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Avalanche Shock Fronts in p-n Junctions

Abstract: The conditions necessary for the formation of avalanche shock fronts, narrow layers of avalanche moving through a diode depletion layer faster than the carrier saturated drift velocity, are shown to be related to the large-signal limits of Read and more general avalanche transit time diode theory. Analysis of shock fronts by a simple analytic method has been used to interpret computer simulations of high efficiency microwave oscillator diodes. The oscillation mode, called the Trapatt mode, involves a compensated electron-hole plasma that is trapped in the depletion layer for a portion of each cycle.

Introduction

Avalanche shock fronts[1] (ASF) can be produced in p-n junctions if, by means of an appropriate external circuit, a voltage much larger than the static breakdown voltage is suddenly applied to the diode. The conditions leading to the formation of such narrow avalanching layers, which travel with velocity in excess of the carrier saturated velocity, can be equated with those occurring under conditions of large-signal operation in conventional microwave power generating diodes when certain normally obeyed limits are exceeded. Microwave avalanche transit-time diodes operating according to the Read[2] or more general Impatt[3,4,5] theories cannot contain an amount of charge in transit greater than a specified amount. When this charge is exceeded, an ASF may form and the diode may, under suitable circuit conditions, fill itself with a compensated electron-hole plasma. A new mode of oscillation that differs from the Read or Impatt mode in that the diode switches periodically from its normal open circuit state to this new, highly conducting state, has been shown[6] to be present at somewhat reduced frequencies, but with much higher power conversion efficiencies. This mode has been called the *Trapatt*[6] mode and has been simulated on a computer through appropriate modelling of a Ge diode and an associated microwave circuit.

We shall herein examine the limits of Read and other Impatt diode operation, discuss the formation of an ASF when they are exceeded, and trace by means of a simple description a complete cycle of the Trapatt diode operation. An ASF analysis is made by making a transformation to an appropriate moving coordinate frame[1] wherein a time-steady state appears to exist, as has been shown by computer simulations of the mode of operation.[7] A simple analytic description of the details of the ASF can then be obtained. The mechanism responsible for sharpening of the ASF is similar to a small-signal gain mechanism previously demonstrated by Misawa,[8] and this fundamental relationship is investigated.

Read and other large-signal Impatt diode limits

Read's original diode design was a specific type of Impatt diode having a narrow avalanching cathode. However, for high-power operation Impatt diodes having broader cathodes have been found superior. The detailed reason for this difference will be made clear shortly, but it stems basically from the fact that larger operating current densities can be obtained from thicker cathodes. Impatt theory deals with the process whereby large charge pulses with the proper phase delay are produced by the cathode under large-current conditions. The large currents not only lead to greater output and efficiency, but are dictated by the practical need for a uniform avalanche in the direction transverse to the current, coupled with the fact that uniformity improves with increasing current. This improvement in uniformity is a natural consequence of the physics of avalanche as is evident from the detailed results of ASF formation.[1]

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Two large-signal limits were recognized by Read as applying to the narrow-cathode diode. These are 1) the requirement, as the diode voltage drops to its minimum in each cycle, that the field in the drift space should not drop below the field E_s required for maintaining velocity saturation, and 2) that the space charge in transit should not cause a rise in field ahead so large as to produce additional avalanche in the drift space. A third large-signal limit, the cathode width requirement of the more general Impatt theory, derives from the space-charge suppression of the field produced by the spatial separation of the newly-generated electrons and holes. This suppression is also the central issue in ASF formation and thus warrants additional consideration. Figure 1(a) illustrates this effect with a specific numerical calculation[7] for a Ge n^+pp^+ diode with acceptor concentration $N_A = 4.2 \times 10^{15}$ cm^{-3} in the p region using the best known parameters of Ge. The small dip at the peak is the only departure of the field profile from the static profile normally associated with such diodes. It results from the carrier space charge and acts to suppress the avalanche. A broad cathode can generate a larger delayed charge pulse, because the carrier transit time is longer and the separation and subsequent suppression take longer to occur.

Exceeding the Impatt limits

For the purpose of ASF formation a large charge pulse is needed because space-charge modulation of the field must be a dominant effect. Figures 1(b) and (c) illustrate the time development at two subsequent times, 10 and 20 psec after the event in Fig. 1(a). The history of what is the start of a Trapatt cycle differs from the normal Read or Impatt behavior, because the circuit maintains a large and approximately constant current (current density J_T) which causes a rapid rise in field in the depleted portion of the diode. (In an Impatt oscillation the terminal current, which leads the voltage by approximately 90°, is instead near a minimum at the time the avalanche is suppressed by the space-charge effects.) The ASF is evident in Fig. 1 as the characteristic relation of avalanche (indicated by the black band) and electron and hole concentration, which progresses to the right in a nearly unchanged form. The front velocity v_F is given[1] through the rate of rise of E and its spatial slope, by the intersection velocity of E with a fixed value E_c, the critical field for impact avalanche:

$$v_F = \frac{J_T - j_{cs}}{qN_A}, \tag{1}$$

where j_{cs} is the residual carrier current in the depleted region. The rise in E results from the displacement current which flows because there are virtually no carriers ahead of the front, and this electrostatic action allows the front velocity to exceed the carrier velocity. Velocity v_F is readily made large by making N_A small. Following the ASF there is a low-field region of increasing width wherein $E < E_s$, and a large concentration of trapped plasma is formed. When it eventually fills the diode, the voltage is reduced to near zero. A description of the complete Trapatt cycle is presented in a later section.

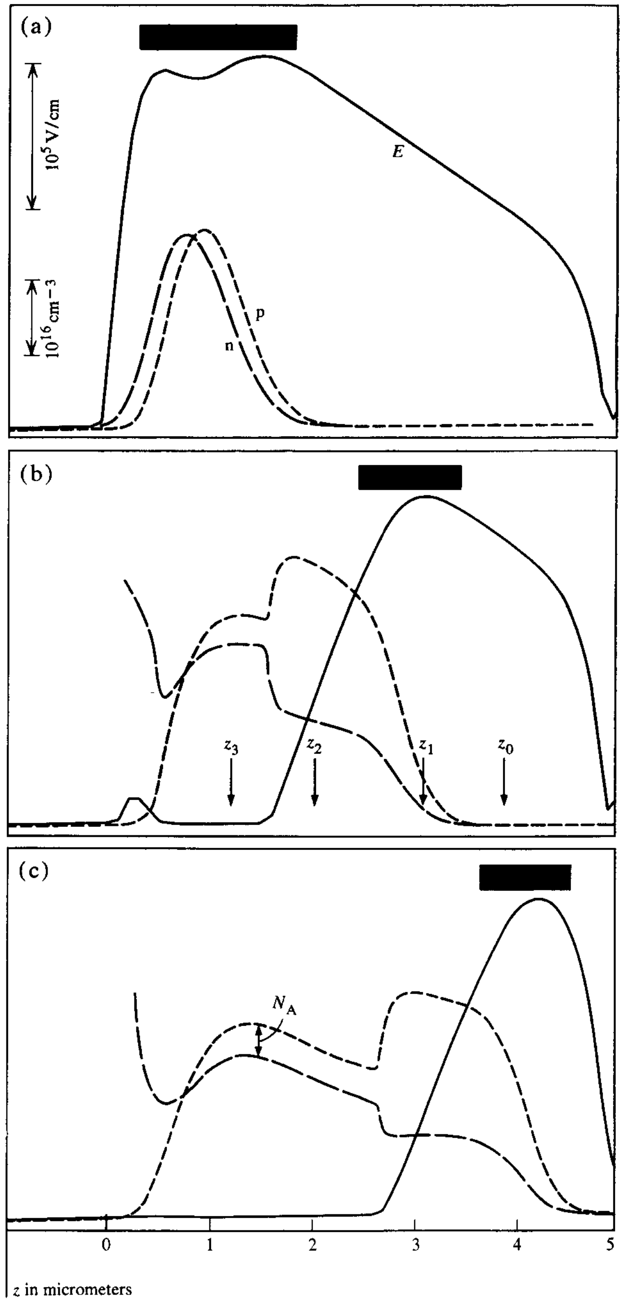

Figure 1 Curves based on portion of computer-generated time sequence of Ge n^+ pp^+ diode oscillation showing electric field (E) and electron (n) and hole (h) concentrations in the depletion layer as functions of distance, z. Time lapse between figures is 10 psec.

Avalanche shock front analysis

By demanding time steady state in a coordinate system moving with velocity v_F, the electron and hole continuity equations for the field range corresponding to saturated velocity v_s are found[1] by using the transformation

$$\frac{\partial}{\partial x} \to \frac{d}{dz}, \qquad \frac{\partial}{\partial t} \to -v_F \frac{d}{dz}:$$

$$\frac{dj_n}{dz}\left(1 + \frac{v_F}{v_s}\right) = -\alpha j_c \tag{2}$$

$$\frac{dj_p}{dz}\left(1 - \frac{v_F}{v_s}\right) = \alpha j_c, \tag{3}$$

where α is the ionization coefficient, $j_c = j_n + j_p$, and electrons and holes are assumed to have identical properties except for charge.

There are four positions, $z_0 \cdots z_3$, indicated in Fig. 1(b). These positions are meant to refer to all positions within each of four characteristic regions and, of course, move with the ASF with velocity v_F. The computer calculation of Fig. 1 has added some detailed variations, particularly around z_2, which are neglected in the following simplified analysis.

Integrating (2) and (3) over a range extending from z_0 ahead of to z_1 in the avalanche, and adding the results gives

$$j_c - j_{cs} = \frac{v_F}{v_s}(j_p - j_n) - \frac{v_F}{v_s}(j_{ps} - j_{ns}), \tag{4}$$

where $j_c = j_{ps} + j_{ns}$ and all j's refer to currents in the fixed reference frame.

The result (4) allows us to establish the equality, in the moving coordinate frame, of Poisson's equation and the expression relating displacement and particle current to the total current. Poisson's equation at point z_1 is

$$\frac{dE}{dz} = \frac{q}{\epsilon}\left(-N_A + \frac{j_p - j_n}{qv_s}\right), \tag{5}$$

while the currents are related by

$$-v_F\epsilon\frac{dE}{dz} = J_T - j_c. \tag{6}$$

By solving (1) for N_A and substituting for N_A in (5) and by substituting for $j_p - j_n$ from (4) we obtain (6) with the exception of an added term $-(j_{ps} - j_{ns})v_F/v_s$. It could well be argued that the space charge represented by this term should properly have been included in the denominator of (1), but for all practical purposes it is negligible with respect to $qv_F N_A$.

The identity of (5) and (6) at the arbitrary position z_1 in terms of (1), which was derived at position z_0, helps explain how the steady-state shape of the ASF is maintained: at velocity v_F there is, in effect, one fewer constraining equation to be satisfied, even in the presence of avalanche. The ASF shape is further restricted outside the range of avalanche because, as we will show on rather general grounds, solutions with $v_F \neq v_s$ in the range $E_s < E < E_c$ must have the form dE/dz = constant. The particle densities in the trailing edge (position $\approx z_2$) are thus ideally independent of z [see (6)]. The currents j_{pm} at z_2 are found by combining $j_{cm} + j_{nm}$ and (4), wherein at z_2 it is usually permissible to neglect j_{cs}, j_{ps}, and j_{ns}:

$$j_{nm} = j_{cm}\frac{1}{2}\left(1 - \frac{v_s}{v_F}\right), \tag{7a}$$

$$j_{pm} = j_{cm}\frac{1}{2}\left(1 + \frac{v_s}{v_F}\right), \tag{7b}$$

where j_{cm}, the maximum particle current, is calculated below. From (7a) it is clear that ASF formation requires $v_F > v_s$, so that from (1)

$$J_T > qv_s N_A - j_{cs}. \tag{8}$$

The slope at z_2 determines in part the required depletion layer width and is

$$\frac{dE}{dz} = \frac{q}{\epsilon} N_A\left(\frac{j_{cm} - J_T}{J_T}\right). \tag{9}$$

In order to relate j_{cm} to the fundamental parameters of the problem one would expect perhaps that this relation would involve $\alpha(E)$. It turns out not to be so for reasons that are made clear below. Combining (2) and (3) gives

$$2\alpha j_c = \left(\frac{v_s}{v_F} - \frac{v_F}{v_s}\right)\frac{dj_c}{dz}. \tag{10}$$

Writing (6) as

$$j_c = v_F\epsilon\frac{dE}{dz}\frac{1}{1 - J_T/j_c}$$

and substituting for j_c on the left of (10), we obtain by integrating from z_0 to z_2

$$\left(\frac{v_F}{v_s} - \frac{v_s}{v_F}\right)\left(J_T \ln\frac{j_{cm}}{j_{cs}} - j_{cm} + j_{cs}\right) = 2\epsilon v_F\int_{E(z_0)}^{E(z_2)}\alpha(E')\,dE'. \tag{11}$$

When $v_F \neq v_s$, j_{cm} is related directly to only J_T and j_{cs} if the RHS of (11) vanishes. The form of the RHS shows that if $\alpha(E)$ is sufficiently nonlinear, so that a distinct E_c can be defined, the integral will indeed vanish. Thus j_{cm} is found by solving

$$j_{cm} = j_{cs}\exp\left[\frac{j_{cm} - j_{cs}}{J_T}\right], \tag{12}$$

which leads to a useful approximate form

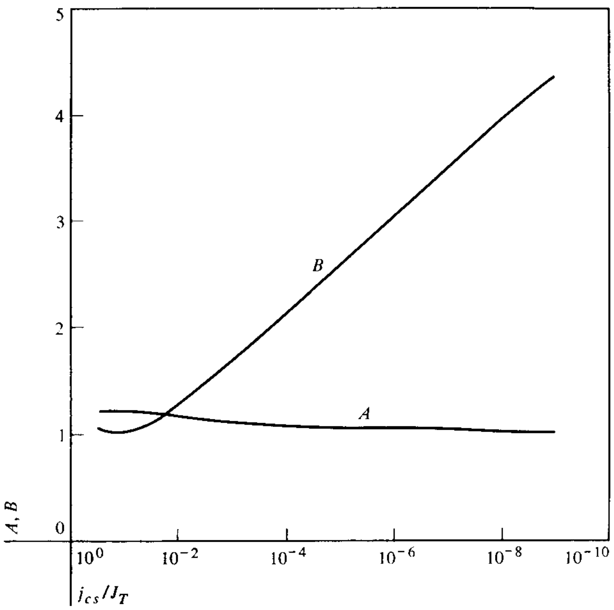

Figure 2 Plot of slowly varying parameters A and B.

$$j_{cm} \approx A J_T \ln\left(2B \frac{J_T}{j_{cs}}\right), \tag{13}$$

where A and B are slowly varying functions if $j_{sc}/J_T \ll 1$. A plot of A and B for various j_{cs}/J_T is shown in Fig. 2.

The final shape of the avalanching region of the ASF cannot be determined without knowledge of a specific $\alpha = \alpha(E)$, but we can find a general expression relating α to the length of this region by integrating α from (10):

$$\int_{z_0}^{z_2} \alpha\, dz = \frac{1}{2}\left(\frac{v_F}{v_s} - \frac{v_s}{v_F}\right) \ln \frac{j_{cm}}{j_{cs}}. \tag{14}$$

From the equations (12) and (13), (14) becomes

$$\int_{z_0}^{z_2} \alpha\, dz = \frac{1}{2}\left(\frac{v_F}{v_s} - \frac{v_s}{v_F}\right)\left(A \ln \frac{2B J_T}{j_{cs}} - \frac{j_{cs}}{J_T}\right). \tag{15}$$

This expression serves much the same function in characterizing $E(z)$ as the similar expression with the RHS of (15) replaced by unity that is appropriate for the static breakdown of a diode. In the present case the limits of integration, besides corresponding to a translation at velocity v_F, are adjustable because of the abovementioned constancy of dE/dz in the range $E_s < E < E_c$. The actual $\alpha(E)$ will usually not allow us to define a strict E_c, but the departure from dE/dz = constant is found from

$$\frac{d^2E}{dz^2} = -\frac{v_s}{\epsilon} \frac{2\alpha j_c}{v_F^2 - v_s^2}, \tag{16}$$

which results from differentiating (6) and using (10). The product αj_c in (16) explains why dE/dz = constant persists to larger E in the leading edge than the trailing edge of the ASF.

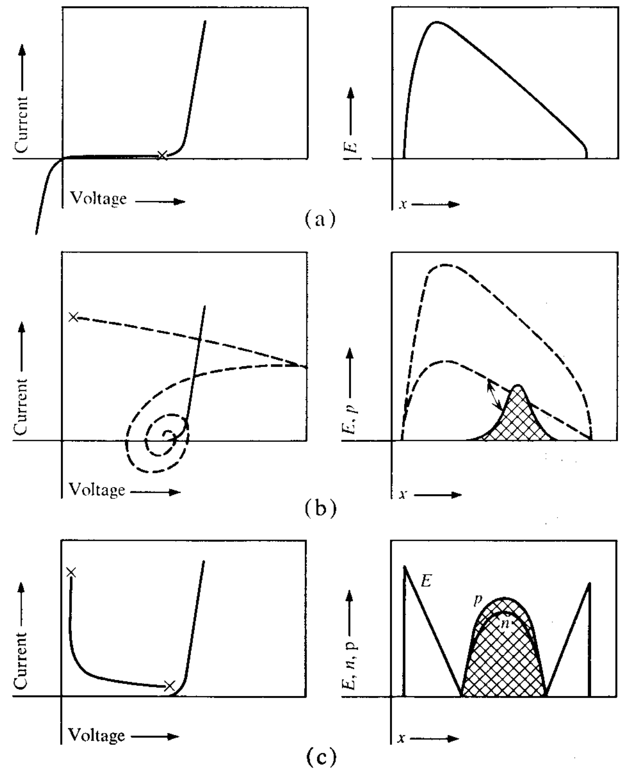

Figure 3 Schematic representation of current-voltage relation with $E(x)$, $p(x)$, $n(x)$ used to describe a Trapatt cycle.

The Trapatt oscillation cycle

The diode under consideration was seen in Fig. 1 to be capable of forming a highly conducting state. Being also just an ordinary p-n junction diode, it can likewise be a good quality open curcuit as indicated schematically by the $\times$ in Fig. 3(a). The Trapatt oscillation leads to high efficiency because neither of these states inherently requires large power dissipation and it is also possible to switch between them without much additional dissipation.

During switching from "off" to "on" [Fig. 3(b)] an Impatt oscillation starts up with the diode placed in a high-Q resonant cavity at this frequency. The light loading allows a rapid amplitude growth and the clockwise phase relation of the depletion-layer capacitance leads to the spiral in Fig. 3(b). The voltage can overshoot the static characteristic by a large amount, typically a factor of two, but at that point the ASF forms and the voltage very rapidly drops to zero. There is little net dissipation because of the quadrature phase relation in the spiral and the rapid speed of the ASF.

Switching "off" [Fig. 3(c)] is accomplished more slowly, but because $I \times V$ is kept relatively small there is again little dissipation. The extraction of the plasma proceeds in the same manner as in the "reverse-recovery transient" of a diode[9] which is rapidly switched from strong forward bias into reverse. The plasma stays trapped at the center, while at the edges space-charge-limited current flows. The final phase of the recovery occurs rapidly as the field rises also in the center and carriers move out with velocity v_s.

If the circuit is still energized at the Impatt frequency, new oscillations proceed to large amplitude very quickly. The complete Trapatt cycle may then require only two or three times the Impatt period. If the circuit is no longer energized a large down shift of the Trapatt frequency will result. The details of this behavior form the subject of a separate investigation.[10]

Growth mechanism—overmultiplication

The mechanism that controls the precipitous formation of an ASF can be characterized as an overmultiplication of secondaries in the avalanche. To make connection with the small-signal theory of Misawa[8] in which growth of space-charge waves takes place, we make use of an analysis in the normal rest frame. Combining the continuity equations and Poisson's equation in terms of the variables x and t, one finds

$$\epsilon \frac{\partial}{\partial x}\left[v(E)\frac{\partial E}{\partial x}\right] = \left[2\alpha(E) - \frac{\partial}{\partial t}\frac{1}{v(E)}\right] \times\left(J_T(t) - \epsilon\frac{\partial E}{\partial t}\right) - qN_A\frac{\partial v(E)}{\partial x}, \qquad (17)$$

where $J_T(t)$ is assumed known. A linearized form of this equation, which immediately yields Misawa's growing wave dispersion relation, is obtained by assuming as he did that $v(E) \approx v_s$, $\alpha(E) \approx \alpha_0 + \alpha' E$ (where $\alpha' = \partial\alpha/\partial E$), and $J_T =$ constant:

$$\epsilon v_s^2 \frac{\partial^2 E}{\partial x^2} = 2\alpha' J_T v_s E - 2\epsilon\alpha_0 v_s \frac{\partial E}{\partial t} + \epsilon\frac{\partial^2 E}{\partial t^2}. \qquad (18)$$

Wave growth is determined by the second term of the RHS of (18) which has a quadrature phase. The first RHS term is often larger, but it is in phase and contributes only to a wave velocity change[8] or, in the limit of very uniform avalanching layers (LHS = 0) to the "avalanche frequency."[4]

In the present analysis of (17) we make no linearization assumptions, but we still assume $J_T =$ constant. In the range $E_s < E < E_c$, (17) reduces to a simple wave equation with characteristic velocity v_s. If, therefore, solutions are imposed by a boundary moving with $v \neq v_s$ only the trivial solutions $\partial E/\partial x =$ constant and $\partial E/\partial t =$ constant will be valid, in agreement with our earlier statement.

For $E > E_s$, (17) allows solutions of the forms (16) and (6). An ASF forms, rather than the steady-state solution with $j_c = J_T$ and $E = E_{ss}(z)$, because of the "overmultiplication" inherent in making the transition from the depleted state: From (6) we see that $E = E_{max}$ when $j_c = J_T$, and from (15) it is evident that $E_{max} > E_{ss}$. There will therefore be an overmultiplication of charge equal to the generation taking place after the passing of E_{max}. By normalizing this charge to the amount generated in reaching E_{max} we define an overmultiplication ratio, R_0, which can be evaluated by integrating (10):

$$R_0 = \frac{j_{cm} - J_T}{J_T - j_{cs}} \approx A \ln\left(\frac{2B}{\exp A^{-1}}\cdot\frac{J_T}{j_{cs}}\right). \qquad (19)$$

When E reaches E_{SS} after the passing of E_{max}, the steady state is no longer possible because of the excess charge, and E continues to fall to E_s and somewhat below.

The upward excursion of E can thus be thought of as the large signal limit of Misawa's gain mechanism which has saturated. The downward excursion of E, on the other hand, is quite different. When $E < E_s < E_c$ normally only the first term of the RHS of (17) containing $v(E) \neq v_s$ makes an important contribution. The role of this term is thus similar to $\alpha(E)$ in creating departures from $dE/dt =$ constant. Dielectric relaxation thus plays an important role in bringing the carrier densities to their required difference of N_A at z_3.

Conclusion

Avalanche shock front formation is a consequence of an overmultiplication of charge when a depleted diode is suddenly strongly reverse biased. The front moves with a velocity several times the saturated drift velocity and can readily be characterized by a steady state condition in a moving coordinate frame. The passing of the ASF causes a rapid change of the diode to a highly conducting state and, in a suitable circuit, high efficiency oscillations in the Trapatt mode are possible.

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