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Incoherent Filtering Using Kinoforms

Abstract: Incoherent optical filtering with the kinoform used as a filtering element is discussed. Kinoform theory is briefly reviewed and initial results with "fan" and correlation filters are presented.

Introduction

The kinoform[1] is a computer-generated wavefront-reconstruction device that may be thought of as a complicated, computer-defined lens, whose surface (for an off-axis point image) is analogous to that of an accurately made blazed grating that diffracts light into a single diffraction order. The characteristics of the kinoform are such that it has potential for use as a filtering element in optical information processing systems. This paper presents the results of our initial experiments using the kinoform as a filter. We investigated two types: "fan" filters and correlation filters. Figure 1 shows the effect of a fan filter. The unfiltered information appears in Fig. 1(a). The photograph in Fig. 1(b) was taken using the same incoherent lighting and camera as for Fig. 1(a), but with a kinoform filter held in front of the camera lens. The result shown in Fig. 1(b) is the convolution of the impulse response function of the filter with the signal. Figure 1(c) shows the effect of rotating the same filter 40° about the axis normal to the kinoform plane. From this example the directional sensitivity of the fan filter is clearly apparent.

Kinoform filters are similar in many respects to hologram filters.[2] Like the holograms discussed by Armitage and Lohmann[3] for use in matched filtering, kinoform filters can be used in incoherent light, and thus have the possibility of wider application in optical data processing systems than filters requiring coherent light. Both incoherent filtering schemes mentioned above should be thought of in terms of convolutional operations since spatial filtering in the Fourier plane is an ill-defined concept in relation to incoherent light.[4] Holographic filters are inefficient in their use of available light and have a limited object field because of the overlapping of diffraction orders. As was first pointed out to us by Lohmann,[5] these disadvantages, characteristic of holograms, can be eliminated if kinoform filters are used. Kinoform filters are not limited to cross-correlation (matched) filters, but can represent any type of symmetric or asymmetric multichannel operator. However, unlike the hologram filters devised for use in coherent light, kinoform filters are limited to impulse response functions that are non-negative. This limitation implies that the kinoform filter must be designed with an impulse response function that gives the best approximation to the frequency response of the ideal bipolar filter.

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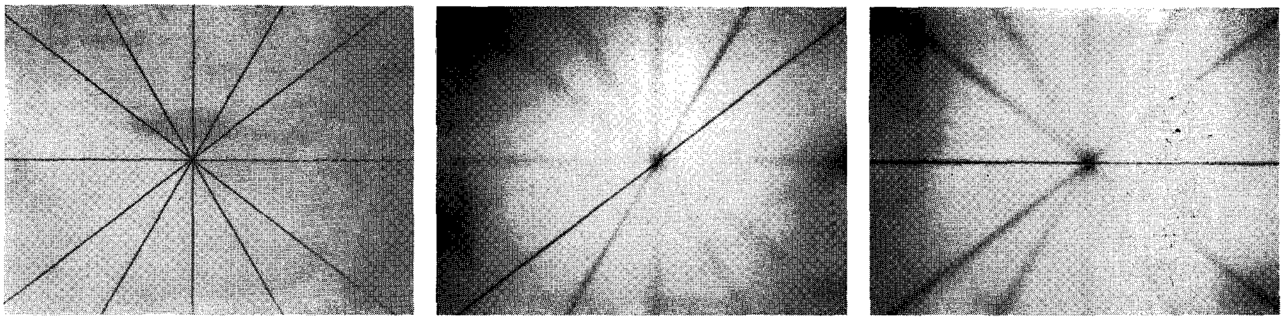
Kinoform filters

• *Kinoforms*

The wavefront reconstructed from a kinoform is the reproduction in the real (or virtual) image plane of a function $|L|^2$, which is a mathematical description of the intensity distribution of a diffusely illuminated object. To achieve accurate reproduction it is necessary to define $|L|^2$ in a manner which assures that the information about the distribution is spread over the whole kinoform plane. In calculating the distribution, $|L|^2$ is represented by a two- or three-dimensional array of point apertures.[5] Each aperture is assigned a complex transmittance L_j,

$$L_j = l_j \exp i\alpha_j \,, \tag{1}$$

where the subscript denotes the jth aperture. The values of l_j represent the transmissivity of the aperture; i.e., zero implies an opaque aperture and unity, a completely transparent one. The values of α_j give a model of the diffuse scattering from the object represented by $|L|^2$, and are chosen in such a way that the information will



(a) (b) (c)

Figure 1 Results of using a kinoform fan filter. (a) An unfiltered test pattern obtained with a view camera in incoherent light; (b) the filtered test pattern obtained by placing a kinoform fan filter over the lens of the camera; (c) the filtered test pattern with the filter rotated 40° about the axis normal to the filter plane.

be spread nearly uniformly throughout the kinoform plane. The filters discussed in this paper were made with two-dimensional arrays using a random number generator to assign values to the α_j.

The actual computations required to calculate the scattered wavefront at the kinoform plane have been described previously.[1,7] In the Fresnel region the wavefront is described by the function $\hat{L}$, the finite Fourier transform of L, if it is assumed that the kinoform will be reconstructed using a point source at the object plane. The wavefront $\hat{L}$ may be written in polar form as

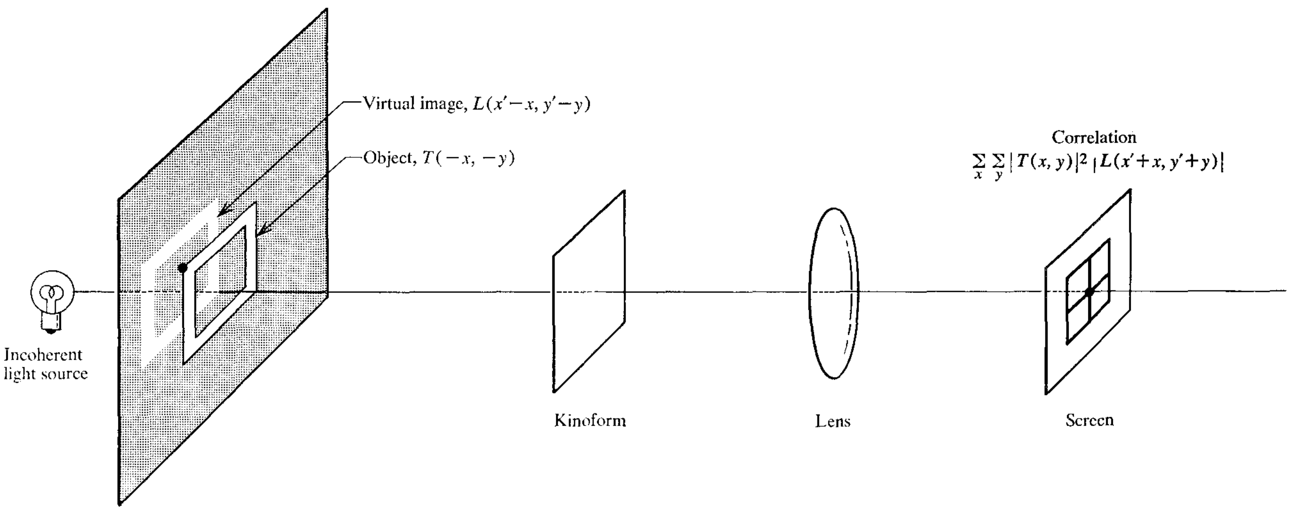
$$\hat{L}_k = A_k \exp i\phi_k , \qquad (2)$$

where the subscript k denotes the kth position in the kinoform plane, A_k is the real amplitude and ϕ_k is the phase. The kinoform technique assumes that the information about $|L|^2$ is sufficiently diffused over the kinoform plane that A_k is a constant A for all values of k. The calculated phase is reduced modulo 2π and plotted photographically using a 32-level grey scale. The large-scale plots are photoreduced to sizes appropriate for illumination with visible light. The photoreduction is etched to obtain a transparent relief image whose surface relief is such that light incident on a region of phase $\phi = 0$ is retarded by one wavelength relative to light incident on a region of $\phi = 2\pi$. The reconstructed image function $|L|^2$ obtained by illuminating the kinoform with light from a point source can be considered to be the impulse response function of a filter.

- *Incoherent filtering with kinoforms*

Coherent filtering with kinoforms is inhibited by the requirement of random phases for the wavelets from the discrete apertures. These random phases will "match" the phases in the wavefront from an object in only the most unusual of circumstances. To be generally effective, the

Figure 2 A kinoform filtering system.



kinoform is limited to applications in which the intensities $|L|^2$ are significant, and in which the phases ϕ_i are irrelevant. This situation is met when incoherent illumination of the object gives an intensity distribution $|L|^2$, and only time-averaged energy is recorded.

Figure 2 indicates schematically an incoherent kinoform filtering system. The object containing the data to be filtered is illuminated by monochromatic, temporally incoherent light. The object, described by the discrete amplitude function $T(-x, -y)$, can be thought of as being made up of many point sources, with each point source having a specified intensity $|T(-x, -y)|^2$ and a phase that varies randomly with time; i.e., the sources are incoherent. If $|L(x', y')|^2$ is the intensity at the point (x', y') of the virtual image, from the kinoform filter, produced by a point source of unit intensity at the point (0, 0), then a source $|T(-x, -y)|^2$ will produce a virtual image whose intensity is $|T(-x, -y)|^2|L(x - x', y - y')|^2$ at the point (x', y'). Since intensities (energy) are additive in incoherent light, the total intensity $G(x', y')$ at the point (x', y') is given by the expression

$$G(x', y') = \sum_x \sum_y |T(-x, -y)|^2 \times |L(x' - x, y' - y)|^2. \quad (3)$$

A lens (eye) can be used to project this virtual pattern onto a screen (retina), as indicated in Fig. 2. If the kinoform is reversed, the pattern formed is the correlation

$$G(x', y') = \sum_x \sum_y |T(x, y)|^2 \times |L(x' + x, y' + y)|^2. \quad (4)$$

Thus the basic kinoform filter can perform, in incoherent light, any mathematical operation that can be expressed as the correlation, or convolution, of two distributions which are everywhere non-negative.

- *The negative coefficient problem*

Bipolar filters are required if general optical processing is to be accomplished. One way in which "negative" intensities can be formed in a single kinoform filter is to introduce a bias level that may be a constant over the extent of the kinoform, or may be a variable dependent upon the spatial coordinates of the kinoform. Figure 3 indicates several ways in which negative intensities may be avoided.

The effect of compensation of negative coefficients (intensities) is distortion and reduction of contrast in the filtered image. For example, a fixed bias [Fig. 3(a)] may be introduced by forming the desired impulse response function $I(x, y)$ whose most negative coefficient has the value $-C$. Then

$$L(x, y) = [I(x, y) + C]^{\frac{1}{2}} \quad (5)$$

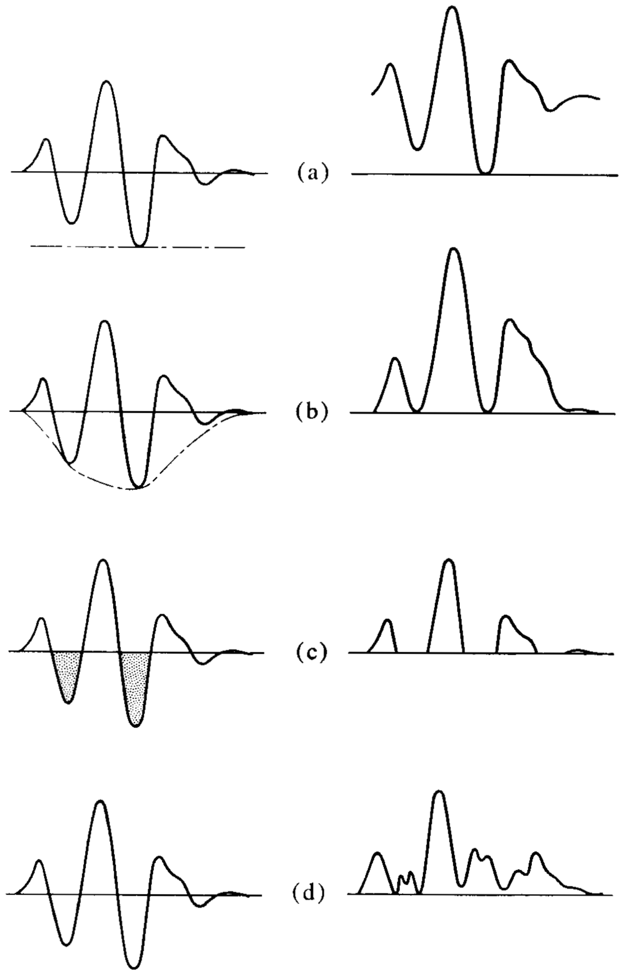


Figure 3 Four methods for approximating bipolar filters with filters whose impulse response is everywhere non-negative. In each case the left-hand curves show the filtered signal $L(x, y)$ and the operation performed on it, and the right-hand curve shows the result of the operation. (a) Addition of a constant bias; (b) addition of a smoothed curve; (c) zeroing of all negative amplitudes; (d) least-squares approximation of the desired frequency characteristics with the constraint that the impulse response be non-negative.

is everywhere non-negative. If the amplitude of the data to be filtered is given by $T(-x, -y)$, then the intensity distribution at the detector screen is (in the convolution mode)

$$G(x', y') = \sum_x \sum_y |T(x, y)|^2 \; |I(x' + x, y' + y)| + \sum_x \sum_y |T(x, y)|^2 \; C. \quad (6)$$

The first term is the desired convolution $|I|^2 * |T|^2$. The second term is a lower frequency term that appears as

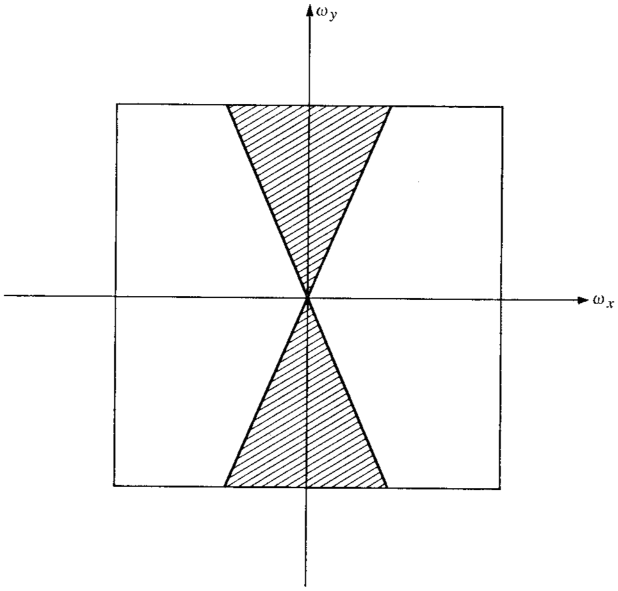


Figure 4 Effect of a fan filter in the frequency plane. The fan is designed to pass spatial frequencies in the shaded area.

a bias level in the filtered image. (It is interesting to note that other incoherent filtering techniques exist[8,9] in which "negative" intensities are formed, and that these experimental techniques also introduce such biases.) The bias is not distracting in some cases; however, it greatly reduces contrast in many important situations and further work is required to make the kinoform filtering method generally useful.

An optimized filter with all coefficients positive can be obtained by finding the filter that gives the best least-squares approximation, under the constraint of non-negative coefficients, to the frequency characteristics of the "perfect" filter that has positive and negative areas in its impulse response function.

Results

Initial results have been obtained with simple fan and correlation filters, and have been studied photographically. Figure 4 shows the desired effect of the fan filter in the spatial frequency domain. The effect, of course, is to pass signals in one spatial orientation, while rejecting signals in other spatial orientations. Fan filters have been applied to digital data, especially in the analysis of seismic data. The classical impulse response of the digital filters [Fig. 5(a)] presents alternate areas of positive and negative values. Figure 5(b) shows the impulse response function for a fan filter having a least-squares approximation to the frequency characteristics of the classical filter, but with the constraint of all non-negative values. Details on the construction of the filter can be found in Ref. 10.

The attenuation spectra in the frequency domain of the conventional and least-squares fan filters are shown in Figs. 6(a) and 6(b), respectively. At first glance, the behavior of the least-squares filter follows that of the classical one reasonably well, but in the areas of low spatial frequency the spectra differ. The least-squares filter cannot discriminate directionwise the low-frequency signals. The result is precisely that predicted by the simple arguments in the previous section: the filtered image is obscured by low-frequency components and, hence, contrast in the filtered image is reduced.

One example of the filtered images obtained with the least-squares fan filter is shown in Fig. 1. Figure 7 shows a second example. The signal, Fig. 7(a), is an entanglement of lines in which it is difficult to discern any intelligible image. The result of applying the fan filter is shown in Fig. 7(b).

Correlation or matched filtering operations are particularly suitable for kinoform filters since the impulse response function $|L|^2$ is always non-negative for these filters. The effect of a simple correlation filter is shown in Fig. 8. The letter B shown in Fig. 8(a) was used as the data $|T|^2$ and a kinoform filter was made with a B-shaped impulse response function $|L|^2$. The result of filtering $|T|^2$ with that filter is shown in Fig. 8(b), which clearly indicates the expected correlation pattern. The correlation obtained by reorienting the filter is shown in Fig. 8(c).

Discussion

It should be emphasized that kinoform filters, in common with all other physical filters, improve the signal-to-noise ratio in an image. In other words, a properly designed filter will suppress the noise and increase the visibility of the desired signal. Also in common with many other filter methods, kinoform filters cause a partial degradation of the signal. Furthermore, these filters are limited to, perhaps, 6-bit precision by the number of discrete grey levels that can be perceived by the human eye, a photographic plate or a television camera.

Kinoform filters are also limited by the narrow range of spatial frequency that can be plotted or successfully obtained in the etched photoreduction. This will then limit the size of the grid over which the filter function $|L|^2$ is defined and the distance from the object at which the function will give satisfactory results; i.e., the focal ratio of the system. The filters described here were defined over a 32×32 grid with 0.02-cm spacing between points and a focusing distance of 50 cm. The focal ratio is limited by our experimental apparatus.

The initial results presented in this paper suggest that the advantages of unrestricted fields, on-axis filtering without stops, and efficient use of incoherent illumination may make kinoform filters useful in many optical data processing applications, in spite of the limitations.

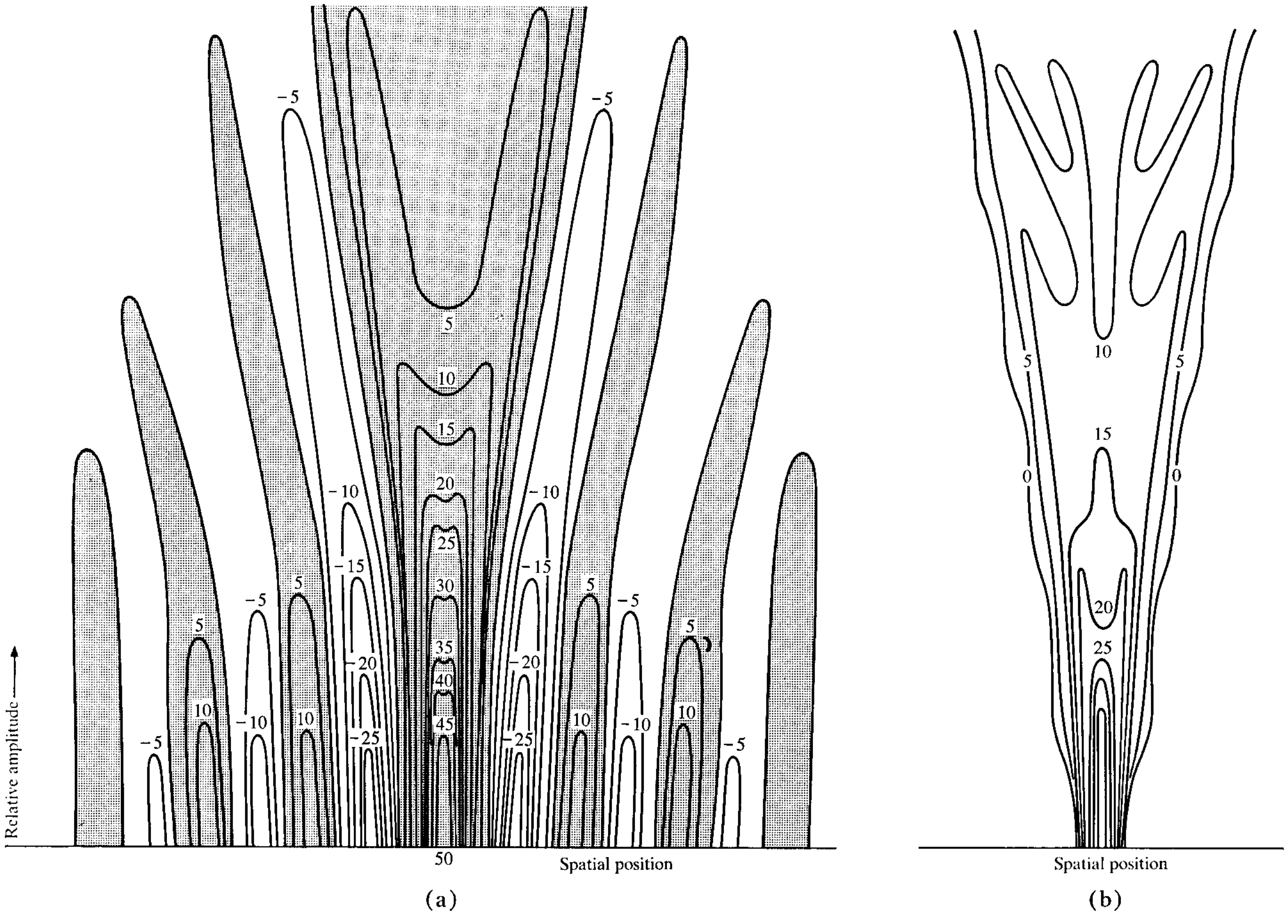
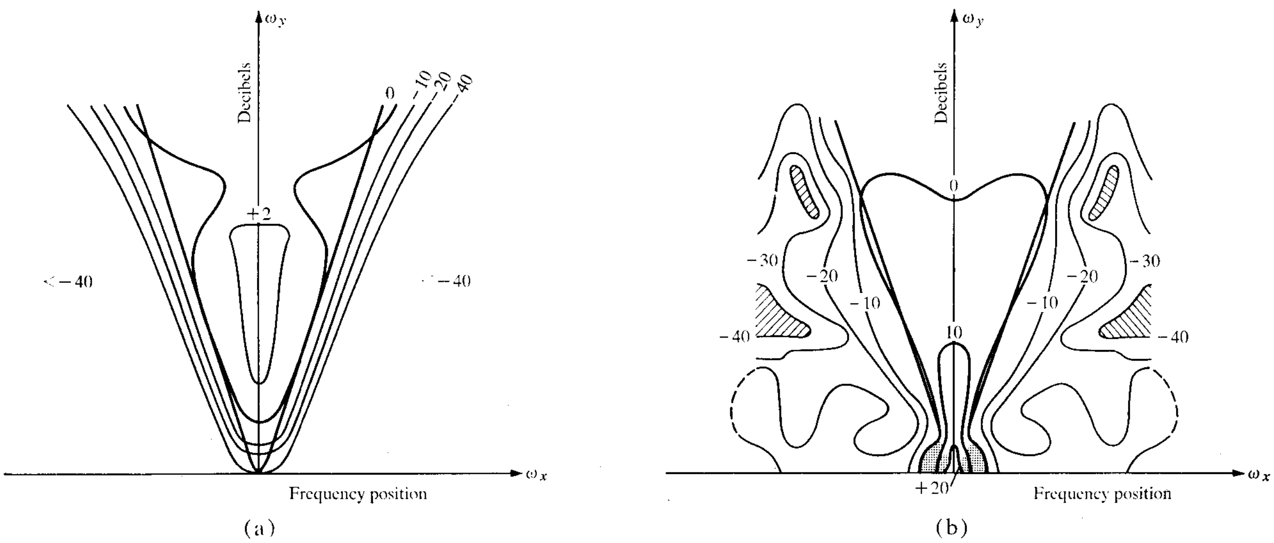
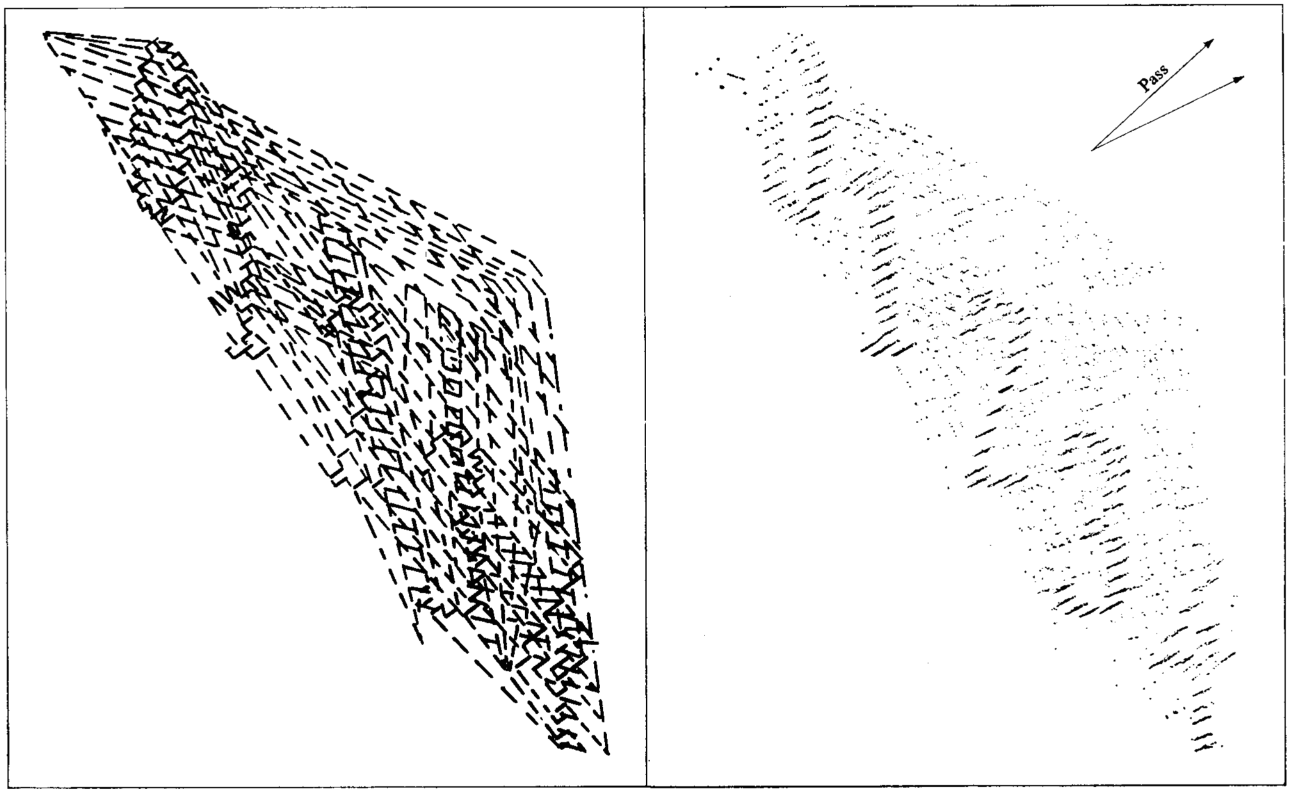


Figure 5 (a) Impulse response of the classical, digital fan filter. (b) Impulse response of the least squares kinoform fan filter. The numbers associated with the curves represent unnormalized weighting values of the impulse responses.

Figure 6 Frequency domain attenuation spectra of the filters shown in Fig. 5. (a) spectrum of the classical digital filter; (b) spectrum of least-squares kinoform filter. The numbers represent centered decibels, i.e., decibel units normalized for the zero-decibel curve to follow the contour of the fan-shaped pass area.

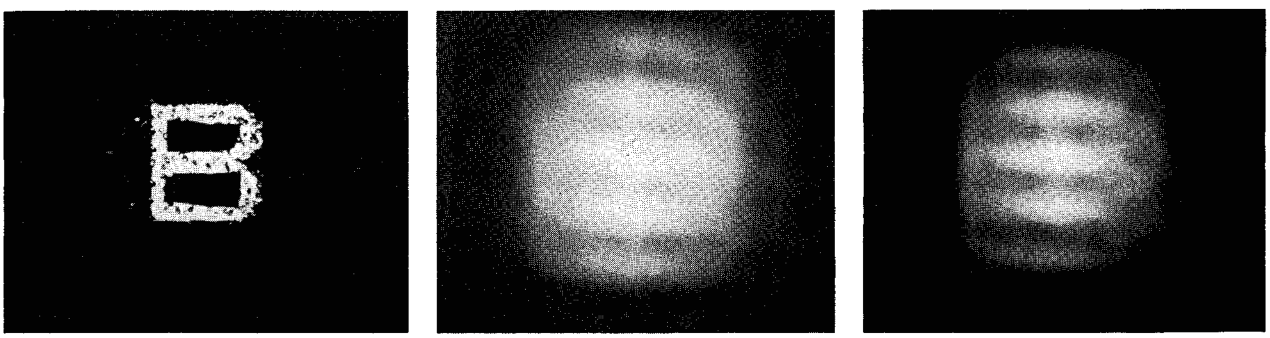




(a) (b)

Figure 7 Example of kinoform filtering for enhancement of data obscured by noise. (a) Unfiltered image; (b) filtered image obtaining using the least-squares kinoform fan-filter.

Figure 8 Results of a kinoform correlation filter. (a) A test image; (b) correlation with a kinoform filter having the letter B as its impulse response; (c) correlation obtained by reorienting filter.



(a) (b) (c)

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