H. Seki
I. P. Batra
W. D. Gill
K. K. Kanazawa
B. H. Schechtman

# A Quasi-steady-state Analysis for the Electrophotographic Discharge Process

**Abstract:** A description of the electrophotographic discharge process is presented for a homogeneous photoconductor characterized by constant free-carrier lifetimes and trapping times. The physical model is described by a set of one-dimensional nonlinear differential equations with appropriate boundary conditions. It is shown that a quasi-steady-state approximation can be derived which is valid for most of the decay process under typical electrophotographic conditions. In this quasi-steady state, the photoconductor layer can be described by an equivalent circuit which consists of a capacitance in parallel with a two-terminal element whose current-voltage characteristic is directly related to the field-dependent photoinjection efficiency. A numerical analysis using large-scale computation of the solutions to the differential equations has been done for a hypothetical but typical material. The results of this computation, which describe the initial transient behavior and the subsequent quasi-steady-state behavior of the photoconductor, are presented. The results of the quasi-steady-state analysis agree with the actual behavior of amorphous Se. It is suggested that measurement of the quasi-steady-state photodischarge process can be used as a convenient technique for obtaining the field-dependent characteristics of the photoinjection process.

## 1. Introduction

Xerography, that part of electrophotography invented by Carlson, [1] is presently a well developed and widely used technology. In its most common implementation [2, 3] a layer of photoconducting material about 10 to 50 $\mu$m thick on a grounded conducting substrate is "sensitized," i.e., charged to a uniform potential by means of a corona discharge in the dark. Then the layer is exposed to an illuminated image to be reproduced. The area exposed to light experiences a rapid decay of the potential because of the transport of photogenerated carriers across the photoconducting layer, while the dark area maintains most of its original voltage, thereby resulting in the formation of a "latent electrostatic image." This paper is concerned primarily with the photodischarge process which is involved in the formation of the electrostatic image for a homogeneous, high-resistivity photoconducting layer having a low density of ionizable donor and acceptor sites. Of the various photoconducting layers in use today amorphous Se and some organic photoconductors are relevant examples.

It has recently been shown by Tabak and Warter [4] that the simple range-limited models [5] are not adequate to describe the photodischarge process of Se. They pointed out that the measured range of holes is much greater than the layer thickness and stressed the importance of field-controlled photogeneration of free carriers. More recently Warter [6] has presented a qualitative discussion of the discharge process taking this into account. The objective of the present work is to derive from the basic physical concepts some quantitative relations that closely describe the photodischarge process in Se-like materials.

In the following section the physical model is discussed and the problem is formulated mathematically. Although a relatively simple model is used, the analysis involves nonlinear differential equations. In Section 3 the existence of a quasi-steady state is demonstrated within the constraints of certain specified conditions. It is shown by a self-consistent analysis that these conditions are realized in most xerographic applications. Some specific consequences of the existence of the quasi-steady state are then considered. The results of a numerical solution of the differential equations are presented in Section 4. These results, including the effects of trapping and detrapping, are obtained for a hypothetical material and give a complete description of both the initial transient behavior and the subsequent quasi-steady-state behavior. Comparisons of the quasi-steady-state analysis with experimental observation are discussed in Section 5. Some

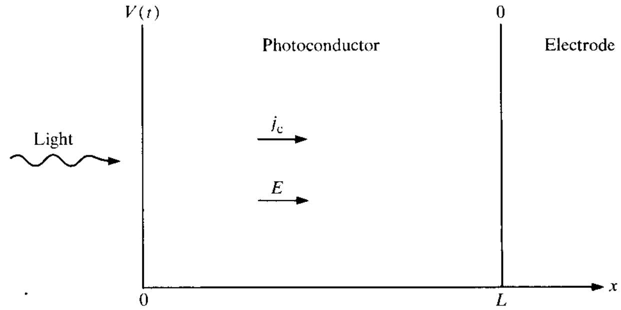


**Figure 1** Diagram showing coordinate and positive vector direction for the electrophotographic model. The thickness of the photoconductor layer is designated as $L$.

important mathematical results are derived in Appendices A and B. The computations involved in this paper were performed on an IBM System/360 Model 91.

## 2. Physical model

The analysis presented here is based on a physical model of the photoconductor and on boundary conditions which assume the following:

1) The deposited corona charge forms an effective blocking contact. By this it is meant that in the dark the deposited charge is immobile on the surface in the form of ions or in traps located at or very near the surface [7].
2) The photoconductor-substrate interface blocks the injection of carriers having polarity opposite to the charge deposited on the surface [7]. A further requirement on the interface, especially if the plate is to be reused, is that the photogenerated carriers that drift across the photoconductor be able to flow out without appreciable buildup of voltage across this interface.
3) The density of ionizable donors and acceptors is negligibly small. This assumption appears to be valid for amorphous Se or some of the organic photoconductors but not for ZnO binder type layers [8].
4) Light is absorbed at the surface of the photoconductor. As will be discussed later, the fact that light is absorbed within a finite thickness is compensated by an effective recombination velocity. This compensation is adequate for materials that exhibit high sensitivity to strongly absorbed light [9].
5) Recombination in the bulk of the photoconductor is negligible. This is a consequence of assumptions 1)–4).
6) The lifetime of free carriers is mainly limited by traps. Here traps are sites in which carriers can be trapped and are neutral when empty. A trap of any given type $i$ is characterized by a constant free carrier lifetime $\tau_i$ with respect to that trap and a constant trapping time $\tau_{T_i}$. The net free carrier lifetime $\tau$ is given by $1/\tau = \sum_i 1/\tau_i$, and in practical situation the $\tau_i$'s and $\tau_{T_i}$'s have values such that most of the carriers involved in the discharge process can traverse the thickness of the photoconductor within xerographic process time.

The problem is treated in one dimension with the coordinates taken as shown in Fig. 1. The field $E$ and the conduction current density $j_c$ are positive in the $x$-direction. Light is incident on the surface ($x = 0$), which has been charged to an initial voltage $V_0$. The differential equations governing the carrier transport in the layer are then as follows:

$$j_c(x, t) = e\mu n(x, t)E(x, t) - eD\frac{\partial n(x, t)}{\partial x}, \tag{1}$$

$$\frac{\partial E(x, t)}{\partial x} = \frac{e}{\epsilon_0\epsilon}\{n(x, t) + \sum_i n_{T_i}(x, t)\}, \tag{2}$$

$$\frac{\partial n(x, t)}{\partial t} = -\sum_i \frac{\partial n_{T_i}(x, t)}{\partial t} - \frac{1}{e}\frac{\partial j_c(x, t)}{\partial x}, \tag{3}$$

$$\frac{\partial n_{T_i}(x, t)}{\partial t} = \frac{n(x, t)}{\tau_i} - \frac{n_{T_i}(x, t)}{\tau_{T_i}}, \tag{4}$$

$$0 = j_c(x, t) + \epsilon_0\epsilon\frac{\partial E(x, t)}{\partial t}. \tag{5}$$

The total voltage across the photoconductor is given by

$$V(t) = \int_0^L E(x, t)\, dx. \tag{6}$$

Equation (1) is the expression for the conduction current density, Eq. (2) is Poisson's equation, Eq. (3) is the continuity equation, Eq. (4) contains the mathematical description of the trap model used, and Eq. (5) reflects the fact that the total current is zero since a floating surface is assumed at $x = 0$. Here $e$ is the charge of the carrier, $\mu$ the carrier mobility, $D$ the diffusion coefficient, $\epsilon_0$ the permittivity of free space, and $\epsilon$ the relative dielectric constant. The density of free carriers is $n$ and the density of carriers in traps of type $i$ is $n_{T_i}$.

Initially the layer is charged to a voltage $V_0$ and the bulk of the layer is neutral, i.e., there are no free or trapped carriers and therefore $n(x, 0) = n_{T_i}(x, 0) = 0$. Then the field is uniform throughout the sample and

$$E(x, 0) = \frac{V_0}{L} = E_0. \tag{7}$$

The carrier drift velocity for this field is termed the initial carrier velocity $v_0$ and is given by

$$v_0 = \mu E_0. \tag{8}$$

The corresponding transit time $t_0$ is given as

$$t_0 = L/v_0. \tag{9}$$

Illumination of the surface ($x = 0$) begins at $t = 0$ and the absorbed photon flux density is taken to be some function of time $f(t)$. The generation efficiency of free carriers is assumed to be a function $g[E(0, t)]$ of both the field at the surface and implicitly the illumination wavelength [4, 9–12]. The recombination rate is taken to be proportional to the free-carrier density at the surface with a recombination velocity $v_r$ (the quantity $v_r$ effectively includes both surface and bulk recombination since light is actually absorbed in a finite volume; thus $v_r$ could also depend on the wavelength of light). The free carriers injected into the bulk of the photoconductor by illumination constitute what might be called the *photoinjected current*. For the case of surface absorption this current is given by

$$j_c(0, t) = e\{f(t)g[E(0, t)] - v_r n(0, t)\}. \tag{10}$$

A photoinjection efficiency [9] $Y$ can be defined as

$$Y \equiv \frac{j_c(0, t)}{ef(t)}. \tag{11}$$

It can be seen (Appendix A) that the photoinjection efficiency is a product of the free-carrier generation efficiency and a recombination loss factor, both of which will depend on the material, field and illumination. Equation (11) thus provides the boundary condition at the illuminated surface for the photodischarge process. An electrophotographic yield for the decay process $Y_x$ can be defined as

$$Y_x(t_L) = \frac{\epsilon_0\epsilon[E(0, 0) - E(0, t_L)]}{e\int_0^{t_L} f(t)\,dt} = \frac{\int_0^{t_L} Y(t)f(t)\,dt}{\int_0^{t_L} f(t)\,dt}, \tag{12}$$

where $t_L$ is the duration of the illumination. For constant photon flux density,

$$Y_x(t_L) = \frac{1}{t_L}\int_0^{t_L} Y(t)\,dt. \tag{13}$$

Note that if at time $t_L$ carriers still remain in the photoconductor (e.g., in deep traps) the electrophotographic yield as defined here in terms of surface field strengths will be somewhat greater than the electrophotographic yield commonly defined in terms of the decay in surface potential.

### 3. The quasi-steady state

Since the differential equations (1)–(4) are involved in basic studies of the transport properties of materials, there have been a number of attempts to obtain usable relations between the measurable parameters and the material parameters under experimentally realizable conditions [13–16]. In the absence of trapping, the problem is greatly simplified and in some cases an exact analysis is possible [17–20]. Here those simplifications which are particularly applicable to the electrophotographic process will be considered.

In what follows, an analysis will be presented that will demonstrate that under certain conditions the system will go into a quasi-steady state. A reasonable simplification that will be made here is to assume that the photon flux density $f(t)$ is a constant. It will be shown that the voltage decay in this quasi-steady state can be directly related to the photoinjection efficiency and that the decay process can be described analytically in terms of basic physical parameters. Relations will be derived which will permit a self-consistent check on the conditions for the establishment of the quasi-steady state.

Let $|(\partial n/\partial t) + \sum_i (\partial n_{T_i}/\partial t)|_{\max}$ denote the maximum value of $|(\partial n/\partial t) + \sum_i (\partial n_{T_i}/\partial t)|$ over the thickness of the sample at any given time. The total number $N_{TOT}(t)$ of charges in the bulk of the photoconductor per unit area at any time is given by

$$N_{TOT}(t) = \int_0^L (n + \sum_i n_{T_i})\,dx. \tag{14}$$

Further the number of charges $N_S(t)$ on the surface at $x = 0$ per unit area is

$$N_S(t) = \frac{\epsilon_0\epsilon}{e}E(0, t). \tag{15}$$

Now assume the following conditions:

a) The total space charge in the photoconductor is very small relative to the charge on the surface, i.e.,

$$N_{TOT}(t) \ll N_S(t). \tag{16}$$

b) The rate of change of the space charge at any point in the photoconductor is much smaller than the rate of change of the surface charge, i.e.,

$$L\left|\frac{\partial n}{\partial t} + \sum_i \frac{\partial n_{T_i}}{\partial t}\right|_{\max} \ll \left|\frac{\partial N_S}{\partial t}\right|. \tag{17}$$

This of course implies that

$$\left|\frac{\partial N_{TOT}}{\partial t}\right| \ll \left|\frac{\partial N_S}{\partial t}\right|.$$

c) The diffusion term in Eq. (1) is negligible.

It can be shown rigorously (Appendix B) that if these conditions are satisfied a quasi-steady state is established in which the field, the conduction current and the free-carrier density are uniform throughout the photoconductor. Since $(\partial n/\partial x) \approx 0$ in the quasi-steady state, it can be said at this stage that condition c) is self-consistent with a) and b). Thus when the conditions for quasi-steady state hold $E(x, t)$, $j(x, t)$ and $n(x, t)$ can be considered as functions of $t$ only. Under these conditions Eqs. (1), (5) and (6) simplify considerably and one writes

$$j_c(t) = e\mu n(t)E(t), \tag{18}$$

$$\frac{dE}{dt} = -\frac{1}{\epsilon_0\epsilon} j_c(t), \tag{19}$$

$$V(t) = LE(t). \tag{20}$$

Also $j_c(t)$ is determined by the boundary condition (11) which gives

$$j_c(t) = efY(E). \tag{21}$$

When combined with Eq. (19) the following differential equation results for $E(t)$:

$$\frac{dE}{dt} = -\frac{e}{\epsilon_0\epsilon} f\, Y(E). \tag{22}$$

Equations (18) and (19) combine to give

$$n(t) = -\frac{\epsilon_0\epsilon}{e\mu}\frac{1}{E}\frac{dE}{dt}. \tag{23}$$

Combining Eqs. (20) and (22) one gets

$$\frac{dV}{dt} = -\frac{eL}{\epsilon_0\epsilon} f\, Y\left(\frac{V}{L}\right), \tag{24}$$

which describes the behavior of the observable $V(t)$. It is interesting to note that in this quasi-steady state the voltage decay process is not directly dependent on the details of the transport process. The mobility is implicitly included in the injection efficiency but the injection efficiency is a characteristic of the sample that can be measured without knowledge of the mobility [16].

Using Eq. (20), Eqs. (19) and (21) can be rewritten as

$$-C\frac{dV}{dt} = j_c(t), \tag{25}$$

$$j_c(t) = ef\, Y\left(\frac{V(t)}{L}\right), \tag{26}$$

where $C$ is the capacitance per unit area of the photoconductor layer ($C = \epsilon_0\epsilon/L$). These two equations demonstrate that the photoconductor in the quasi-steady state can be represented by an equivalent circuit consisting of a capacitor $C$ in parallel with a two-terminal element. The current-voltage characteristic of the two-terminal element is determined by the photoinjection efficiency through Eq. (26). In the special case where the injection efficiency is linearly dependent on the field, the photoconductor will appear as an $RC$ circuit.

The assumptions a) and b) stated earlier were written in terms of variables that cannot be measured directly. To further examine these conditions, it is necessary to consider the trapped-carrier density. Since the trapped-carrier density is initially zero it tends to increase with time until it reaches the equilibrium value of $(\tau_{T_i}/\tau_i)n$. Therefore in the quasi-steady state

$$N_{TOT}(t) \leqq L\left(1 + \sum_i \frac{\tau_{T_i}}{\tau_i}\right)n(t), \tag{27}$$

and

$$\frac{dn}{dt} + \sum_i \frac{dn_{T_i}}{dt} \approx \left(1 + \sum_i \frac{\tau_{T_i}}{\tau_i}\right)\frac{dn}{dt}. \tag{28}$$

Hence using (15), (22), (23), (27) and (28), one obtains

$$\frac{N_{TOT}}{N_S} \leq \left(1 + \sum_i \frac{\tau_{T_i}}{\tau_i}\right)\frac{Lef}{\mu\epsilon_0\epsilon}\frac{Y(E)}{E^2}, \tag{29}$$

and

$$\frac{L\left|\frac{dn}{dt} + \sum_i \frac{dn_{T_i}}{dt}\right|}{\left|\frac{dN_S}{dt}\right|} \approx \left(1 + \sum_i \frac{\tau_{T_i}}{\tau_i}\right)\frac{Lef}{\mu\epsilon_0\epsilon}\frac{1}{E^2}\left|E\frac{dY(E)}{dE} - Y(E)\right|. \tag{30}$$

The expressions on the right-hand side in relations (29) and (30) are in terms of imposed external conditions and material properties, and provide a means of stating self-consistent conditions under which the quasi-steady state can be expected. Thus, sufficient conditions for the existence of a quasi-steady state are

$$\left(1 + \sum_i \frac{\tau_{T_i}}{\tau_i}\right)\frac{Lef}{\mu\epsilon_0\epsilon}\frac{Y(E)}{E^2} \ll 1, \tag{31}$$

$$\left(1 + \sum_i \frac{\tau_{T_i}}{\tau_i}\right)\frac{Lef}{\mu\epsilon_0\epsilon}\frac{1}{E^2}\left|E\frac{dY}{dE} - Y\right| \ll 1. \tag{32}$$

If these conditions hold for a reasonable period after illumination begins, the system can be expected to establish a quasi-steady state after an initial transient. The duration of this transient period will depend largely on the transport parameters, $\mu$, $\tau_i$ and $\tau_{T_i}$. After the quasi-steady state is once established, it is possible that conditions (31) and (32) may break down at lower fields if $Y(E)$ decreases less rapidly than $E^2$. If the sums in Eqs. (31) and (32) include deep traps with large $\tau_{T_i}/\tau_i$, but for which $\tau_i$ is much larger than the initial carrier transit time ($t_0$), then these traps will not affect the initial portion of the decay. In such cases these traps can be excluded from conditions (31) and (32) and it is possible for the discharge process to achieve a quasi-steady state for a limited period, after which this is no longer true due to buildup of excess space charge of carriers in these deep traps.

### 4. Case study of the quasi-steady state

Using the results obtained in the previous section, it will now be demonstrated that in typical electrophotographic applications the quasi-steady-state approximations are valid. This is best done by examining specific cases. Measurements of photoinjection efficiency are relatively scarce. In most instances, reported measurements of the free-carrier generation efficiency turn out to be measurements on the photoinjection efficiency. These generally indicate that $Y$ is a monotonically increasing function of $E$ [4, 9–12]. Since such curves are relatively smooth and any segment of the curve can be approximated quite well by a power-law dependence on the field, it is instructive to consider the quasi-steady-state decay process under constant illumination assuming

$$Y(E) = Y_0 E^m, \tag{33}$$

where for reported data $m$ has had values between 0.5 and 2.5 [4, 9–12]. Then Eqs. (22) and (23) now become, respectively,

$$\frac{dE}{dt} = -\frac{ef}{\epsilon_0\epsilon} Y_0 E^m, \tag{34}$$

$$n(t) = \frac{f}{\mu} Y_0 E^{m-1}. \tag{35}$$

Conditions (31) and (32) can be expressed as

$$\left(1 + \sum_i \frac{\tau_{\mathrm{T}_i}}{\tau_i}\right) \frac{ef}{\mu C} \frac{Y}{E^2} \ll 1, \tag{36}$$

$$|m - 1| \left(1 + \sum_i \frac{\tau_{\mathrm{T}_i}}{\tau_i}\right) \frac{ef}{\mu C} \frac{Y}{E^2} \ll 1. \tag{37}$$

Integration of Eq. (34) gives for $m \neq 1$

$$E(t) = E_0 \Big/ \left\{1 + (m - 1) \frac{e Y_0}{\epsilon_0\epsilon} E_0^{m-1} ft\right\}^{1/(m-1)}, \tag{38}$$

For $m = 1$ we have

$$E(t) = E_0 \exp\left(-\frac{e Y_0}{\epsilon_0\epsilon} ft\right), \tag{39}$$

where $E_0 = E(0)$. Through Eqs. (20), (34), (38) and (39) one obtains

$$\frac{dV}{dt} = -\frac{ef}{\epsilon_0\epsilon} Y_0 L^{(1-m)} V^m, \tag{40}$$

with solutions for $m \neq 1$

$$V(t) = V_0 \Big/ \left\{1 + (m - 1) \frac{e Y_0}{\epsilon_0\epsilon} \left(\frac{V_0}{L}\right)^{m-1} ft\right\}^{1/(m-1)}, \tag{41}$$

and for $m = 1$

$$V(t) = V_0 \exp\left(-\frac{e Y_0}{\epsilon_0\epsilon} ft\right), \tag{42}$$

where $V_0 = V(0)$. Equations (40)–(42) give the time-dependent behavior during the quasi-steady-state photodischarge process.

Note that the form of Eq. (41) includes two distinct families of decay curves, $V(t) = V_0/(1 + |A|\ t)^{|P|}$ or $V(t) = V_0(1 - |A|\ t)^{|P|}$ depending on whether $m > 1$ or $m < 1$. Here $A$ and $P$ are constants defined as

$$A \equiv (m - 1) \frac{e Y_0}{\epsilon_0\epsilon} \left(\frac{V_0}{L}\right)^{m-1} f,$$

$$P \equiv 1/(m - 1).$$

It is seen from Eq. (40) that the slope of log $|dV/dt|$ versus log $V$ will give the value of $m$ and that plots of such curves should coincide for decays which start from various voltages. The latter property can be used as an experimental check as to whether the quasi-steady state was established.

At this point a matter of considerable concern in the imaging sciences can be noted, i.e., reciprocity, by which is meant the property of obtaining the same image level independent of the light intensity so long as the integrated light input is the same. Electrophotographically this means that the same voltage is obtained in the latent electrostatic image for a given integrated light input. It is clearly seen from Eqs. (41) and (42) that, starting from the same initial voltage, the final surface voltage is determined by the integrated light input $ft_L$. Hence, so long as the injection efficiency is independent of the light intensity, reciprocity can be expected when the photodischarge process takes place in the quasi-steady state.

## 5. Numerical analysis

Since the preceding analysis is limited to the assumed conditions a) and b) of Section 3 and ignores diffusion, the problem has also been investigated by numerical computation. The behavior for transient situations, very high light intensity and severe trapping are the situations for which this approach is most useful. The problem is essentially an initial value problem to be analyzed by the difference method. The basic variables $E$, $n_{\mathrm{T}_i}$, and $n$ are calculated through numerous iterations of small time increments using the first two terms of the Taylor series.

$$F(x, t + \Delta t) = F(x, t) + \frac{\partial F}{\partial t} \Delta t. \tag{43}$$

The relevant time derivatives can be derived from the differential equations and are

$$\frac{\partial E}{\partial t} = -\frac{e\mu}{\epsilon_0\epsilon} nE + \frac{e}{\epsilon_0\epsilon} D \frac{\partial n}{\partial x}, \tag{44}$$

$$\frac{\partial n_{\mathrm{T}_i}}{\partial t} = \frac{n}{\tau_i} - \frac{n_{\mathrm{T}_i}}{\tau_{\mathrm{T}_i}}, \tag{4}$$

and

$$\frac{\partial n}{\partial t} = -\sum_i \frac{\partial n_{\mathrm{T}_i}}{\partial t} - \mu E \frac{\partial n}{\partial x} - \frac{e\mu}{\epsilon_0\epsilon} n\left(n + \sum_i n_{\mathrm{T}_i}\right) + D \frac{\partial^2 n}{\partial x^2}. \tag{45}$$

Some care must be used in the choice of the spatial grid in relation to $t$ and in the calculation of the space deriva-

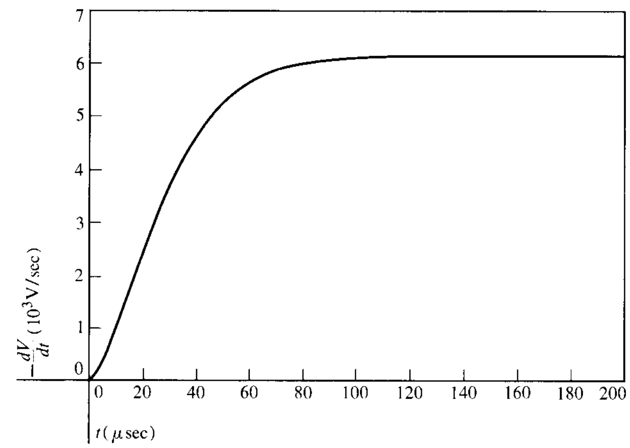


**Figure 2** Initial time dependence of $-(dV/dt)$ from the numerical computation.

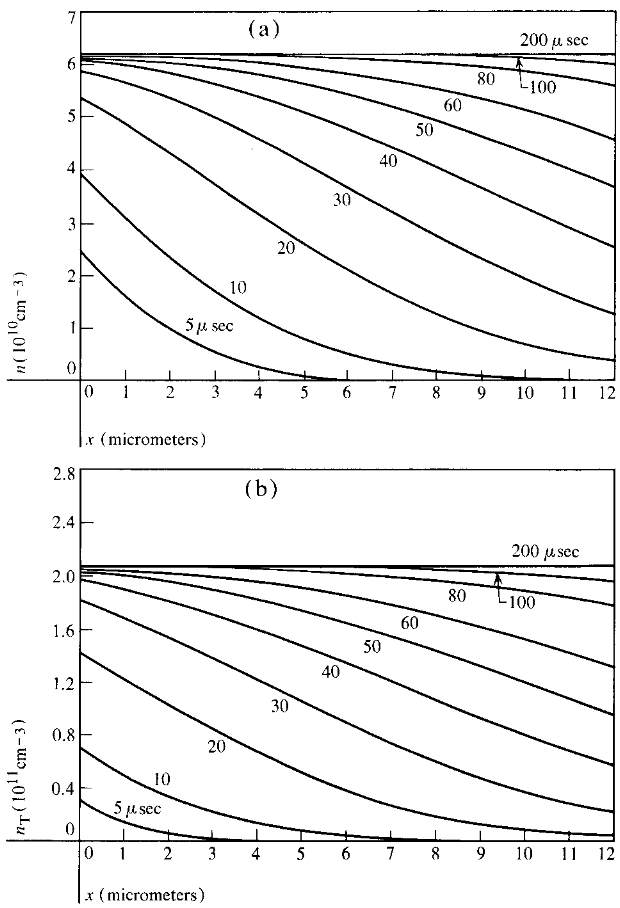


**Figure 3** Two spatial distributions for various times during the transient portion of the numerical computation.
(a) Distribution of free carrier density.
(b) Distribution of trapped-carrier density.

tives to avoid computational instabilities. The result is an array of $E(x_i, t_k)$, $n_{T_i}(x_i, t_k)$ and $n(x_i, t_k)$ from which the other parameters can be calculated. Here $x_i$ and $t_k$ represent, respectively, the discrete values taken in the computation for the continuous space and time variables. The advantage of the numerical approach is that it is considerably more flexible with regard to the problems it can handle. For example, it can easily incorporate time-dependent variations of the incident light, arbitrary boundary conditions at the two surfaces of the photoconductor layer, and arbitrary initial conditions. In effect, it provides a numerical laboratory in which the more complex aspects of the photodischarge process can be investigated.

Here some computational results concerning a hypothetical material are presented as an illustration of the numerical analysis technique. The results also serve to confirm the validity of the previous quasi-steady-state analysis. In this example a material with one type of trap is considered and the relevant material parameters are chosen as follows:

$$\epsilon = 3$$

$$\mu = 3.3 \times 10^{-4}\ \mathrm{cm^2/V\ sec}$$

$$\tau = 2 \times 10^{-6}\ \mathrm{sec}$$

$$\tau_T = 6.7 \times 10^{-6}\ \mathrm{sec}$$

$$L = 12 \times 10^{-4}\ \mathrm{cm}$$

$$Y(E) = 3.16 \times 10^{-10}\ E^{1.5}\ \text{carriers/photon.}$$

The values given represent a situation in which there is considerable trapping and detrapping. The layer is charged initially to 500 V and the charge carriers are holes. The trap-free transit time of the leading carriers is then 8.73 $\mu$sec. It is assumed that the electrode is non-injecting and that photoinjection is the sole source of space charge in the photoconductor layer. The illumination absorbed by the photoconductor is given by

$$f(t) = 10^{14}\{1 - \exp(-10^5 t)\} \qquad (\text{photons/cm}^2\ \text{sec}).$$

A finite risetime is used for computational purposes to describe the essentially step-function illumination. The Einstein relation is assumed for the diffusion coefficient and the calculation is for $T = 300°$K.

In Fig. 2 the initial transient behavior of $dV/dt$ is plotted as a function of time. An important fallout of numerical analysis is that it allows the investigation of parameters which may not be easily measured. Examples of this are shown in the next two figures. Figure 3(a) presents the spatial profile of the free carrier density for various times during the initial transient period and Fig. 3(b) presents the spatial profile of the trapped-carrier density. It should be noted that the scales for the carrier density on the two figures are different. At 5 $\mu$sec the leading edge of the

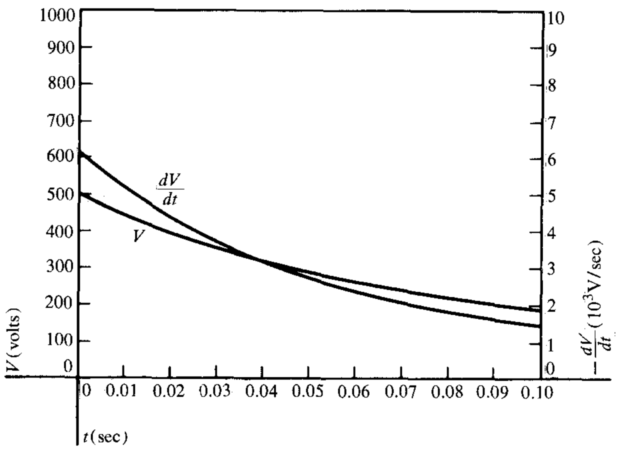


**Figure 4** The time dependence of $V$ and $-(dV/dt)$ for most of the photodischarge process in the numerical computation.

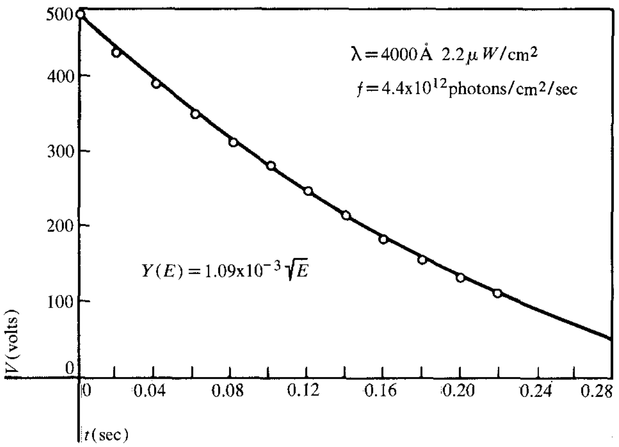


**Figure 5** Comparison of the measured surface potential vs time relation (points) in a photodischarge process of amorphous Se with quasi-steady-state theory (solid curve). The curve is calculated from Eq. (41) setting $V_0 = 480$ and using $f$ and $Y(E)$ as indicated in the figure.

free carriers is at $x = 6.9 \times 10^{-4}$ cm. The total number of free carriers is rapidly increasing and it is seen in Fig. 2 that $dV/dt$ is also rising rapidly at this stage.

The absorbed light is essentially saturated to $10^{14}$ photons/cm$^2$ sec by 60 $\mu$sec and beyond this point, if the appropriate values in this example are substituted into Eqs. (36) and (37), one obtains

$$\frac{N_{\mathrm{TOT}}}{N_{\mathrm{S}}} \leq \frac{0.3}{\sqrt{E}},$$

$$\frac{L\left\{\left|\frac{dn}{dt}\right| + \left|\frac{dn_{\mathrm{T}}}{dt}\right|\right\}}{\left|\frac{dN_{\mathrm{S}}}{dt}\right|} \leq \frac{0.15}{\sqrt{E}},$$

which indicates that the system should reach quasi-steady state. Indeed it is seen in Fig. 2 that at roughly 160 $\mu$sec, which corresponds to about 20 transit times, $|dV/dt|$ reaches a maximum and subsequently decreases relatively slowly. In Figs. 3(a) and 3(b) the free- and trapped-carrier densities are seen to be essentially uniform by this time. The ratio of trapped carriers to free carriers is essentially $\tau_{\mathrm{T}}/\tau$, as expected from Eq. (4), and the quasi-steady state is established. The subsequent behavior of the system is seen in Fig. 4, where $V$ and $dV/dt$ are plotted against a longer time scale. The logarithmic plot of $V$ vs $-dV/dt$ consists of a straight line with a slope of $1/m$, which agrees with Eq. (40).

### 6. Comparison with actual behavior

The transport properties of amorphous Se and its photoinjection efficiency have been reported [4]. Using this information the prediction of the quasi-steady-state analysis is now compared to the photodischarge behavior of amorphous Se samples prepared in this laboratory. Figure 5 is a linear plot of the voltage versus time for a sample 50 $\mu$m thick discharged by 2.2 $\mu$W/cm$^2$, $\lambda$ = 4000 Å illumination from an initial voltage of about 500 V. Taking $\mu = 0.16$ cm$^2$/V sec and $\epsilon = 6$ the solid curve was calculated from Eq. (41) letting $Y_0 = 1.09 \times 10^{-3}$ and $m = 0.5$ which is in close agreement with the reported square root field dependence (for $E > 10^4$ V/cm) of the injection efficiency for $\lambda$ = 4300 Å [4]. A more direct comparison between the decay process and the injection efficiency can be made by plotting $dV/dt$ vs $V$ and this is especially convenient when the field dependence of the injection efficiency is not a simple power law. Figure 6 is a logarithmic plot of $dV/dt$ vs $V$ in which the dashed curve is the experimental result obtained by use of a differentiating circuit. In this case the same sample was charged to an initial voltage of about 600 V and discharged by 2.5 $\mu$W/cm$^2$, $\lambda$ = 5500 Å illumination. The solid curve was calculated using Eq. (24) based on the reported injection efficiency data [4] for $\lambda$ = 5500 Å. The higher voltage portion of the curves differ by a slight vertical shift. This shift most likely represents error in the measurement of the light intensity and the agreement is good, considering that the injection efficiency and decay rate measurements were on two different samples in different laboratories. Below 200 V the measured photodischarge rate decreases to less than the calculated value. The voltage at which this departure takes place increases with the starting voltage. This is interpreted to be due to the buildup of positive space charge in deep traps as was discussed at the end of Section 3. Evidence for the presence of such deep traps in amorphous Se has been reported recently [21]. Measure-

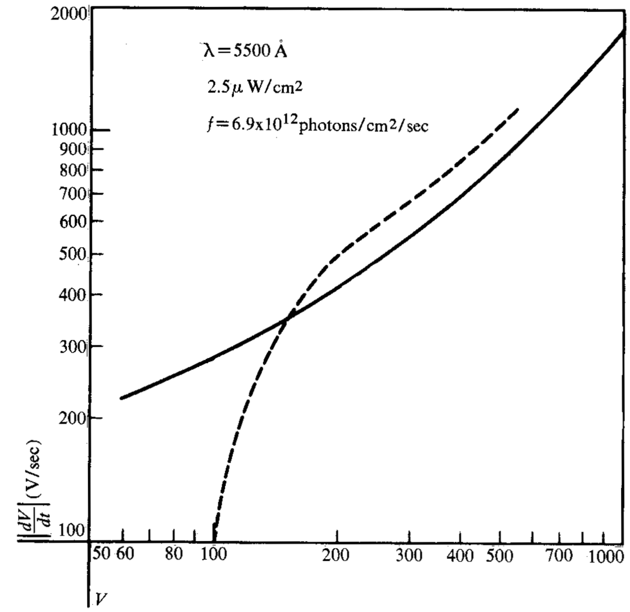


**Figure 6** Comparison of the measured $dV/dt$ vs $V$ (dashed curve) in a photodischarge process of amorphous Se with quasi-steady-state theory (solid curve). The curve is calculated from Eq. (24) using published data on the photoinjection efficiency.

ments on samples of different thickness gave similar agreement but the effect of deep traps increased with the thickness.

## 7. Discussion

It has been shown by theoretical analysis and by numerical computation that the electrophotographic discharge process can take place in the quasi-steady state. The conditions for the establishment of the quasi-steady state have been clearly derived in terms of external conditions of the electrophotographic process and basic material properties of the photoconductor. It can be seen by examining these conditions that in prevailing electrophotographic applications the quasi-steady-state analysis will describe the essential features of the photodischarge process in most cases. As a direct consequence of this state the voltage decay rate or conduction current is proportional to the photoinjection efficiency. Since the conduction current is limited by the photoinjection process the transport properties of the carriers play only a secondary role in the decay process. As seen in Eqs. (24) and (A5) the mobility and recombination affect the decay process through the photoinjection efficiency but details of the trapping dynamics play a relatively minor role.

An important ramification of the quasi-steady-state analysis and its verification in the case of amorphous Se is that it shows that measurement of the electrophotographic discharge process provides an effective and simple technique to directly measure the photoinjection efficiency. Unlike the pulse technique [4, 9–11] this approach gives the injection efficiency as a continuous function of the electric field. This technique is being used in this laboratory on an organic photoconductor with considerable success [22]. Since the photoinjection efficiency is related to important basic phenomena such as photogeneration of free carriers and recombination [9], the direct measurement of its field dependence should prove quite valuable. It should be noted that in such a measurement care must be taken to avoid misinterpretation which may result from the effects of deep traps, as was discussed at the end of Section 3. One method is to take a series of measurements at various starting voltages. The envelope or overlap of the resulting family of curves of $dV/dt$ vs $E$ will give the desired field-dependent photoinjection efficiency.

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## Appendix A. Photoinjection efficiency

The photoinjection efficiency is derived here for the case where the light is totally absorbed at the surface ($x = 0$) and the recombination process is characterized by a recombination velocity $v_r$. When Eq. (10) is combined with Eq. (1) the surface-free carrier density is given by

$$n(0, t) = \frac{f(t)g[E(0, t)] + D\dfrac{\partial n(0, t)}{\partial x}}{v_r + \mu E(0, t)}, \tag{A1}$$

and using Eq. (5) the time derivative at the surface for $E$ is

$$\frac{dE(0, t)}{dt} = -\frac{e}{\epsilon_0\epsilon}\,\frac{f(t)g[E(0, t)]E(0, t) - \dfrac{v_r}{\mu}D\dfrac{\partial n(0, t)}{\partial x}}{E(0, t) + v_r/\mu}. \tag{A2}$$

The photoinjection current density is

$$j_c(0, t) = \frac{ef(t)g[E(0, t)]E(0, t) - e\dfrac{v_r}{\mu}D\dfrac{\partial n(0, t)}{\partial x}}{E(0, t) + v_r/\mu}. \tag{A3}$$

When the diffusion term is negligible these equations are greatly simplified, e.g., Eq. (A2) gives the differential equation for $E(0, t)$

$$\frac{dE}{dt} = -\frac{e}{\epsilon_0\epsilon} f(t)g(E)\frac{E}{E + v_r/\mu}, \tag{A4}$$

and using Eq. (11) the photoinjection efficiency becomes

$$Y(E) = g(E)\frac{E}{E + v_r/\mu}. \quad \text{(A5)}$$

It should be noted that, when the diffusion term is negligible, the photoinjection efficiency is a product of the generation efficiency and a recombination loss factor. It is seen that at fields where $E \gg v_r/\mu$ the loss factor approaches unity and the photoinjection efficiency approaches the free-carrier generation efficiency.

### Appendix B. Quasi-steady state

Here it is useful to let

$$E(x, t) = E(0, t) + \Delta E(x, t) \quad \text{(B1)}$$

$$j_c(x, t) = j_c(0, t) + \Delta j_c(x, t) \quad \text{(B2)}$$

$$n(x, t) = n(0, t) + \Delta n(x, t) \quad \text{(B3)}$$

Then by Eqs. (2) and (14)

$$\frac{\Delta E(x, t)}{e} = \frac{1}{\epsilon_0\epsilon}\int_0^x (n + \sum_i n_{T_i})\, dx \leq \frac{1}{\epsilon_0\epsilon} N_{TOT} \quad \text{(B4)}$$

and by Eqs. (15) and (16)

$$\frac{\Delta E(x, t)}{E(0, t)} \ll 1. \quad \text{(B5)}$$

From Eqs. (3) and (4)

$$\left|\frac{\Delta j_c(x, t)}{e}\right| = \left|\int_0^x \frac{\partial}{\partial t}(n + \sum_i n_{T_i})\, dx\right| \leq L \left|\frac{\partial n}{\partial t} + \sum_i \frac{\partial n_{T_i}}{\partial t}\right|_{max}. \quad \text{(B6)}$$

Then by Eqs. (5), (15) and (17)

$$\left|\frac{\Delta j_c(x, t)}{j_c(0, t)}\right| \ll 1. \quad \text{(B7)}$$

It follows from Eq. (1), with diffusion neglected, that

$$\left|\frac{\Delta n(x, t)}{n(0, t)}\right| \ll 1.$$

Thus, in steady state the deviations of $n$, $E$, and $j_c$ from their values at the surface are negligible. In other words, these quantities are spatially uniform.

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*The authors are located at the IBM Research Laboratory, San Jose, Cal. 95114.*