D. Gabor

Laser Speckle and Its Elimination

Abstract: "Objective" speckle arises from the uneven illumination of an object with a multiplicity of waves that interfere at its surface. "Subjective" speckle arises at rough objects even if they are illuminated evenly by a single wave. The noise in the image is caused by the interference of the point-figures, which have random phases. Subjective speckle cannot be reduced except by extending the aperture. On the other hand the "objective" speckle in a plane, for instance in the plane of a transparency, can be reduced, and in the limit made invisible, by a special type of wide-angle illumination. This consists of a one-parameter family of plane waves, which can be produced by diffraction at a special grating, or two crossed gratings, close to the object plane. This makes it possible to produce multiple holograms, with the same insensitivity to dust or scratches as diffused holograms, but without any visible speckle in the reconstruction.

Introduction

Laser speckle noise is a direct consequence of the high coherence of laser light and has been long recognized as the Enemy Number One of holography. When a hologram is taken of a transparency illuminated with a single plane or spherical wave nothing is lost of the information content. The resolution is given by the angle subtended by the hologram as seen from the object. But the reconstructed image of the object is marred by the schlieren of the optical system; every speck of lens cement, every particle of dust shows up as a system of interference fringes. Only lensless Fourier holograms are free of the schlieren, but here, too, dust or scratches on the hologram mar the reconstruction. On the other hand, a hologram taken of a transparency in diffused illumination does not show up the schlieren, and the reconstruction is highly insensitive to dust or scratches in the hologram, but the resolution is very strongly reduced by speckle noise.

I wish to distinguish between two types of laser speckle, "objective" and "subjective." "Objective" laser speckle arises from uneven illumination of the object; it is really *there*, and a photographic emulsion spread over the surface of the object would show it up. Even a perfect optical system cannot do better than to reproduce it exactly. On the other hand, "subjective" speckle arises in the case of an evenly illuminated rough object, by the imperfection of the optical reproduction, whether this is produced directly or via a hologram. It was first described, almost as soon as lasers had become available, by Oliver[1] and by Rigden and Gordon,[2] as the "sparkly" or "granular" appearance of uniformly illuminated rough surfaces. These authors gave also a correct qualitative explanation of the phenomenon. It arises by a diffraction effect at the receiving end or, more exactly, by the limitation of the amount of light admitted for image formation by the optical instrument or by the eye.

This limitation has two causes. The dimples and projections of a macroscopically rough object can act as small reflectors with a complicated system of narrow lobes, and only a part of these will be caught by the objective. This, however, is a small effect, negligible in most cases. It would be the same if the object were illuminated with almost monochromatic noncoherent light. The second effect is far more important. The dimples and projections, even if they are microscopically small (so that by themselves they emit almost spherical waves), give randomly distributed phases to these wavelets, and their interference produces a strong noise in the image and spoils the resolution, even if the optical system has sufficient aperture for good imaging in incoherent light. A full mathematical theory of this "subjective" speckle was given by Enloe.[3]

Enloe's mathematical results have a simple interpretation, which I owe to unpublished notes of G. W. Stroke. *An objective that is good enough for incoherent light is not good enough for coherent light.* In incoherent light the absolute squares of the amplitude point-spread values are summed. These are always positive, and they decrease sharply with the distance from the geometrical point-image; in the case of a round aperture, with the 3rd power. But the amplitudes themselves decrease only with the 3/2 power, and they are positive or negative. If there are other points near the one considered, the amplitudes sometimes add, sometimes subtract, and the result is strong speckle noise. This becomes even stronger if the aperture is a thin annulus. In this case the intensity

The author is a Staff Scientist at the CBS Laboratories, Stamford, Connecticut.

falls off with the first power of the distance, the amplitude only with the one-half power, and photographs taken through such an aperture in coherent light lose all likeness to the object.

As Enloe has recognized, the only remedy to subjective speckle is to widen the aperture. I will show later that there is no escaping from this conclusion, because if by any a posteriori manipulation we could correct the subjective speckle, we could break through the "information barrier."

On the other hand, I will show that the "objective" speckle, which arises in the diffuse illumination of plane transparencies, can be completely eliminated by a special type of illumination that preserves the advantages of multiple holograms: the almost uniform distribution of information and the consequent insensitivity of the reconstruction to dust or scratches. Gerritsen, Hannan and Ramberg,[4] and before them Upatnieks,[5] have already made progress in this direction by using certain phase-gratings in the plane of the object. I will show that in principle one can approach the advantages of uniform distribution almost beyond any limit, though the realization may require considerable experimental skill.

One-dimensional speckle

For simplicity consider first N plane waves striking a plane $z = 0$, with their wave normals all in the x-z plane. Their interference phenomenon in the zero plane is the "speckle." In the usual small-angle approximation we would obtain exactly the same result with spherical waves of equal curvature; that is to say, the same effect would occur if the waves originated not at infinity but at any $z =$ constant plane, because these too would give straight interference fringes in the y-direction.

The resulting amplitude of these waves is

$$A(x) = \sum_{n-1}^{N} A_n \exp [i(k_n x - \omega t)], \tag{1}$$

where the A_n are complex.

The resulting local intensity is

$$\begin{aligned} I(x) &= A(x) A^*(x) \\ &= \sum_n \sum_m A_n A_m^* \exp [i(k_n - k_m)x]. \end{aligned} \tag{2}$$

The energy collected in an interval X, centered on x_0 is

$$\begin{aligned} E(X, x_0) &= \int_{x_0 - \frac{1}{2}X}^{x_0 + \frac{1}{2}X} I(x)\, dx \\ &= X \sum_n \sum_m A_n A_m^* [\operatorname{sinc} \tfrac{1}{2}(k_n - k_m) X] \\ &\quad \times \exp [i(k_n - k_m)x_0]. \end{aligned} \tag{3}$$

As is usual in statistical problems, we are asking first for the mean-square fluctuation of this quantity, which will give us a sufficient insight into the problem. We must therefore calculate

$$\langle [E(X, x_0) - \langle E \rangle]^2 \rangle = \langle E^2(X) \rangle - \langle E \rangle^2,$$

where the averaging is over the x_0, at given X. We first calculate the square of Eq. (3), which is

$$\begin{aligned} E^2(X, x_0) &= \sum_n \sum_m \sum_{n'} \sum_{m'} A_n A_m^* A_{n'} A_{m'}^* \\ &\quad \times [\operatorname{sinc} \tfrac{1}{2}(k_n - k_m) X][\operatorname{sinc} \tfrac{1}{2}(k_{n'} - k_{m'}) X] \\ &\quad \times \exp [i(k_n - k_m - k_{n'} + k_{m'})x_0]. \end{aligned} \tag{4}$$

Averaging this over x_0 by integrating over the whole plane turns the last factor into a delta function, which is zero except for

$$|k_n - k_m| = |k_{n'} - k_{m'}|. \tag{5}$$

The condition given by Eq. (5) is automatically satisfied for identical pairs n,m, but it is also satisfied whenever two intervals between wave numbers coincide so that they give the same fringe spacing. This latter I call *degeneracy*. If it occurs, it can make the speckle much worse, but, as will be shown later, it can also make it much better.

We can now write the average of Eq. (4) in the form

$$\begin{aligned} \langle E^2(X) \rangle &= X^2 (\sum_n I_n)^2 + X^2 \sum_{n \neq m} \sum_m I_n I_m (1 + g_{mn}) \\ &\quad \times \operatorname{sinc}^2 \tfrac{1}{2}(k_n - k_m) X, \end{aligned} \tag{6}$$

where $I_n = A_n A_n^*$ is the intensity of the nth beam by itself, and g_{mn} is the degeneracy factor of the pair n,m:

$$g_{nm} = (A_n A_m^* \sum_{n'} \sum_{m'} A_{n'} A_m^*) / I_n I_m, \qquad (n', m' \neq n, m). \tag{7}$$

The summation here has to be carried out only over those n',m' pairs that are *not* identical with n,m but for which $|k_n - k_m| = |k_{n'} - k_{m'}|$.

We now obtain the relative mean-square energy fluctuation in intervals of length X in the form

$$\begin{aligned} [\langle E^2 \rangle - \langle E \rangle^2] / \langle E \rangle^2 &= (\sum_n I_n)^{-2} \sum_{n \neq m} \sum_m I_n I_m (1 + g_{nm}) \\ &\quad \times \operatorname{sinc}^2 \tfrac{1}{2}(k_n - k_m) X. \end{aligned} \tag{8}$$

For $X = 0$ the sinc^2 factor becomes unity, and Eq. (8) assumes its maximum value, which is

$$\sum_{n \neq m} \sum_m I_n I_m (1 + g_{nm}) / (\sum_n I_n)^2.$$

In the absence of degeneracy (i.e., if $g_{nm} = 0$) for N waves of equal intensity this value is given by $(N - 1)/N$ and, for many waves, it approaches the value unity. This

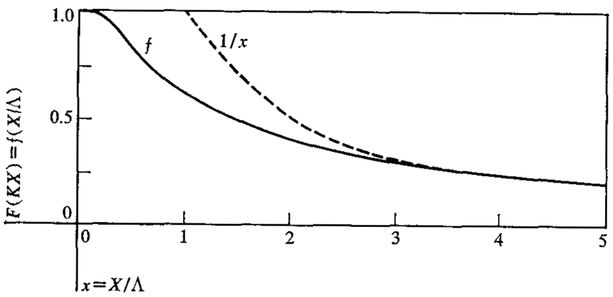

Figure 1 Relative mean square fluctuation of intensity in coherent light as a function of the sample interval X, in one dimension, with random illumination uniform within an angle $(\lambda/2\pi)K$.

happens to be exactly the value for the mean-square fluctuation in a simple fringe system with intensity $\cos^2 Kx$.

Extension to two dimensions

If the same calculation is carried out for two dimensions, for waves characterized by wave numbers k_{n_x} and k_{n_y}, nothing essential changes, except that for degeneracy we have now a double condition

$$|k_{n_x} - k_{m_x}| = |k_{n'_x} - k_{m'_x}|, \text{ and}$$

$$|k_{n_y} - k_{m_y}| = |k_{n'_y} - k_{m'_y}|, (n', m' \neq n, m),$$

which means that the fringes produced by two different pairs must coincide not only in spacing, but also in direction. This, evidently, will be very rarely the case with frosted glass and the like; hence, degeneracy can be neglected except for artificially produced diffusers, such as will be discussed later. Therefore, in the two-dimensional case, the relative mean-square energy fluctuation over areas XY will be simply the product of two expressions of the form (8), with $g_{mn} = 0$.

Application to a random scatterer

We now consider a plane illuminated by a rectangular frosted glass or the like, so that in the area of interest the waves are *uniformly* distributed in an angular range $0 < k < K$ or between $\pm\frac{1}{2}K$, both in the x and y directions. According to the argument given in the last section we can neglect the degeneracy and carry out the calculation in one dimension, taking its square in the end. As now we deal with (practically) a continuum of waves, we can replace the sum in Eq. (8) by an integral, and there is no need to exclude the case $n = m$, as its contribution is vanishing. We have therefore to calculate the x-factor of the relative mean-square fluctuation, which is

$$F(KX) = \frac{\int_0^K \int_0^K [\mathrm{sinc}^2 (k_n - k_m)X]\, dk_n\, dk_m}{\int_0^K \int_0^K dk_n\, dk_m}. \tag{9}$$

We introduce the new variables $x = \frac{1}{2}(k_n + k_m)X$, $y = \frac{1}{2}(k_n - k_m)X$. The integrand is a function of y alone; moreover it is even in y. We can therefore use the formula that replaces integration over a square by integration over two triangles:

$$X^2 \int_0^K \int_0^K f(k_n, k_m)\, dk_n\, dk_m = 4 \int_0^{\frac{1}{2}KX} f(y)\, dy \int_0^{KX-2y} dx,$$

which gives

$$\begin{aligned} F(KX) &= 4(KX)^{-2} \int_0^{\frac{1}{2}KX} \mathrm{sinc}^2\, y\, dy \int_0^{KX-2y} dx \\ &= (\tfrac{1}{2}KX)^{-2}\Big[KX \int_0^{\frac{1}{2}KX} \mathrm{sinc}^2\, y\, dy \\ &\quad - 2 \int_0^{\frac{1}{2}KX} (\sin^2 y/y)\, dy \Big] \qquad (10) \\ &= (\tfrac{1}{2}KX)^{-2}\{ KX[\mathrm{Si}\,(KX)] \\ &\quad - \mathrm{Cin}\,(KX) - 2\sin^2 \tfrac{1}{2}KX\}, \qquad (11) \end{aligned}$$

where Si is the sine integral, and

$$\mathrm{Cin}\, z = \int_0^z [(1 - \cos t)/t]\, dt,$$

a function tabulated, for instance, by Abramovitz and Stegun.[6] This function is plotted in Fig. 1 in units of the smallest interference-fringe spacing $\Lambda = 2\pi/K$ which arises in the range of illuminating waves 0 to K or, $-\frac{1}{2}K$ to $+\frac{1}{2}K$.

The essential result is very simple. Beyond about 4 or 5 such fringe spacings the function $F(KX)$ assumes the asymptotic form, for one dimension,

$$F(KX) \sim 2\pi/KX = \Lambda/X, \tag{12}$$

and in two dimensions the mean-square fluctuation is the square of this. (This result has been already obtained by Enloe[3] and by Gerritsen, Hannan and Ramberg.[4])

The significance of this result is explained in Fig. 2. The object plane is illuminated by a square diffuser in the angular range $K = (2\pi/\lambda)(\sin\phi_{\max} - \phi_{\min})$. Consider a square area in this plane, $X'X'$. An objective picks up an angular range K', and images the area into $X''X''$. As KX is an invariant, $K'X' = K''X''$. We can consider Λ, the fringe spacing of the extreme waves admitted by the objective, as the resolution limit. (The factor 0.5 or 0.6 that is usually added is somewhat unreal, as it gives the extreme limit of discriminability.) We can then express the result as follows.

Random illumination within a certain solid angle (square in our case, but evidently it would not make much dif-

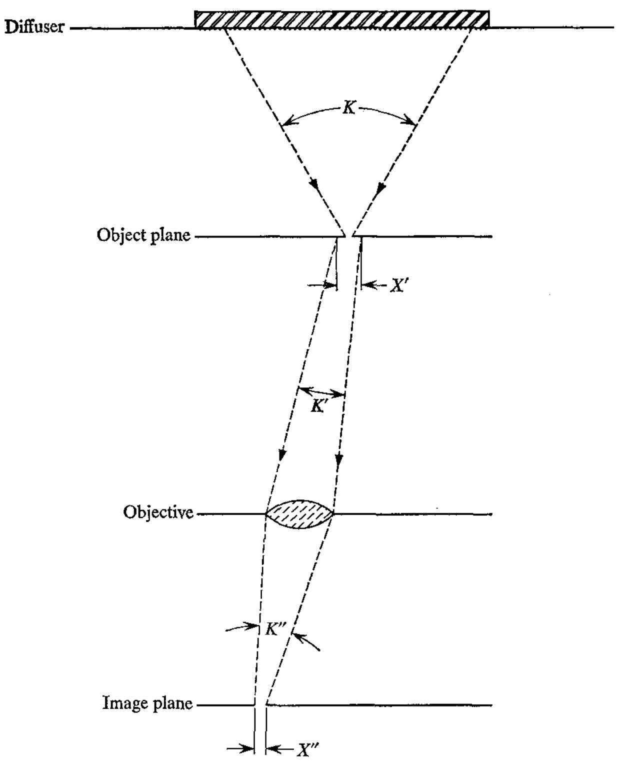

Figure 2 Interpretation of the law of relative intensity fluctuation. The mean square intensity fluctuation is a function of the invariant $K'X' = K''X''$.

ference if it were round) *with coherent light produces a relative mean-square speckle noise that is the inverse of the resolution elements contained in the sampled areas.*

This becomes even more illuminating if we consider that experience shows that in a picture of good television or photographic standards the root-mean-square noise per resolved element must be 20 dB down on the signal, and the mean-square noise by 40 dB, that is to say, by a factor of 10^4. This means that the "sufficiently noiseless" elementary area must contain 10^4 elements which would be otherwise resolvable; hence, *random laser speckle spoils the linear resolution by about two orders of magnitude.* (A little less in the case of moving pictures.)

Eliminating laser speckle in the imaging of plane objects

Diffused holograms have great advantages over holograms taken with regular (plane or spherical) illumination. They have made it possible to view holograms of three-dimensional objects with two unaided eyes, and the information in them is *redundantly* recorded, hence the reconstruction is insensitive to dirt or scratches in the hologram. One has, of course, to pay for this redundancy by imperfect utilization of the photographic emulsion, but a part of this loss is regained by the fact that the entropy of a diffused hologram comes near to the theoretical maximum. The entropy of diffused holograms is probably 10 times that of television pictures and possibly 50 times that of printed matter.* Can we not realize at least a part of this gain without the loss caused by the speckle?

A diffused hologram can be interpreted as a random repetition of a basic pattern. I will show that at least in the case of plane objects we can eliminate the speckle by the basic holograms distributed not at random, but in a regular lattice, with approximately equal intensities, provided that certain phase conditions are accurately observed.†

The key to speckle-free illumination is the degeneracy factor g_{nm}. In order to make good use of it, we make all wave number intervals k equal. Taking first again the one-dimensional case, let us illuminate the object plane with N equally spaced waves with complex amplitudes a_n, and a resulting amplitude

$$A(x) = \sum_n a_n \exp(inkx). \tag{13}$$

The total width $(N-1)k = K$ must be fitted to the aperture of the optical lens system or to the size of the hologram.

The resulting intensity is

$$I(x) = AA^* = \sum_n \sum_m a_n a_m^* \exp[i(n-m)kx]. \tag{14}$$

This contains interference fringes with wave numbers from k to $(N-1)k$. We can now annul all these fringes, with the exception of the last (finest) system, which arises by the interference of the marginal waves. These fringes are innocuous if they are finer than the finest detail we want to resolve. The conditions for speckle-free illumination are, therefore,

$$\begin{aligned} a_1a_2^*+a_2a_3^*+a_3a_4^*+\cdots+a_{N-2}a_{N-1}^*+a_{N-1}a_N^*&=0\\ a_1a_3^*+a_2a_4^*+\cdots+a_{N-3}a_{N-1}^*+a_{N-2}a_N^*&=0\\ &\vdots\\ a_1a_{N-1}^*+a_2a_N^*&=0. \end{aligned} \tag{15}$$

These are $N-2$ equations for the $N-1$ essential amplitudes a_n. For practical applications the symmetrical solutions are of interest, with N odd, so that n runs from $-\frac{1}{2}(N-1)$ to $\frac{1}{2}(N+1)$. For $N = 3$ and 5 the solutions

* In fact both by holography with skew reference beams at various angles of incidence[7] and by "carrier-frequency photography," i.e., by placing a line screen on top of the emulsion and giving it various orientations, one can increase the entropy of the record to such an extent that up to 60 good line-drawing pictures can be recorded without appreciable crosstalk.[8]

† The ninefold repetition of the hologram, as used with good results by Gerritsen, Hannan and Ramberg[4] can be considered as a first step in this direction.

Table 1 Amplitudes for speckle-free illumination for several values of N.

N	$-a_4$	$-a_3$	$-a_2$	$-a_1$	a_0	a_1	a_2	a_3	a_4
1					1				
3				1	ip	1			
5			1	ip	$-\frac{1}{2}p^2$	ip	1		
7		1	ip	$-\frac{1}{2}p^2$	$-\frac{1}{8}ip(p^2-4)$	$-\frac{1}{2}p^2$	ip	1	
9	1	ip	$-\frac{1}{2}p^2$	$-\frac{1}{8}ip(p^2-4)$	$-\frac{1}{64}(p^4-24p^2+16)$	$-\frac{1}{8}ip(p^2-4)$	$-\frac{1}{2}p^2$	ip	1

p is a real parameter with arbitrary value, assigned to even out the absolute values of amplitude. $p = \frac{5}{4}$ is a suitable value.

are easily found (see Table 1). This suggests how to proceed to larger N. In progressing from N to $N+2$ only one new amplitude has to be calculated, the new a_0, which results from the previous a_0 and a_1 from the rule

$$new\ a_0 = -a_0(a_0a_0^* - a_1a_1^*)/(a_0a_1^* - a_0^*a_1).$$

How does one produce such a wave complex? The obvious solution is to make a filter so that on illuminating it with a plane wave the diffracted light shall have the amplitudes and phases we have calculated. This means simply taking a filter with the amplitude transmission function

$$t(x) = A(x) = \sum_n a_n \exp(inkx).$$

To realize such a filter we must represent $t(x) = A(x)$ in the form $R \exp(i\phi)$, so that it consists of a pure amplitude filter with transmission R, and a pure phase filter with phase shift $\phi(x)$.

Taking into account the Eqs. (15), we obtain for the amplitude filter the transmission

$$\begin{aligned} t_1(x) &= (AA^*)^{\frac{1}{2}} \\ &= [(\sum_n a_na_n^*) + 2a_1a_N^* \cos(N-1)kx]^{\frac{1}{2}} \qquad (16) \end{aligned}$$

because all other fringes have dropped out, except the marginal ones. Only these will be visible in the object plane.

For the pure phase filter we obtain the transmission

$$\begin{aligned} t_2(x) &= \exp[i\phi(x)] \\ &= \sum_n a_n \exp(ikx)/[(\sum_n a_na_n^*) \\ &\quad + 2a_1a_N^* \cos(N-1)kx]^{\frac{1}{2}}. \qquad (17) \end{aligned}$$

We can now cross two such filters at right angles, and obtain one which will produce N^2 overlapping holograms in a regular lattice. Detailed calculation shows that the transmittance of this system will again consist of the dc terms plus the two marginal fringe systems at right angles, plus two new systems $\cos(N-1)k(x+y)$ and $\cos(N-1)k(x-y)$, at 45° to the main axes. These are also harmless, as their spacing is below the resolution limit.

These rules are valid if the filter is in contact with the object plane. It would not be difficult to put the amplitude transmission $t_1(x)t_1(y)$ into this plane, but the two crossed phase filters may have to be some distance away. In this case a correction will have to be applied to the phases by extrapolating the waves to the filter plane.

I would not recommend producing the phase filter photographically. It is doubtful whether the required accuracy could be obtained. I would rather recommend computing the profile $\phi(x)$, drawing it out on a large scale, reducing it to the right size and producing by some photomechanical process a tool with which the grating can be ruled, in a material such as a transparent plastic.

Subjective speckle

"Objective" speckle can be eliminated by a special type of illumination, which at the same time retains the main advantage of diffuse holograms: redundant information at high entropy; that is to say, the "noiselike" aspect of the diffuse hologram is retained. The situation is different with what I have called "subjective" speckle, i.e. the noise and distortion arising from insufficient apertures.

The reason is that, if we could achieve this, we could break through the "information barrier" of optical transmission. This may be illustrated with an example. Goodman[9] has recently drawn attention to an interesting paradox. We take an aperture consisting of N small holes. These can be distributed in such a way that they have $\frac{1}{2}N(N-1)$ different spacings, approximately evenly distributed over different directions. Each *pair* gives a Young diffraction pattern with a distinct Fourier component; hence, we have in the image $\frac{1}{2}N(N-1)$ data. How can we transmit $\frac{1}{2}N(N-1)$ data with N point-apertures?

The answer is that the experiment would succeed with incoherent light, but not with coherent light. The intensity of the Fourier component corresponding to one pair is the geometrical mean of the intensities at the two point-apertures, say A and B, multiplied by the coherence factor γ_{AB}, which is a function of *two* variables. Hence if we know γ_{AB} and γ_{AC} we cannot predict γ_{BC}. But in coherent light γ_{AB} = cos (A, B) is the function of *one* variable only, the phase; hence, γ_{BC} = cos [(A, C) − (A, B)]. We need only determine the phases (for instance, with a hologram) in order to carry out this prediction. Hence the $\frac{1}{2}N(N-1)$ Fourier components carry in fact only N data of information.*

The frustration of such attempts by coherent light manifests itself in speckle. A set of N small apertures distributed at random over a circular disc is a fair approximation to Goodman's set. Experiments carried out in CBS Laboratories by F. Weindling have shown that 360 pinholes give a point-figure that appears acceptable. But as soon as one tries the experiment with, say, 10 to 20 image points close together the speckle becomes intolerable. This, incidentally, is an example of the general experience that "structural" information theory, which *counts* the degrees of freedom, can lead to very misleading results unless it is supplemented by "metrical" information theory, which takes account of the noise. Nature always finds a way for frustrating those who want to break through the information barrier!

* One can consider these remarks as a generalization of a result recently found by Toraldo di Francia[10] that in incoherent illumination the optical information capacity of an infinitely thin annulus is equal to that of the full disc. No optical method has been disclosed yet for achieving this, but in radio astronomy, Wild[11] has shown that N antennas, arranged in a ring, can produce an image with $\frac{1}{2}N^2$ independent points.

References

1. B. M. Oliver, "Sparkling Spots and Random Diffraction," *Proc. IEEE* **51**, 220 (1963).
2. J. D. Rigden and E. I. Gordon, "The Granularity of Scattered Optical Maser Light," *Proc. IRE* **50**, 2367 (1962).
3. L. H. Enloe, "Noise-like Structure in the Image of Diffusely Reflecting Objects in Coherent Illumination," *Bell System Tech. J.* **46**, 1479 (1967).
4. H. Gerritsen, W. Hannan and E. Ramberg, "Elimination of Speckle Noise in Holograms with Redundancy," *Appl. Opt.* **7**, 2301 (1968).
5. J. Upatnieks, "Improvement in Two-dimensional Image Quality in Coherent Optical Systems," *Appl. Opt.* **6**, 1905 (1967).
6. M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, Washington, 1964.
7. E. N. Leith and J. Upatnieks, *J. Opt. Soc. Am.* **53**, 1377 (1963).
8. F. Bestenreiner and R. Deml, "Trägerfrequenz-Photographie," *Optik* **28**, 263 (1968).
9. J. W. Goodman, "Three-dimensional Imaging with Incoherent Light," USA-Japan Seminar on Information Processing by Holography, to be published in *Applications of Holography*, E. Barrakette, Ed., Plenum Press, New York, 1970.
10. G. Toraldo di Francia, "Some Recent Progress in Classical Optics," Proc. of the Florence Inaugural Conf. of the European Physical Society, "The Growth Points of Physics," *Revista del Nuovo Cimento* **1**, special issue, 460 (1969).
11. J. P. Wild, "A New Method of Image Formation with Annular Apertures and an Application in Radio Astronomy," *Proc. Roy. Soc. A* **286**, 499 (1965); also, the *Culgoora Radioheliograph*, C.S.I.R.O., Sydney, Australia (1969).

Further bibliography on laser speckle

1. L. Allen and D. G. C. Jones, "An Analysis of the Granularity of Scattered Optical Laser Light," *Phys. Lett.* **7**, 321 (1963).
2. P. S. Considine, "Effects of Coherence on Imaging Systems," *J. Opt. Soc. Am.* **56**, 1001 (1966).
3. L. I. Goldfischer, "Auto-Correlation Function and Power Spectral Density of Laser Produced Speckle Patterns," *J. Opt. Soc. Am.* **55**, 247 (1965).
4. A. L. Jones, "On the Imaging of Small Diffuse Objects with Coherent Light," IBM TR 01.1131, Systems Development Division Laboratory, Endicott, New York 1968.
5. E. N. Leith and J. Upatnieks, "Imagery with Pseudo-Randomly Diffused Coherent Illumination," *Appl. Opt.* **7**, 2085 (1968).
6. T. J. Skinner, "Energy Considerations, Propagation in a Random Medium and Imaging in Scalar Coherence Theory," Thesis, Boston University, 1964.
7. T. Suzuki and R. Hioki, "Coherence of Light from Random Medium," *Japan. J. Appl. Phys.* **5**, 807 (1966).
8. T. Suzuki and R. Hioki, "Speckled Diffraction Pattern and Source Effect on Resolution Limit in Holography," *Japan. J. Appl. Phys.* **5**, 814 (1966).

Received January 6, 1970