subsidence of the land surface in the Tronchetto area of Venice, due to relatively local extraction of water from the subsoil, to be about 9.6 cm. If the geological subsidence in the Po River valley has a long-range effect extending to Venice, our theoretical estimate might be increased to about 11.4 cm. These two values are indicated by the parallel dashed lines on Fig. 2.

The comparison between theory and experiment is quite acceptable. Although the numerous assumptions which were necessary permit us to put only limited trust in the accuracy of the subsidence value yielded by the calculation, the order of magnitude of the result is such that it fully justifies the concern of those who hold that the uncontrolled extraction of water from the subsoil is the principal cause of the sinking of Venice. In fact, it will be difficult in the future to improve our knowledge of some of the quantities that are of determining importance in the model (e.g., previous decreases in piezometric levels in the strata), and for others (e.g., the stratigraphy), radical changes are not to be expected. Thus we believe the model will reliably serve to estimate the subsidence to be attributed to extraction of water from the aquifers, until such time as the drilling operations currently under way at Tronchetto reveal, with greater accuracy, the stratigraphic characteristics of the area and the subsequent laboratory tests furnish adequate values for the parameters that define the hydrologic and mechanical properties of the subsoil.

Eventually, our model could also be used as a forecasting tool, which would require preliminary study of a hydrologic model in order to suggest the trend of the piezometric levels as a consequence of possible future intervention by man, for example, the closing of the more active wells. That project is ambitious and to realize it a further collection of data would be necessary (beyond those which come from the well at Tronchetto), data that would not be easy to obtain. In any event, we believe that a hydrologic model should be developed to direct in a qualitative manner what ecological choices should be made.

## Acknowledgment

We are grateful to the National Research Council, Laboratory for the Study of the Dynamics of Large Masses, Venice, Italy, for supplying stratigraphic and piezometriclevel data in advance of publication.

## References and note

- 1. M. Fuller, "Summary of the Controlling Factors of Artesian Flows," U.S. Geol. Survey Bull., 319 (1908).
- H. E. Minor, "Goose Creek Oil Field, Harris County, Texas," Am. Assoc. Petrol. Geol. Bull. 9(2), 286 (1925).
- 3. W. E. Pratt and D. W. Johnson, "Local Subsidence of the Goose Creek Field," J. Geol. 34(7), 577 (1926).
- 4. G. Gambolati, (a) "Study of the Possibility of Formulating a Mathematical Model of Subsoil Relevant to the Phenome-

- non of Subsidence," *Tech. Rep. CVR001* (in English); (b) "A One-dimensional Model of the Subsoil for Computing the Surface Subsidence," *Tech. Rep. CRV002* (in English), IBM Scientific Center, Venice, Italy, 1970.
- C. E. Jacob, "The Flow of Water in an Elastic Artesian Aquifer," Trans. Am. Geophys. Union 21, 574 (1940).
- S. W. Lohman, "Compression of Elastic Artesian Aquifers," Paper 424-B, U.S. Geol. Survey, 1961.
- K. Terzaghi, "Die Berechnung des Durchlässigkeitsziffer des Tones aus dem Verlauf der Hydrodynamischen Spannungserscheinungen," Sitz. Akad. Wiss. Wien, Abt. IIa 132, 125 (1923).
- 8. O. E. Meinzer and M. A. Hard, "The Artesian Water Supply of the Dakota Sandstone in North Dakota with Special Reference to the Edgeley Quadrangle," *Paper 520-E*, U.S. Geol. Survey, 1925.
- K. Terzaghi and R. B. Peck, Soil Mechanics in Engineering Practice, second edition, John Wiley and Sons, Inc., New York 1967
- D. W. Taylor, Fundamentals of Soil Mechanics, John Wiley and Sons, Inc., New York 1948.
- 11. G. A. Leonards, "Engineering Properties of Soils" in Foundation Engineering, edited by G. A. Leonards, McGraw-Hill Book Co., Inc., New York 1962.
- 12. B. E. Lofgren and R. L. Klausing, "Land Subsidence Due to Ground-water Withdrawal—Tulare-Wasco Area, California," *Paper 437-B*, U.S. Geol. Survey, 1969.
- J. F. Polan and G. H. Davis, "Land Subsidence due to Withdrawal of Fluids," Reviews in Engineering Geology, Vol. II, Geological Society of America, 1969, p. 187.
- 14. (a) L. Alberotanza, V. Favero, P. Gatto, G. Mozzi, F. Pianetti, R. Serandrei-Barbero, "Situazione Idrogeologica del Sottosuolo di Venezia: Ricostruzione dei Livelli Aquiferi Soggetti a Sfruttamento, sulla Base dei Dati Relativi ai Pozzi Censiti ed ai Rilievi Diretti (The Hydrological Situation of the Venetian Subsoil: Reconstruction of the Exploited Aquifer Systems on the Basis of the Data Relevant to Known Wells and Direct Surveys)"; (b) "Ricostruzione Lito-stratigrafica del Sottosuolo nella Zona di Venezia, sulla Base della Documentazione di 120 Serie Stratigrafiche (Lito-stratigraphic Reconstruction of the Subsoil in the Venetian Area on the Basis of the Information of 120 Stra-

- tigraphies)"; (c) "Considerazioni sulla Interdipendenza tra l'Evoluzione della Pressione Idrostratica negli Acquiferi e l'Abbassamento del suolo a Venezia (Considerations on the Interdependence between the Evolution of the Hydrostatic Pressure in the Aquifers and the Subsidence in Venice)," technical reports of the National Research Council, Venice (to be published).
- G. Brighenti, "Sulla Comprimibilita di Rocce Serbatoio di Giacimenti di Idrocarburi (About the Compressibility of Reservoirs of Hydrocarbons)," La Ricerca Scientifica 6(1), 17 (1964).
- 16. G. Evangelisti and B. Poggi, "Sopra i Fenomeni di Deformazione dei Terreni da Variazione della Pressione di Strato (About the Phenomena of Compaction of Strata due to Variation of the Neutral Pressure)," Atti della Accademia delle Scienze, Bologna, 1970.
- Assoc. Geotecnica Italiana, "Il Sottosuolo di Venezia, Mestre e Marghera. Argomenti Vari (The Subsoil of Venice, Mestre and Marghera; Miscellaneous)," VIII Convegno di Geotecnica, Cagliari, 1967.
- 18. For the clays in which  $\sigma_z < 10 \, \text{kg/cm}^2$ , the value of  $c_b$  corresponding to  $\sigma_z = 10 \, \text{kg/cm}^2$  was used. For the sands, the values in Table 4 were roughly halved relative to the curve of Fig. 5 because the latter is based on samples that, in the laboratory compression tests, were free to expand in all directions. We need, however, the vertical compressibility when lateral expansion is impeded. This was obtained [16] by multiplying the values in Fig. 5 by the factor  $\frac{1}{3}(1+\nu)/(1-\nu) \approx 0.55$ , where  $\nu$  is Poisson's modulus, taken to be 0.25.
- M. Caputo, G. Folloni, A. Gubellini, L. Pieri, and M. Unguendoli, "Survey and Geometric Analysis of the Phenomenon of Subsidence in the Region of Venice and its Hinterland," *Tech. Rep.* 9 (in English), National Research Council, Venice, 1971.

Received July 6, 1971; revised August 17, 1971

The author is located at the IBM World Trade Corporation Scientific Center, San Polo 1364, 30125 Venice, Italy.