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A Theory of Granularity and Bleaching for Holographic Information Recording*

Abstract: The Kelly three-stage model of photographic information recording is a mathematical model of black-and-white silver-halide film with granularity neglected. In the work reported here, the theory is extended for use in phase-holographic applications. Granularity effects are contained in the fourth and final stage, a two-dimensional, nonhomogeneous, filtered Poisson process. The output of this stage is a sample function of the random process that describes the pattern of modification of the emulsion. Formulas for the signal-to-noise ratio of a hologram and the optimum granular behavior are derived as examples of the use of the granularity model.

Introduction

A decade ago D. H. Kelly[1] formulated a mathematical model of photographic information recording in terms of a sequence of three operations or stages connected in tandem, the output of one being the input of the next. To ensure that each operation was a separate entity, free of interaction with the other two, he separated the operations in space and time. That is, he noted that first the radiant-flux pattern incident on an emulsion sifts its way through the turbid emulsion and perhaps even bounces off the back of the film before it strikes the silver-halide crystals in the emulsion. It is only when the incident photons strike the crystals that they create latent-image centers on and in the crystals. And it is much later that the activated crystals, and perhaps their near neighbors, are developed into silver grains at a rate which depends on the local concentration of developer at the crystals. Thus, Kelly was led to describe the photographic process in terms of 1) an optical-diffusion stage functioning in the emulsion and its backing during exposure, 2) a sensitometry stage functioning in and on the crystals during the formation of the latent image and 3) a chemical-diffusion stage functioning during development (see Fig. 1).

The present work is based on Kelley's original concepts with modifications of interest in phase holography.[2] To begin with, the optical-diffusion formulation is modified for exposure with coherent light. Secondly, the sensitometry and chemical-diffusion stages are discussed in terms of developed-silver density rather than optical density. Lastly, a fourth stage is added following the chemical-diffusion stage to account for granularity and bleaching or etching of the emulsion.

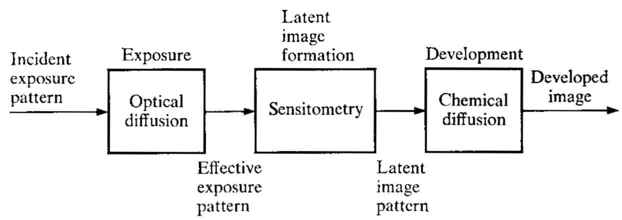

Figure 1 The structure of Kelly's model of photographic information recording, showing the decomposition of the over-all operation into three spatially and temporally separated stages.

The design of the granularity stage is based on both conceptual and experimental considerations. Conceptually, it is reasonable to expect the effects of individual bleached-silver grains to appear in a mathematical model as carriers as well as corrupters of information. (No grains, no hologram.) The similar dual role played by the electrons in a phototube suggests that the mathematical model of granularity effects should be closely related to the theory of shot noise in vacuum tubes.[3,4] Those additional random fluctuations of amplitude transmission and phase shift in the hologram that are independent of granularity effects can be accounted for as additive or multiplicative noise.

Admittedly, in a phototube all of the electrons are identical, whereas in a hologram all of the grains are different. The clouds of bleached or etched emulsion surrounding the bleached-out grains are also all different.

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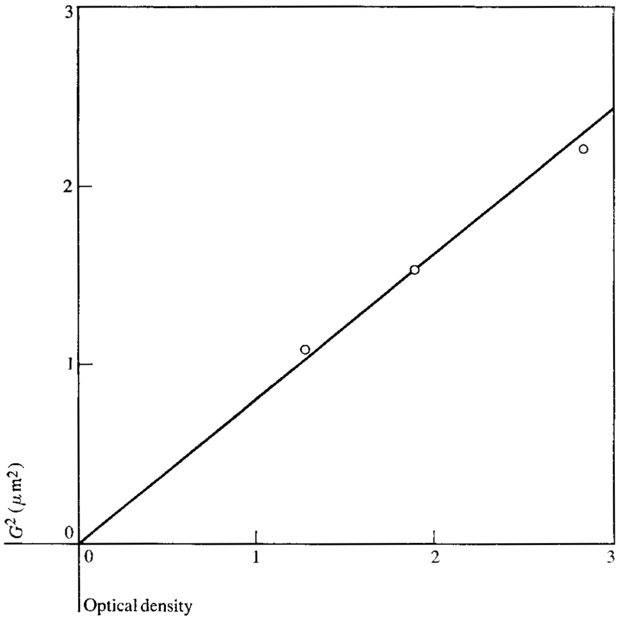

Figure 2 Square of the Selwyn-granularity G for Royal-X Pan film plotted to show the approximate direct proportionality of the variance and the mean of the optical density. The optical density is approximately proportional to the developed-silver density.

Fortunately, however, this situation has been treated mathematically for the time domain by Parzen[5] in his book on stochastic processes. Parzen's treatment is used here. The extension to two dimensions is based in part on the methods of Hurwitz and Kac.[4]

Experimentally, it has been found that in certain cases the variance of the optical density of photographic film is directly proportional to the average density for uniform exposures (see Fig. 2). This direct proportionality is characteristic of shot-noise processes. In addition, using an 80 cycle/mm exposure pattern, Kelly[6] has shown that increased chemical-diffusion effect, with gamma held nearly constant, can result in greater strength for high spatial-frequency image components and relatively increased freedom of these components from the disturbing effects of granularity. This result is consistent, not only with the shot-noise type of model but also with the notion that granularity can be accounted for in terms of a fourth stage appended to the model following the chemical-diffusion stage (see Fig. 3). In this fourth stage, the response of the emulsion layer to the bleaching or etching away of a developed-silver grain is treated in a manner analogous to the response of the electrical circuit of a phototube to the impact of an electron on the anode.

In the development that follows, the four stages, in sequence, are considered quantitatively. A discussion is included in which the signal-to-noise ratio and frequency-domain implications of the granularity theory are considered.

Stage 1: Optical diffusion

The input to Stage 1 is the incident complex-scalar disturbance pattern u; the output of the stage is the real, non-negative effective exposure pattern e. The effective exposure of the film over the time interval (t_1, t_2) at the point $\mathbf{r} = (x, y)$ in the plane of the emulsion is given by the equation

$$e(\mathbf{r}') = \int_{t_1}^{t_2} \left\{ \iint h_1(\mathbf{r}' - \mathbf{r}_1) h_1^*(\mathbf{r}' - \mathbf{r}_2) \times u(\mathbf{r}_1, t) u^*(\mathbf{r}_2, t)\, d^2\mathbf{r}_1\, d^2\mathbf{r}_2 \right\} dt. \quad (1)$$

The complex optical-diffusion point-spread function h_1 is normalized subject to the condition

$$\iint h_1(\mathbf{r}' - \mathbf{r}_1) h_1^*(\mathbf{r}' - \mathbf{r}_2)\, d^2\mathbf{r}_1\, d^2\mathbf{r}_2 = 1. \quad (2)$$

When limits are not specified, the integration is over the entire plane. The asterisk (*) is used to indicate the complex conjugate. A discussion of the properties of Eqs. (1) and (2) is provided by Raney.[7] The quantity in braces is the time-dependent effective irradiance falling on the emulsion. (Reciprocity-law failure has been neglected here but it could be taken into account through a modification of Eq. (1) of the form

$$e(\mathbf{r}') = \int_{t_2}^{t_1} f(\{\cdot\})\, dt,$$

where the argument of the function f is the quantity in braces in Eq. (1).

Stage 2: Sensitometry

This stage is a point-by-point, nonlinear, nonstochastic stage whose input is the effective exposure at any point $\mathbf{r}'$ and whose output is directly proportional to the expected effective number of latent-image centers created by the exposure, per unit of plate area, at $\mathbf{r}'$. The effective exposure at $\mathbf{r}'$, $e(\mathbf{r}')$, and the resultant expected latent-image-center density $\tau(\mathbf{r}')$ are related by the sensitometry function s. This functional relationship can be defined in terms of the equation

$$\tau(\mathbf{r}') = s[e(\mathbf{r}')]. \quad (3)$$

The graph of the function s is closely related to the familiar d vs log e curve of conventional photographic sensitometry.[8] The expected latent-image-center density $\tau(\mathbf{r}')$ resulting from the effective exposure $e(\mathbf{r}')$ can be defined as being equal to the spatially averaged developed-silver density $\langle\sigma\rangle$ resulting from a spatially uniform exposure equal to $e(\mathbf{r}')$ in the limit as the area of the exposed

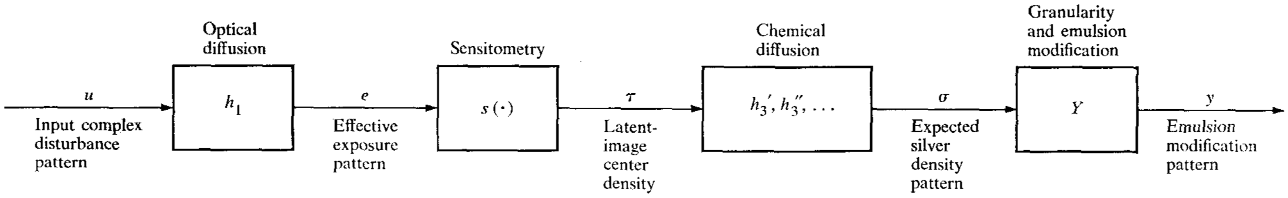

Figure 3 Four-stage model of phase-holographic information recording. The granular pattern y of emulsion modification is shown as having been generated by the Poisson random process Y. In turn, the process Y is shown being modulated by the non-granular expected developed-silver density pattern σ. The four stages are described mathematically in terms of the functions printed in the blocks. The spatial patterns that convey information from stage to stage are described mathematically in terms of the functions printed above the connecting lines.

and developed plate becomes infinite. Thus, the sensitometry function is macroscopic rather than microscopic. The developed-silver density is expressed in units of mass per unit of plate area.

Stage 3: Chemical diffusion

This stage is responsible for the various adjacency effects.[9] Its behavior has been attributed to the restricted flow of solution within the emulsion during development. The input to the stage is the effective latent-image-center density pattern τ, and the output is σ, the pattern of expected mass of developed silver per unit of plate area.

There is no fundamental reason why the relationship between the input and output patterns, σ and τ, must be linear. Consequently, following Volterra,[10] let us express the silver density at $\mathbf{r}$ in terms of the input pattern τ by means of the expansion

$$\begin{aligned}\sigma(\mathbf{r}'') &= a_0 + a_1 \int h_3'(\mathbf{r}'' - \mathbf{r})\tau(\mathbf{r})\, d^2\mathbf{r} \\ &\quad + a_2 \iint h_3''(\mathbf{r}'' - \mathbf{r}_1, \mathbf{r}'' - \mathbf{r}_2) \\ &\quad \times \tau(\mathbf{r}_1)\tau(\mathbf{r}_2)\, d^2\mathbf{r}_1\, d^2\mathbf{r}_2 \\ &\quad + a_3 \iiint h_3'''(\mathbf{r}'' - \mathbf{r}_1, \mathbf{r}'' - \mathbf{r}_2, \mathbf{r}'' - \mathbf{r}_3) \\ &\quad \times \tau(\mathbf{r}_1)\tau(\mathbf{r}_2)\tau(\mathbf{r}_3)\, d^2\mathbf{r}_1\, d^2\mathbf{r}_2\, d^2\mathbf{r}_3 + \cdots . \end{aligned} \tag{4}$$

This expansion is a generalization of the familiar power series.

In his pioneering work Kelly[1,6] assumed that chemical diffusion had a purely linear effect. More recently, however, Nelson et al.[11] have reported using a variant of the second-order form consisting of a linear and a quadratic term. For latent-image-center density patterns of low contrast, it can be shown that the second-order form of Nelson et al. reduces to the linear form of Kelly.

Stage 4: Granularity and bleaching

It can be deduced from five rather reasonable assumptions that this stage is a two-dimensional, filtered Poisson random process[5] designated as Y. The five basic assumptions are designated (A) through (E) below.

The input to Stage 4 is the silver density pattern σ. The output is y, which is the granular pattern of modification of the emulsion and a sample function of the random process Y. This modification may be a relief pattern due to bleaching or etching, or a pattern of modification of the refractive index, or some combination of these. Following Ooue and Hatanaka,[12] let us assume that each developed-silver grain is replaced by a localized but diffuse microscopic cloud of modification, centered at the grain location. Let us also assume that the total modification at a point $\mathbf{r}$ on the hologram is the sum of the modifications at that point due to the individual grain contributions.

These remarks can be put in mathematical form as follows. Let the number of developed grains on the hologram be n, the value of a random variable N. Let the location of the kth grain be $\boldsymbol{\rho}_k$, the value of a random position vector $\mathbf{P}_k$. Let the modification at $\mathbf{r}$ due to the kth grain be $w(\mathbf{r} - \boldsymbol{\rho}_k; a_k)$, where w is the general grain-modification function and a_k is an ordered set of parameters, the values of an ordered set of random variables A_k that account for variations in w from grain cloud to grain cloud.

(A) The total modification at $\mathbf{r}$ is

$$y(\mathbf{r}) = \sum_{k=1}^{n} w(\mathbf{r} - \boldsymbol{\rho}_k; a_k), \tag{5}$$

the value of the random variable

$$Y(\mathbf{r}) = \sum_{k=1}^{N} w(\mathbf{r} - \mathbf{P}_k; A_k). \tag{6}$$

(B) For any single grain-cloud k,

$$\mathrm{E}_{A_k}\left[\int w(\mathbf{r} - \mathbf{P}_k; A_k)\, d^2\mathbf{r}\right] = e, \tag{7}$$

where $\mathrm{E}_{A_k}[\cdot]$ indicates the expectation taken over the random parameters A_k and where e is a constant, independent of $\mathbf{P}_k$.

(C) The locations of individual silver-halide crystals are mutually independent, and whether or not a developed grain forms from a crystal is independent of whether or not a developed grain forms on any other crystal, except through chemical diffusion.

(D) Nowhere on the plate can one be certain about the presence of a grain, without looking.

(E) Probabilities $\Pr\{\cdot\}$ of grain locations are given in terms of the expected silver-density pattern by the equation

$$\Pr\{\text{the } k\text{th grain on the hologram is located on a surface element } d^2\mathbf{r} \text{ at } \mathbf{r}\} = \sigma(\mathbf{r})\, d^2\mathbf{r} \Big/ \int \sigma(\mathbf{r}')\, d^2\mathbf{r}'.$$

Assumption (A) is a statement to the effect that, during reconstruction of the image, the phase-shift pattern of the hologram is the sum of the contributions of the individual grain-clouds. Assumption (B) states merely that the plate and its processing are spatially uniform. Assumption (E) requires that the expected emulsion modification be proportional to the expected silver density and, together with Assumptions (C) and (D), requires that the grain locations be statistically independent and identically distributed with a finite probability density function whose value at $\mathbf{r}$ is

$$\sigma(\mathbf{r}) \Big/ \int \sigma(\mathbf{r}')\, d^2\mathbf{r}'.$$

Two results that can be derived from these assumptions are

$$E[Y(\mathbf{r})] = a \int h_4'(\mathbf{r} - \boldsymbol{\varrho})\sigma(\boldsymbol{\varrho})\, d^2\boldsymbol{\varrho} \tag{8}$$

and

$$\operatorname{Var}[Y(\mathbf{r})] = a \int h_4''(\mathbf{r} - \boldsymbol{\varrho})\sigma(\boldsymbol{\varrho})\, d^2\boldsymbol{\varrho}, \tag{9}$$

where a is a constant of proportionality and where, by Eq. (7),

$$h_4'(\mathbf{r} - \boldsymbol{\varrho}) = E_A[w(\mathbf{r} - \boldsymbol{\varrho}; A)]/e \tag{10}$$

and

$$h_4''(\mathbf{r} - \boldsymbol{\varrho}) = E_A[|w(\mathbf{r} - \boldsymbol{\varrho}; A)|^2]/|e|^2. \tag{11}$$

The subscript k on the random set of parameters A_k has been removed since the grain clouds are all the same insofar as their statistical properties are concerned. Equations (8) through (11) indicate that for a sequence of uniform exposures, such as those made with a step wedge, the variance of the emulsion modification is directly proportional to its mean value. Equations (8) and (10) account for the limitation on resolution in the hologram plane imposed by the finite average size of the grain clouds. The function h_4' may be regarded as a granularity point-spread function.

Discussion: Granularity effects in the frequency domain

It must be admitted that the modification random process Y is nonstationary, if only because the hologram is of finite size. Yet it is often useful to employ the concepts of the Wiener spectrum, which has been defined only for stationary processes, and of spatial filtering, in which all regions on the hologram plane or on the image plane are affected equally. One technique, which is used widely in the study of granularity, is to impose stationarity by assuming spatially uniform exposure and processing of the emulsion, which is assumed to be infinite in extent. An alternative scheme, used here, spares the analyst who wishes to use stationary-process concepts from having to neglect the pattern recorded on the emulsion.

In effect, one manufactures a stationary process, U' let us say, from a nonstationary process U first by suppressing all absolute position references so that U is free to drift at random over the entire plane. One then takes expectations such that $\phi_{U'U'}$, the autocorrelation function of the manufactured process, is the inverse Fourier transform of $\tilde{\phi}_{U'U'}$ which is $(2\pi)^{-2}$ times the expectation of the absolute square of the Fourier transform of the original nonstationary process. The Wiener spectrum γ_{UU} of the nonstationary process is then defined as being the Wiener spectrum of the manufactured process, $\tilde{\phi}_{U'U'}$. The same technique can be used for cross-correlation functions $\phi_{U'V'}$ and Wiener cross spectra $\gamma_{UV} = \gamma_{U'V'}$, with the U and V set free to drift at random over the plane but kept rigidly fixed relative to each other.

From the preceding discussion it follows that the Wiener spectrum γ_{YY} of the modification random process Y is defined such that

$$\gamma_{YY}(\boldsymbol{\omega}) = (2\pi)^{-2}E[|\tilde{Y}(\boldsymbol{\omega})|^2]. \tag{12}$$

Similarly, for the individual grain cloud, one has

$$\gamma_{ww}(\boldsymbol{\omega}) = (2\pi)^{-2}E_A[|\tilde{w}(\boldsymbol{\omega}; A)|^2]. \tag{13}$$

Let the object whose hologram is being recorded be described by an ordered set of parameters o, the value of an ordered set of random variables O. Then the expected silver-density pattern $\sigma(\cdot; o)$ is a sample function of the random process $\Sigma(\cdot; O)$. The Wiener spectrum of the expected silver-density process Σ is therefore defined such that

$$\gamma_{\Sigma\Sigma}(\boldsymbol{\omega}) = (2\pi)^{-2}E_O[|\tilde{\Sigma}(\boldsymbol{\omega}; O)|^2]. \tag{14}$$

Here the tilde over the symbol for a function indicates its Fourier transform, and $\omega = (\omega_x, \omega_y)$ designates a point in the spatial-frequency plane. The expectations are taken over all relevant random variables. Following Rice,[3] it can be shown that the Wiener spectrum of the emulsion modification pattern satisfies the equation

$$\gamma_{YY}(\omega) = \bar{n}\gamma_{ww}(\omega) + \bar{n}^2[\gamma_{\Sigma\Sigma}(\omega)/\gamma_{\Sigma\Sigma}(0)]\gamma_{ww}(\omega), \quad (15)$$

where $\bar{n}$ is the expected number of grain clouds making up the hologram.

An intuitive explanation for the form of Eq. (15) is not difficult to provide. Since the properties of the individual grain clouds (in particular, their locations on the hologram) are assumed to be statistically independent, the Wiener spectrum of the exposure-pattern-independent (i.e., noise) contributions of the aggregate of grain clouds may be expected to be just $\bar{n}$ times the Wiener spectrum of a single cloud. Thus, the contributions of the individual grain clouds to the noise content of the hologram add incoherently.

On the other hand, the exposure-pattern-dependent (i.e., signal) contributions of the individual grain clouds to the Wiener spectrum of the hologram add coherently, hence the factor $\bar{n}^2$. Moreover, the signal component of the emulsion-modification pattern of the hologram is provided by the grain clouds, not by the silver-density process Σ. Consequently, the contribution of the silver-density process enters the expression in a normalized fashion, as $\gamma_{\Sigma\Sigma}(\omega)/\gamma_{\Sigma\Sigma}(0)$. In this way, only the *form* of Σ affects the Wiener spectrum of the emulsion-modification pattern.

Additional random variations in the hologram plate that are independent of the locations and other properties of the grain clouds can be accounted for as additional additive or multiplicative noise.

Neglecting these additional noise components, it is easy to see from Eq. (15) that the signal-to-noise ratio of a hologram can be defined by the equation

$$(S/N) = \bar{n}\,\frac{\int [\gamma_{\Sigma\Sigma}(\omega)/\gamma_{\Sigma\Sigma}(0)]\gamma_{ww}(\omega)\, d^2(\omega)}{\int \gamma_{ww}(\omega)\, d^2(\omega)}. \quad (16)$$

Following the rule that maximization of (S/N) results in optimized performance, one is led to conclude that 1) the more grains the better and 2) if $\omega = \omega_{\max}$ is the spatial frequency for which $\gamma_{\Sigma\Sigma}(\omega)$ is maximum, then the optimal grain clouds have Wiener spectra that look like delta functions centered at $\omega_{\max}$. The first conclusion seems reasonable enough; the second is absurd. Clearly, a signal-to-noise ratio criterion used by itself is not adequate for optimizing a holographic system.

Considering the granularity stage alone, one could reasonably desire that the output pattern y represent the input silver-density pattern σ with minimum mean-squared-error (MSE). The remainder of this paper deals with the determination of grain-cloud parameters to meet the minimum MSE condition.

The granular properties of a holographic plate are approximately uniform over the plate. It is reasonable, therefore, to define J, the MSE, in terms of the manufactured (primed) stationary processes introduced earlier in this section. Thus J is defined in accordance with the equation

$$J = \mathrm{E}[|Y'(\mathbf{r}) - a\Sigma'(\mathbf{r})|^2] \text{ for all } \mathbf{r}. \quad (17)$$

The minimization of J can be accomplished through the use of Wiener filter theory.[13,14]

Let a spatial-frequency filter be inserted following the granularity stage in the model diagrammed in Fig. 3. The input to this filter is considered to be a sample function of the manufactured process Y'. Let the value of the transfer function of the filter at the spatial frequency ω be $g(\omega)$ and let the filter-output stationary process be designated $a\hat{\Sigma}'$. Now, let the filter be of the Wiener type, designated such that $\mathrm{E}[|a\hat{\Sigma}'(\mathbf{r}) - a\Sigma'(\mathbf{r})|^2]$ is minimum for all $\mathbf{r}$. This implies that the transfer function of the filter must satisfy the condition

$$g(\omega) = \gamma_{a\Sigma Y}(\omega)/\gamma_{YY}(\omega), \quad (18)$$

where $\gamma_{a\Sigma Y}$ is the Wiener cross-spectrum of $a\Sigma$ and Y. The Wiener cross-spectrum is defined such that

$$\gamma_{a\Sigma Y}(\omega) = (2\pi)^{-2}\mathrm{E}[a\tilde{\Sigma}^*(\omega)\,\tilde{Y}(\omega)]. \quad (19)$$

As with Eq. (15), it can be shown, following Rice,[3] that the Wiener cross-spectrum satisfies the equation

$$\gamma_{a\Sigma Y}(\omega) = (2\pi)^{-2}a\bar{n}\mathrm{E}_A[\tilde{w}(\omega;\, A)]\mathrm{E}_O[|\tilde{\Sigma}(\omega;\, O)|^2/\tilde{\Sigma}(0;\, O)]. \quad (20)$$

Since the task of minimizing the MSE has been assigned to the grain clouds rather than to a separate filter, the Wiener-filter optimality condition must be reformulated such that the best filter is no filter at all; that is, Eq. (18) must be rewritten as

$$\gamma_{YY}(\omega) = \gamma_{a\Sigma Y}(\omega). \quad (21)$$

It is convenient to assume that the total mass of developed silver, as well as the number of developed-silver grains, is negligibly dependent on the set of object parameters o. Then the quantity $\tilde{\Sigma}(0;\, O)$ can be removed from the expectation brackets and rewritten as $\tilde{\sigma}(0) = \int \sigma(\mathbf{r})d^2\mathbf{r}$. Consequently, by Eqs. (14), (15) and (20), Eq. (21) can be written as

$$\frac{\gamma_{ww}(\omega)}{\mathrm{E}_A[\tilde{w}(\omega;\, A)]} = \frac{a}{\tilde{\sigma}(0)}\,\frac{\gamma_{\Sigma\Sigma}(0)\gamma_{\Sigma\Sigma}(\omega)}{\gamma_{\Sigma\Sigma}(0) + \bar{n}\gamma_{\Sigma\Sigma}(\omega)}. \quad (22)$$

This is the desired result. If it were not for the statistical variations from grain cloud to grain cloud, the left-hand side of Eq. (22) would be proportional to the transfer function of the granularity stage.

The right-hand side of Eq. (22) reduces to a constant, $(a/\bar{n})\gamma_{\Sigma\Sigma}(0)/\bar{\sigma}(0)$, for all spatial frequencies ω for which $\gamma_{\Sigma\Sigma}(\omega)/\gamma_{\Sigma\Sigma}(0) \gg 1/\bar{n}$. Since typically $\bar{n} \approx 10^7$ or more, this condition may be expected to hold for almost all spatial frequencies of interest, except when extreme care is taken to ensure that the zeroth- and first-order images are kept separate. When the inequality signs are reversed, the right-hand side of Eq. (22) reduces to $[a/\bar{\sigma}(0)]\gamma_{\Sigma\Sigma}(\omega)$. Thus, for example, if $\tilde{\Sigma}(\omega)$ tends to decay as ω^{-1} at high spatial frequencies $[\omega = (\omega_x^2 + \omega_y^2)^{\frac{1}{2}}]$, then $\tilde{w}(\omega)$ should tend to decay as ω^{-2}.

When the grain clouds are optimized such that Eq. (21) is satisfied, the MSE is a minimum and is equal to

$$(2\pi)^{-2} \int \{a^2\gamma_{\Sigma\Sigma}(\omega) - \gamma_{YY}(\omega)\}\, d^2\omega. \qquad (23)$$

By Eqs. (14), (15), (20) and (21), this expression reduces to

$$(2\pi)^{-2} \int \gamma_{\Sigma\Sigma}(\omega)\left\{1 - \frac{\gamma_{\Sigma\Sigma}(0)}{\bar{\sigma}^2(0)} \times \frac{\mathrm{E}_A[\tilde{w}(0;\, A)]}{\gamma_{ww}(0)} \times \mathrm{E}_A[\tilde{w}(\omega;\, A)]\right\} d^2\omega. \qquad (24)$$

The frequency-domain properties of real grain clouds obtained with commercially available emulsions and processes may be inferred from experiments reported by Smith.[15]

Summary

Kelly's tandem-stage approach to the formulation of a mathematical model of photographic information recording has been modified for phase holography and extended to include granularity effects. Following Kelly, the first three stages—optical diffusion, sensitometry and chemical diffusion—have been formulated in terms of nonstochastic functions. In contrast to Kelly's treatment, however, the emulsion has been taken as being exposed to coherent light, with u being the total (object wave plus reference wave) incident complex-amplitude pattern. In addition, whereas Kelly had described chemical diffusion as a linear operation on the optical-density pattern, in the present work the formulation has been generalized to account for recent findings of nonlinearity,[11] and the stage has been assumed to operate on the silver-density pattern. Effects of granularity have been discussed in terms of the theory of nonhomogeneous filtered Poisson processes, on the strength of the observed variation of Selwyn granularity with density, an experimental result reported by Kelly, widely-held conceptual considerations (grains as carriers as well as corrupters of information), and the plausibility of a set of five assumptions which have been shown elsewhere to be sufficient for the theory to hold. The granularity theory has been formulated in such a way as to make use of the valuable frequency-domain concepts of stationary-process theory without forcing the analyst to assume a uniform, featureless exposure pattern. As examples of its versatility, the granularity theory has been used 1) to define the signal-to-noise ratio of a phase hologram and 2) to determine the optimal frequency-domain characteristics of a grain cloud in the bleached emulsion.

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