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Wave Propagation in Negative Differential Conductivity Media: n-Ge

Abstract: A study has been made of transverse electromagnetic wave propagation in the negative differential resistance medium provided by suitably oriented n-type germanium at 77°K. The wave frequency is chosen to fall below the critical scattering rates in this system (1 GHz), and the sample dimensions are maintained below the critical length for domain formation. Thus when the electric vector is oriented parallel to the dc biasing field, and the propagation vector is normal to the biasing field, growth of the wave is to be expected and evidence of it is presented. In addition, the real and imaginary parts of the conductivity are evaluated throughout the dc bias field range from zero to 3.5 kV/cm, for a range of resistivities.

Introduction

Transverse electromagnetic (TEM) waves are attenuated in resistive media as a consequence of the real (in-phase) conductivity. If this conductivity is negative, then the conclusion that the wave will experience negative attenuation or gain readily follows. Although there are no known total negative resistance phenomena, a variety of negative differential resistances have now been reported. Thus it is of interest to study the propagation of TEM waves within media which are biased into a region of negative differential conductivity. In addition to the obvious device potential of such experiments in providing an asynchronous amplification mechanism,* they also provide an opportunity to study the transport process that underlies the negative differential conductivity (NDC) employed.

The experimental work described here has been carried out in n-type germanium at 77°K, both because of the interest that surrounds the origin of this NDC and because of the weakness of the effect; a weak NDC has the attendant advantage of a more spatially uniform electric field. However any device interest will clearly be directed towards a stronger NDC process such as that occurring in GaAs.

Choice of experimental regime

• *Wave propagation and round trip effects in a specimen having a negative differential conductivity.*

A propagation coefficient ($k_R + ik_i$) corresponding to the real frequency, ($\omega/2\pi$), is readily derived from Maxwell's equation in terms of the complex conductivity ($\sigma_R - i\sigma_i$).

$$k_R = k_0\sqrt{\frac{\epsilon_L}{2}}\left[1 + \frac{\sigma_i}{\epsilon_L\omega} + \sqrt{\left(1 + \frac{\sigma_i}{\epsilon_L\omega}\right)^2 + \frac{\sigma_R^2}{\epsilon_L^2\omega^2}}\right]^{\frac{1}{2}} \tag{1}$$

and

$$k_i = +\frac{\sigma_R}{|\sigma_R|}k_0\sqrt{\frac{\epsilon_L}{2}}\left[-1 - \frac{\sigma_i}{\epsilon_L\omega} + \sqrt{\left(1 + \frac{\sigma_i}{\epsilon_L\omega}\right)^2 + \frac{\sigma_R^2}{\epsilon_L^2\omega^2}}\right]^{\frac{1}{2}} \tag{2}$$

for a transverse electromagnetic wave having a periodicity of exp $(-i\omega t + i\mathbf{k}\cdot\mathbf{z})$.

The complexity of the transport process is contained within the terms σ_R and σ_i. Thus an experimental study of k_R and k_i provides information concerning the transport process.[1] Furthermore we observe that in a suitable material, and subject to certain other conditions discussed later, a dc bias field E_0 may be applied of such a magnitude that σ_R becomes negative, implying inversion of the sign of k_i in Eq. (2). TEM waves propagating across E_0, and having an $\mathbf{E}$ vector oriented parallel to E_0 will therefore grow.

In a sample with flat and parallel surfaces multiple reflections must be taken into account. The surface reflection coefficient

$$rr^* = \frac{(k_R - k_0)^2 + k_i^2}{(k_0 + k_R)^2 + k_i^2}$$

* Contrast the acoustic amplifier where synchronous motion of electrons and phonons is necessary for maximum interaction.

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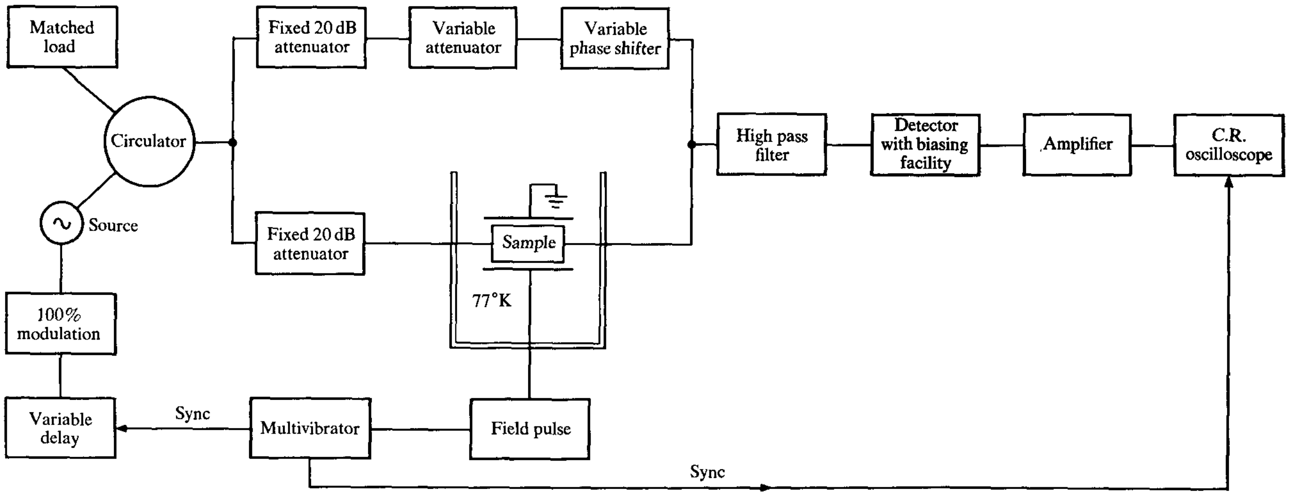

Figure 1 Schematic diagram of experimental system.

is, for the frequencies and specimen resistivities of interest, strongly dependent upon free-electron concentration, and may approach unity. Within the cavity formed by the sample we can therefore distinguish three operational regimes:

(a) Broadband amplification with matched or Bloomed specimen surfaces. [$rr^* = 0$, Gain $= e^{-2k_i l}$, l being the specimen length in the propagation direction].

(b) Narrow-band amplification [$rr^* e^{-2k_i l} < 1$]. The signal is multiply reflected within the specimen cavity, experiencing gain irrespective of direction of propagation owing to the reciprocal nature of the gain process.

(c) Oscillation [$rr^* e^{-2k_i l} > 1$]. The signal reflected within the cavity experiences round-trip gain.

The experimental work described here falls into the second category.

• *Instability and field nonuniformity*

The foregoing remarks have neglected the problem of applying a biasing field to a specimen with an NDC. Domain formation is inhibited by a suitable choice of NL product, L being the specimen length measured parallel to E_0 and N, the free carrier concentration. However, it is well known that as a consequence of the NDC the electric field must then be spatially non-uniform. This behavior becomes less marked as the strength of the NDC is decreased. Thus n-Ge at 77°K is chosen for the present experiment in spite of the attendant disadvantages of small effects, requiring large specimen dimensions in the propagation direction and therefore low specimen impedances. Because of the spatial non-uniformity the experimental evaluation of σ_R will be valid up to threshold, but above threshold the deduced value of σ_R can only be a minimum value.

• *Frequency limitation*

We turn now to the complexities of the transport process contained within the terms σ_R and σ_i. The microwave electric fields are small in comparison to the bias field in the present experiment, and therefore a local conductivity appropriate to a particular bias field may be used in Eqs. (1) and (2). It is well known that this conductivity will not necessarily be equal to the "slope" conductivity, as defined by the gradient of the velocity-field characteristic. Indeed, the present experiment resembles the work of Gibson[1] et al. in which the microwave conductivity of a positive resistance sample was studied. Following the philosophy of that work, we conclude that the frequency of the wave must lie below the onset of frequency dependent effects.

A number of workers have shown that in the warm electron regime the microwave conductivity of n-Ge at 77°K is frequency dependent at 9 GHz[2] but not at 2.85 GHz.[3] At the higher electric-field strengths necessary to enter the NDC region, McGroddy[4] has observed oscillations at frequencies around 1 GHz. Thus in the work described here a frequency of 3.1 GHz has been employed in order that the measurement may be valid both above and below the threshold field.

Experimental system

The experimental system is shown schematically in Fig. 1. The germanium specimen is mounted in a microstrip transmission line in one arm of a microwave bridge. The rf power (3.1 GHz) is 100% modulated, thus providing a reference zero for balancing of the bridge, and the high-field bias pulse synchronized to appear within the rf pulse. Measurements of phase and amplitude of the

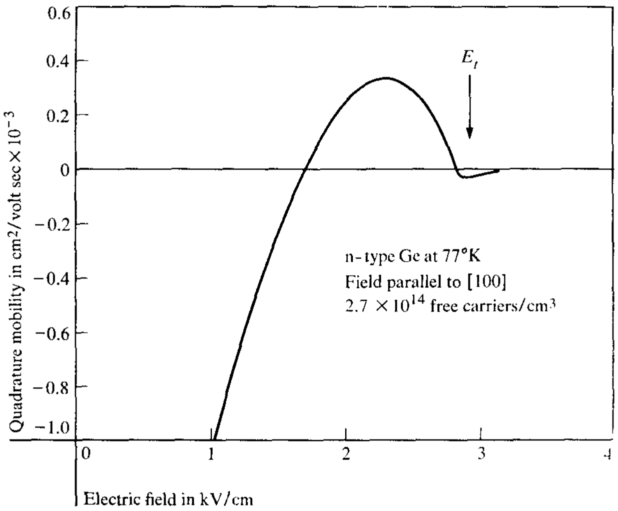

Figure 2 Quadrature mobility as a function of electric field for 5.7 Ω-cm, n-Ge at 77°K. The electric field is applied in the [100] direction, and the measurement performed at 3.1 GHz.

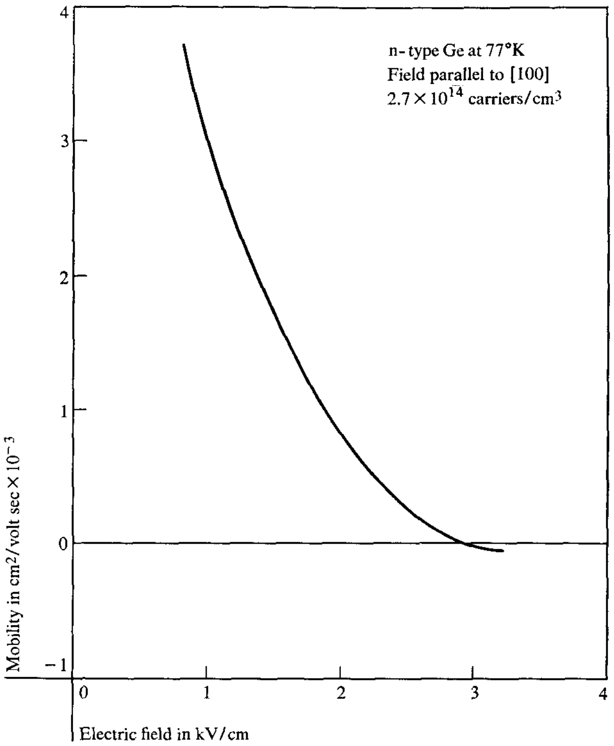

Figure 3 Mobility as a function of electric field for 5.7 Ω-cm, n-Ge at 77°K. The electric field is applied in the [100] direction.

transmitted signal are made as a function of the electric field strength $\mathbf{E}_0$.

The specimen of 5.7 Ω-cm, n-Ge, is cut so that its length, parallel to the bias field and [100] axis, is just below the critical value for domain formation (0.15 cm). Its transverse dimensions are such that it is (a) slightly narrower then the microstrip width (0.4 cm) thus avoiding the infinite fields at the strip edges and (b) a maximum in the propagation direction (0.45 cm) consistent with problems of power dissipation and the specimen impedance presented to the pulse forming network. Contact areas are large in this experiment and, consequently, it was necessary to use antimony doped gold plated Kovar in making the alloy junctions in order to produce a thermal match.

Results and discussion

The observed phase and amplitude of transmitted signal is analyzed as described in the Appendix, which gives the imaginary and real parts of the mobility shown in Figs. 2 and 3 respectively. Before commenting upon these we first introduce Fig. 4 where the real part of the mobility has been integrated and is compared with the experimental velocity field curves of Chang and Ruch,[5] Elliott, Gunn and McGroddy[6] and, incidently, the theoretical curve of Paige.[7] This comparison suggests that in the region of positive differential resistance the rf measurement is following the dc values within the accuracy of existing experimental data. However, the change from inductive to capacitative behavior of the free-carrier system, shown in Fig. 2, is indicative of frequency dependent effects. This would seem to imply that the drift velocity in the region beyond 1.6 kV/cm of Fig. 4 is somewhat high.

The real part of the mobility changes sign at 2.93 kV/cm in Fig. 3. Beyond this point Fig. 2 indicates that frequency dependent effects have disappeared, though the added complexity of a spatially nonuniform electric field is to be expected. Thus the observed negative mobility is a minimum value. Nevertheless the observation of a negative mobility of at least 30 cm^2/volt sec implies that the rf energy within the specimen is growing at least as shown in Fig. 5. Furthermore it should be noted that due to the restriction of both the frequency and specimen length in the propagation direction it was not possible to use a normal mode of the specimen cavity in this experiment. Thus the round trip gain effects have not been used to their full advantage.

In this paper the amplification of TEM waves propagating within the negative differential conductivity of n-Ge has been reported. The measurement has also provided a minimum value of the negative mobility, and a value for the threshold field. However, the value of this technique as a diagnostic tool is limited owing to the non-uniform electric field accompanying an NDC. Outside the region of NDC values of the complex mobility have been obtained which suggest that there are some frequency dependent effects occurring for electric field strengths in the region 1.5 to 2.5 kV/cm.

Acknowledgment

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Appendix

- *Microstrip analysis*

It is well known that a microstrip transmission line approximates the TEM propagation required, but has significant leakage paths for radiation outside the volume enclosed by the ground and signal planes (i.e., outside the region occupied by the specimen in this experiment*). The following description of the microstrip, although approximate, has been found satisfactory when the specimen insertion loss is not large.

The complex plane lying normal to the propagation direction in the microstrip system is transformed into a new complex plane, ω[8] given by

$$\frac{\pi z}{h} = \frac{\pi(x+iy)}{h} = 1 + \omega + \ln(\omega)$$

$$= 1 + re^{i\theta} + \ln(re^{i\theta}),$$

where h is the height of the strip above the earth plane, in the direction y, and x lies normal to y in the z plane. In the ω plane the flux lines lie upon the circumference of circles of radius r and the specimen boundary approximates to a circle on transformation, when the strip is close to the earth plane. The fraction of the total power in the specimen is

$$\frac{\epsilon\pi\beta/2h}{(\epsilon-1)\pi\dfrac{\beta}{2h} + 1 + \dfrac{\pi B}{2h} + \ln\left(1+\dfrac{\pi B}{2h}\right)},$$

where ϵ is the relative permittivity of the specimen, β the specimen width in the x direction and B the strip width.

The capacity per unit length of the microstrip is therefore

$$(\epsilon - 1)\frac{\beta}{2h} + \frac{1}{\pi} + \frac{B}{2h} + \frac{1}{\pi}\ln\left(1+\frac{\pi B}{2h}\right). \tag{3}$$

Following Wheeler's[9] analysis of the parallel strip transmission line we use Eq. (3) to obtain the effective permittivity of the heterogeneously loaded microstrip, and thus evaluate the scattering matrix of the system. Obtaining ϵ from Eqs. (1) and (2), each matrix element is expressed in terms of the complex conductivity, which can therefore be deduced from experimental measurements of the phase and amplitude of the transmitted signal.

* In fact the specimen is made somewhat narrower than the strip in order to avoid large inhomogeneous fields at the strip edges. Underneath the unloaded strip the field rapidly approaches the homogeneous field[8].

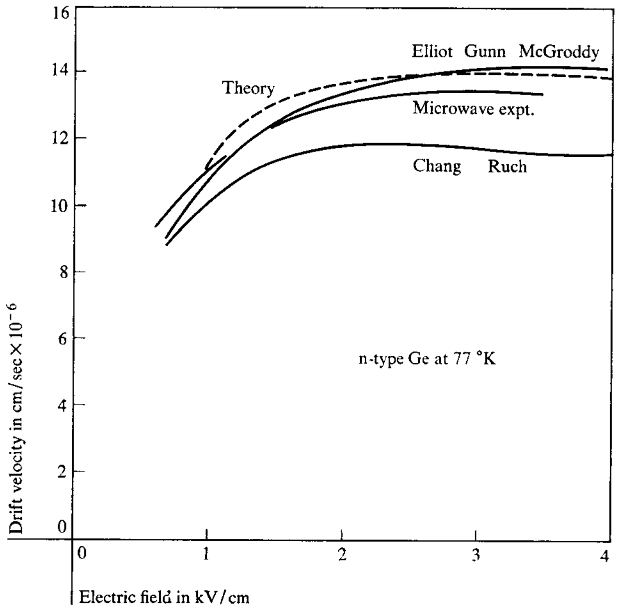
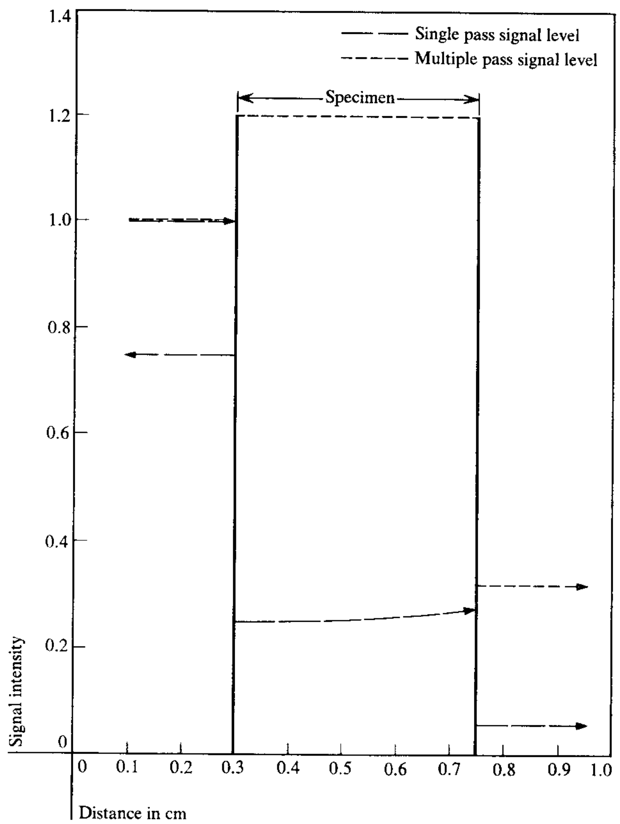


Figure 4 Drift velocity as a function of electric field. The broken curve is obtained by Paige in the preceding paper, and the curve labeled "Microwave expt." obtained by integration of the results shown in Fig. 3.

Figure 5 Schematic diagram of the signal level within the germanium cavity.



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