

Negative Conductivity Effects and Related Phenomena in Germanium. Part II

Abstract: This paper is the second part of a two-part review of recent work on current instabilities and related properties of germanium in high electric fields. In this part the transferred carrier mechanism for producing bulk negative differential conductivity (BNDC) in a semiconductor is discussed. Experimental work on instabilities related to three realizations of this effect, in uniaxially compressed n and p-type Ge, and in n-type Ge at low temperatures with field and current in a $\langle 111 \rangle$ direction, is reviewed. Theoretical understanding of these effects, which is largely qualitative at this time, is discussed. In an appendix a list of materials in which BNDC effects have been observed is presented, with some of the relevant properties of these materials.

Introduction

In a previous paper¹ (hereafter referred to as I) we have discussed the bulk negative differential conductivity (BNDC) which occurs in cooled, unstrained n-type Ge of appropriate orientation, and which gives rise to microwave current modulation under appropriate conditions of sample size and doping, and external circuitry. Knowledge of this effect has made necessary a new analysis of a number of old high field transport experiments in germanium, such as measurements of velocity-field curves and longitudinal anisotropy of the high field conductivity. These topics were also discussed in part I, along with the Erlbach effect, a case of transverse negative conductivity.

BNDC effects have also been seen in Ge under several other sets of conditions.^{2,3,4} Contrary to the situation discussed in I, the causes of these phenomena, which in each case are some variations of the transferred-carrier mechanism, have been understood from the outset; indeed, in each case the effect was found as a result of a deliberate experimental search after the appropriate conditions had been suggested by consideration of the transferred-carrier mechanism.

In this paper we first discuss the transferred-carrier mechanism^{5,6} for producing BNDC, which is responsible for the Gunn effect in GaAs⁷ and other polar semiconductors. Then we discuss the realizations of this phenomenon in n-type³ and p-type² Ge, in which carrier trans-

fer is between stress-split states of the conduction or valence band. Ridley and Watkins⁵ suggested the possibility of finding BNDC in strained p-Ge, and the associated current oscillations found by Kastal'skii and Ryvkin² were actually the first of the BNDC effects to be reported in Ge. For clarity, we have chosen to discuss the effects in strained n-Ge first, however. These results are useful in discussion of the effect in p-Ge, which is in the following section. The most recently discovered effect, the oscillations in $\langle 111 \rangle$ oriented n-Ge at very low temperatures,⁴ is due to a variation of the intervalley transfer mechanism. Here the transfer is between valleys with minima at the same energy, but having different conductivity effective masses, so that the heating rates are different in the different valleys, and field-induced transfer proceeds from the hotter to the cooler valleys. This is the final topic considered in the body of this paper. In an appendix we give a list of materials in which BNDC and microwave frequency current oscillations have been observed, along with some discussion of the relevant properties of these materials.

Transferred carrier mechanism

In the beginning of this decade, Ridley and Watkins⁵ and Hilsum⁶ considered the possibility of attaining BNDC in a semiconductor with band structure characterized by a low-mass, high-mobility valley lying lower in energy than a high-mass, low-mobility valley. If the energy separation is large enough compared to thermal energy,

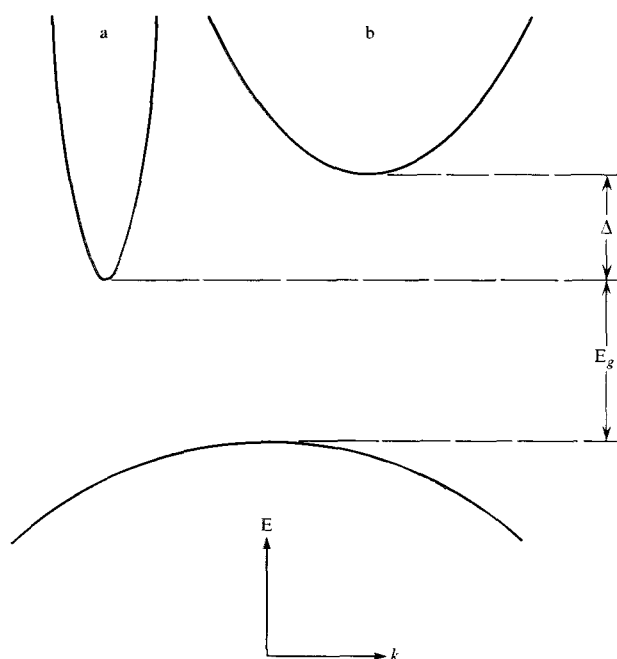


Figure 1 Band structure for BNDC due to the transferred carrier mechanism.

the carriers are all in the lower energy valley at low electric-field strengths. As the field is increased, the carriers gain energy until they can scatter into the upper valley. If the mobility difference is sufficiently great, and if the population shift occurs over a sufficiently narrow range of field, then the current decreases with increasing field and there is a region of negative conductivity.

To put this on a somewhat more quantitative basis, consider the band structure shown in Fig. 1, and assume the material represented is lightly doped and n-type. If $\Delta \gg kT_i$, where T_i is the lattice temperature, then at low field strengths all of the electrons are in the lower energy valley. At higher field strengths where some of the carriers have transferred to valley *b*, the upper valley, the conductivity is given by

$$\sigma = e(n_a\mu_a + n_b\mu_b) \quad (1)$$

and its field derivative by

$$\frac{d\sigma}{dF} = e(\mu_a - \mu_b) \frac{dn_a}{dF} + en_a \frac{d\mu_a}{dF} + en_b \frac{d\mu_b}{dF}, \quad (2)$$

where we have assumed that $n_0 = n_a + n_b$, the total carrier density, is constant. The condition for BNDC is that the field derivative of $j = \sigma F$ be negative:

$$\frac{dj}{dF} = \sigma + F \frac{d\sigma}{dF} < 0, \quad (3)$$

Using (1) and (2), this becomes

$$\frac{F \left[(\mu_a - \mu_b) \frac{dn_a}{dF} \right] + F \left[n_a \frac{d\mu_a}{dF} + n_b \frac{d\mu_b}{dF} \right]}{n_a\mu_a + n_b\mu_b} < -1. \quad (4)$$

The mobility of the lower valley is higher in the model band structure chosen, so in the first term $(\mu_a - \mu_b)$ is positive. The factor $(+F dn_a/dF)$ is negative, tending towards zero at zero field and very high field, so the first term favors BNDC. If the single-valley velocity-field curves are sublinear in F so that $d\mu/dF < 0$, the second term also favors BNDC. If, following Ridley and Watkins, we assume $\mu \propto F^p$, we are led to their condition for BNDC:

$$\left[\left(\frac{\mu_a - \mu_b}{n_a\mu_a + n_b\mu_b} \right) \times \left(-F \frac{d\mu_a}{dF} \right) \right] - p > 1. \quad (5)$$

If the single-valley velocity-field curves tend towards saturation, (p approaches -1) this gives the weakest possible condition on the first term in Eq. (5).

The assumption that n_0 is constant gives the additional requirement that there be no impurity levels ionizing in the field range of the BNDC, as such increase in carriers would tend to swamp the BNDC provided by the transferred-carrier mechanism. Likewise, we require that the band gap E_g be such that there is no possibility of the formation of electron-hole pairs in the field range of interest. If the band gap is sufficiently smaller than Δ , formation of electron-hole pairs is possible. This might prevent electrons from reaching energies high enough to allow transfer to the lower-mobility valley. Furthermore, the positive contribution of the field derivative of the carrier concentration might be large enough to overcome the negative differential mobility, giving a differential conductivity which is always positive; no BNDC.

Strained n-type germanium

The valleys lying lowest in energy of the many-valley conduction band of Ge are those lying along the $\langle 111 \rangle$ directions of momentum space at the Brillouin zone faces. The constant energy surfaces near these points are prolate ellipsoids of revolution about the $\langle 111 \rangle$ directions, with effective mass anisotropy of 19.07.⁸ The valleys, normally degenerate in energy, are split by application of uniaxial stress in any direction except a $\langle 100 \rangle$ direction. Because of the effective mass anisotropy, a current direction can be chosen in strained n-Ge to give a band structure appropriate for BNDC due to the transferred-carrier mechanism described above. The sign of the deformation potentials, the nature of the valley anisotropy, and experimental considerations require that the current and compressive stress be perpendicular.

Smith³ first reported current oscillations due to this effect in the configuration where current was in the $[211]$ direction and compressive uniaxial stress in either the

Table 1^a Conductivity effective masses and valley edge splittings for the $\langle 111 \rangle$ conduction band valleys in strained germanium.

The configurations of stress and current directions in which current instabilities occur in n-type Ge are listed.

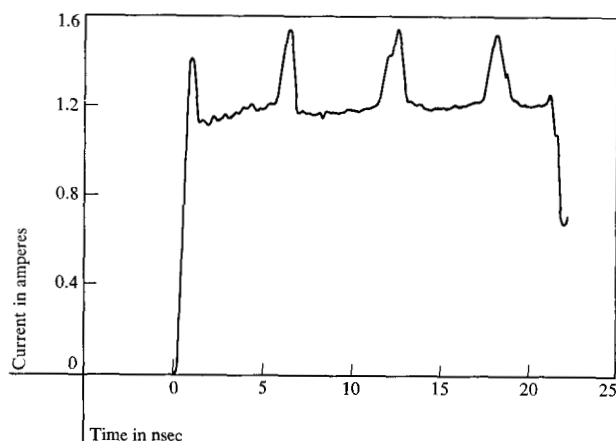
Current direction	Stress direction	Splitting ^b	m_e/m	Masses ^c m_i/m	m_h/m
[211]	$\bar{[111]}$	0.102	1.00(1)	—(0)	1.27(2)
[211]	[01 $\bar{1}$]	0.076	1.27(2)	—(0)	6.44(1)
[111]	$\bar{[211]}$	0.102	1.11(1)	1.11(2)	19.07(1)
[111]	[01 $\bar{1}$]	0.076	1.11(2)	—(0)	1.11(1)
[421]	$\bar{[112]}$	0.102	1.02(1)	1.68(1)	1.12(1)
[110]	$\bar{[110]}$	0.076	1.00(2)	—(0)	2.72(2)
[110]	$\bar{[111]}$	0.102	1.00(1)	—(0)	1.00(1)
					2.72(2)

^a From J. E. Smith, Jr., J. C. McGroddy and M. I. Nathan, *Phys. Rev.*, to be published.

^b The splitting indicated is that between the lowest valley or valleys and the next lowest for a stress of 10^4 kg/cm², in eV.

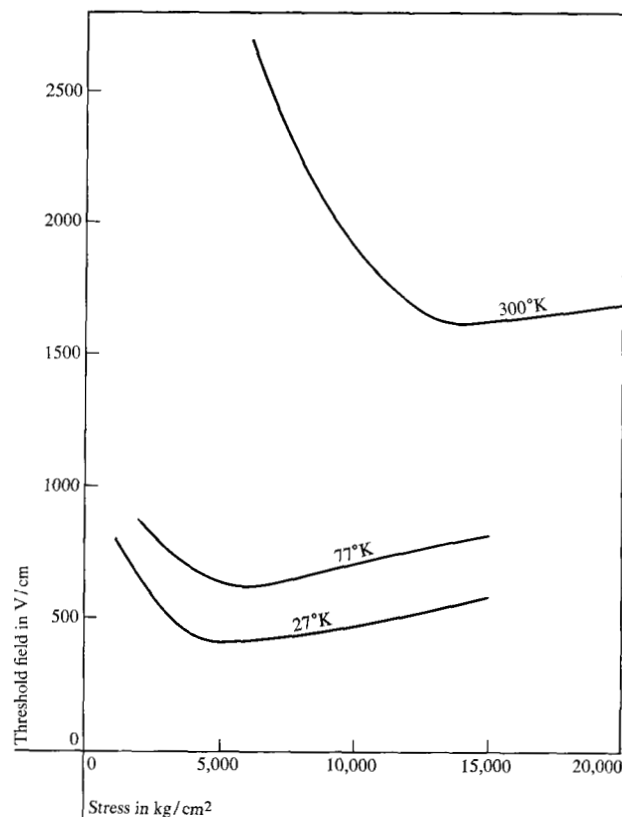
^c m_e is the conductivity effective mass of electrons in the valley(s) lying lowest in energy, m_i intermediate in energy, and m_h highest. m_t is the transverse mass; we take $K = 19.07$.

Figure 2 Current waveform measured in n-type germanium with a stress of 1.6×10^4 kg/cm² along the [111] direction and current along the $\bar{[211]}$ direction. The temperature was 27°K, and the material had room temperature resistivity 6 Ω -cm and carrier concentration 2.6×10^{14} cm⁻³.



$\bar{[111]}$ or [01 $\bar{1}$] direction. Subsequently the authors^{9,10} have considered a number of other configurations and reported experimental results for them. However, the [211] current, $\bar{[111]}$ stress has proven experimentally to be most favorable, giving the lowest threshold electric fields, largest amplitude current oscillations, and operation at the highest temperatures. Only this configuration will be discussed in detail in this review; others are summarized in Table 1.

Figure 3 Threshold field versus uniaxial stress for n-type Ge with room temperature resistivity 1 Ω -cm and carrier concentration 1.9×10^{15} cm⁻³. Current is in the $\bar{[211]}$ direction and stress in the [111] direction; this stress splits the valleys at the rate of 0.102 eV for a stress of 10^4 kg/cm². (From reference 10).



Compressive uniaxial stress along a $\langle 111 \rangle$ direction lowers the energy of the valley lying along that particular $\langle 111 \rangle$ direction relative to the other three at the rate of 0.102 eV for a stress of 10^4 kg/cm². Experimentally, the fracture stresses for Ge under uniaxial compression have typically been of the order of 2×10^4 kg/cm², so at zero field, even at room temperature, all of the electrons can be transferred into the lowest $\langle 111 \rangle$ valley quite easily. If the current direction is chosen to be the $\langle 211 \rangle$ direction normal to the stress direction, then the conductivity effective mass of electrons in the lowest valley is simply m_t , the transverse mass. The three remaining valleys are degenerate in energy, two of them having mass $1.27 m_t$ and the remaining one $6.44 m_t$, so we have a situation favorable for the transferred carrier mechanism: at low fields the electrons are in the low mass, high-mobility valley, and at higher fields they can transfer into one of the three lower mobility valleys. However, neither the mobility differences nor the density-of-states ratios are so favorable as those found in GaAs. The single-valley velocity-field curves of n-Ge are strongly sub-linear,¹¹ which favors the success of the transferred-carrier mechanism in producing BNDC.

Experimental results for the $[211]$ current, $[\bar{1}11]$ stress configuration are shown in Figs. 2 and 3. The spiked current waveform, characteristic of the Gunn effect, is shown in Fig. 2. At 27°K the amplitude of the oscillations is comparable to, but slightly smaller than, that found in GaAs at 300°K; the circuitry employed in the Ge experiments does not allow precise comparison of the amplitudes. At temperatures above 27°K the oscillations are weaker, indicating a weaker BNDC. Threshold field versus stress is shown in Fig. 2 for 27°, 77°, and 300°K. The minimum threshold field observed, about 275 V/cm at 27°K, is more than an order of magnitude smaller than found in GaAs at room temperature. This is a reflection of the higher mobility in Ge at low temperatures as well as the smaller splitting between the higher- and lower-mobility valley edges, since the peak drift velocity is about the same as in GaAs.

Perhaps the most interesting feature of these curves, however, is the minimum in each one. F_t at first decreases with increasing stress, passes through a minimum, and then rises. This behavior is found in all configurations and at all temperatures, with the minimum occurring at higher stresses and fields at higher temperatures. Similar behavior has been reported in GaAs for F_t versus hydrostatic pressure and has been attributed to the thermal population of the upper valleys at low fields.¹² In Ge, however, the minimum occurs at 5000 kg/cm² at a lattice temperature, T_l , of 27°, which corresponds to a valley splitting of 0.05 eV, or more than $20 kT_l$. Thus thermal population of the upper valleys is essentially zero, and this mechanism is irrelevant here. The

authors have suggested the following explanation of the minima: the drift velocity in each valley is strongly sub-linear at high field, so that the transfer of electrons to lower-velocity valleys is most effective at producing negative conductivity at high fields (see Eq. (5)). At small stress or separation of the valleys, most of the transfer occurs at lower fields where it is insufficient to cause BNDC. In the more nearly saturated region occurring at higher fields, the condition for BNDC is weaker, as discussed in the last section, and the small amount of transfer still occurring is sufficient to give a region of negative conductivity. At stresses near the minimum of F_t , most of the transfer occurs at intermediate fields, and negative conductivity occurs there, too. As the stress is increased further, transfer cannot occur until the carriers in the lower-energy valley are well into the sublinear region of the single-valley velocity-field curve, and the BNDC begins as soon as transfer begins, giving the intuitively expected region where threshold field rises with increasing valley separation. The authors⁹ have calculated the stress dependence of F_t on the basis of this simple model and found semiquantitative agreement with experiment.

As mentioned above, BNDC effects have been observed in a number of other configurations of current and stress direction, with the reported results summarized in Table 1. In every configuration which gives a qualitatively correct situation for the intervalley transfer mechanism, current oscillations have been observed. In some more marginal cases the temperatures must be lowered to make the BNDC sufficiently strong for the appearance of the oscillations. In general the $[211]$ current, $[\bar{1}11]$ stress configuration has been most successful, producing the largest oscillations and lowest threshold fields at all temperatures.

The current oscillations in strained n-Ge are qualitatively well understood. The transferred electron mechanism is able to account for all of the experimental results in at least a qualitative fashion; but quantitative calculations of threshold fields, magnitude of the BNDC, etc., have not been performed at this date. It seems likely, however, that Monte Carlo and similar calculations,¹³ which have had some success in treating the BNDC in GaAs as well as in cooled unstrained $\langle 100 \rangle$ n-Ge, are capable of treating this problem quantitatively although the calculation would be a most laborious task.

p-type germanium

BNDC can also be produced in p-Ge by application of uniaxial stress, as first suggested by Ridley and Watkins.⁵ Pikus and Bir¹⁴ have discussed in detail the effect of uniaxial stress on the valence-band structure of Ge family semiconductors. For compression along either the $\langle 100 \rangle$ or $\langle 111 \rangle$ direction, the degeneracy at the top of the valence

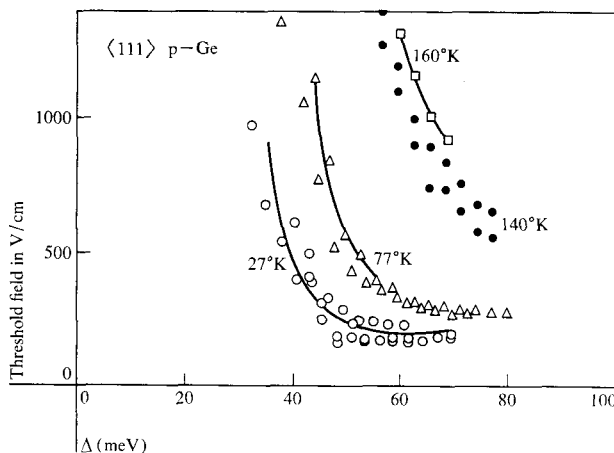
Table 2 Relative effective masses for uniaxially compressed p-type Ge. The subscripts $\parallel$ and $\perp$ on the effectiveness masses refer to current parallel to and normal to the stress direction: $m(1)$ is the effective mass of the highest (electron) energy sub-band and $m(2)$ the lowest, in units of the free electron mass.

Stress Direction	Inverse ^a Masses		Splitting Rates ^b
$\langle 111 \rangle$	$m_{\parallel}^{-1}(1) = 23.6$	$m_{\parallel}^{-1}(2) = 2.4$	$\frac{\delta\epsilon}{x} = \frac{S_{44}d}{\sqrt{3}} = 4.0 \times 10^{-6} \text{ eV cm}^2/\text{kg}$
	$m_{\perp}^{-1} = 7.7$	$m_{\perp}^{-1}(2) = 18.3$	
$\langle 100 \rangle$	$m_{\parallel}^{-1}(1) = 21.9$	$m_{\parallel}^{-1}(2) = 4.1$	$\frac{\delta\epsilon}{x} = 2(S_{11} - S_{12})b = 3.6 \times 10^{-6} \text{ eV cm}^2/\text{kg}$
	$m_{\perp}^{-1}(1) = 8.6$	$m_{\perp}^{-1}(2) = 17.5$	

^a G. E. Pikus and G. L. Bir, *Fiz. Tverd. 1*, 1642 (1959) [*Sov. Phys.—Solid State 1*, 1502 (1960)].

^b M. Cardona, private communication.

Figure 4 Threshold field versus valence band sub-band splitting for p-type Ge. Uniaxial stress and current are along a $\langle 111 \rangle$ direction. The material has room temperature resistivity 3 $\Omega\text{-cm}$ and carrier concentration $1.1 \times 10^{15} \text{ cm}^{-3}$. (From Ref. 10).



band is lifted and the resulting constant energy surfaces are ellipsoids of revolution about the stress direction. The constant energy surfaces of the higher-lying sub-band (here we refer to electron energies) are oblate ellipsoids, and those of lower-lying sub-band are prolate. So if an electric field is applied parallel to the stress direction, the hole masses are suitably ordered for the occurrence of BNDC due to transfer of holes between the two sub-bands. The relevant effective masses and sub-band splitting rates for both $\langle 100 \rangle$ and $\langle 111 \rangle$ stresses are given in Table 2. The strong nonparabolicity of the valence band under these conditions also favors this effect.

Kastal'skii and Ryvkin² have reported observation of current oscillations due to this effect at 4.2°K in lightly doped samples of p-Ge for both the $\langle 100 \rangle$ and $\langle 111 \rangle$ orientations. More recently the authors¹⁰ have reported a study of the effect at temperatures between 27° and 160°K, the highest temperatures at which oscillations were observed. Even at 27°K no oscillations were observed in the $\langle 100 \rangle$ orientation. As can be seen from Table 2, this orientation is expected to be less favorable than $\langle 111 \rangle$ because both the sub-band splitting rate and the effective mass difference are smaller.

Typical threshold fields are shown as a function of pressure for several temperatures in the 27° to 160°K range in Fig. 4. In most of the curves F_t decreases with increasing stress (increasing sub-band separation) even at the highest stress obtained; at 27°K the minimum in the curve is observed. The splitting per unit stress is smaller in p-Ge than in n-Ge, so this result is consistent with the n-Ge results; the minimum simply occurs at somewhat higher stresses, and only at 27°K are we able to go beyond the stress corresponding to the minimum. Oscillations might also be observed above 160°K if higher stresses could be applied before the samples were destroyed.

The properties of the valence band of InSb are even more favorable for observation of this effect than those of p-Ge. The effective masses of the split sub-bands differ by a larger factor. However, no instability is found in p-type InSb. Instead, for fields above about 500 V/cm a type of breakdown effect is observed. This effect is probably due to impact ionization by the small residual electron population, which rapidly increases the free-carrier concentration. Since the fields required for appreciable ionization by electrons are much lower than for holes, the electron avalanche dominates.

Kastal'skii and Ryvkin n-type germanium

Kastal'skii and Ryvkin⁴ have observed BNDC effects in unstrained, $\langle 111 \rangle$ oriented n-Ge at low temperatures due to intervalley transfer of carriers between valleys with the same minimum energies. To understand the origin of this effect, let us assume that intervalley transfer is entirely by emission of a phonon. With the current and field in the $\langle 111 \rangle$ direction, one of the four valleys has conductivity effective mass $19.07 m_i$, and the other three are significantly lighter, with $m^* = 1.11 m_i$. At low fields the electrons are distributed uniformly among the four valleys, and the conductivity is given by

$$\sigma = n_h e \mu_h + 3 n_l e \mu_l, \quad (6)$$

where n and μ are the carrier concentrations and mobilities in the heavy (h) and light (l) valleys. The heating rate of electrons in a valley is proportional to $1/m^*$, so electrons in the three light valleys are excited to energies necessary for emission of an intervalley phonon at lower fields than those in the heavy valleys. Since intervalley scattering from one valley to any of the three remaining valleys is equally probable, one third of the electrons which scatter end up in the heavy valley, lowering the average conductivity given in Eq. (6). If this transfer occurs over a sufficiently narrow range of field, the differential conductivity will be negative in this range.

Intervalley transfer can also occur by way of impurity scattering, which has the opposite energy dependence and therefore tends to equalize the populations of electrons in the four valleys. Because of this, the effect is observed in relatively pure n-Ge. Scattering by absorbing a phonon also tends to overcome the desired repopulation, so the effect is observed at low temperatures where the density of intervalley phonons is very low. At lower temperatures the phonon emission process is a stronger function of the average energy of electrons in a valley, producing a greater amount of transfer to the heavy valley. Finally, electron-electron collisions tend to equalize the average energies of electrons in all of the valleys, and so must be minimized. This is also favored by choosing lightly doped samples.

The considerations outlined above led Kastal'skii¹⁵ to suggest that the effect would be found in uncompensated n-Ge doped to the order of several times 10^{12} electrons/cm³ with shallow donors, and in the temperature range of 4.2° to 40°K. Current-voltage characteristics measured by Kastal'skii and Ryvkin⁴ are shown in Fig. 5. They observed BNDC at temperatures from 4.2° to 30°K, above which there was no instability. In the case of the 4.2°K data, the sample was constantly illuminated to produce electron-hole pairs. At low illumination levels, i.e., low carrier concentrations, the I - V curve shown was measured; here the carrier concentration was too low to allow formation of domains in samples of the

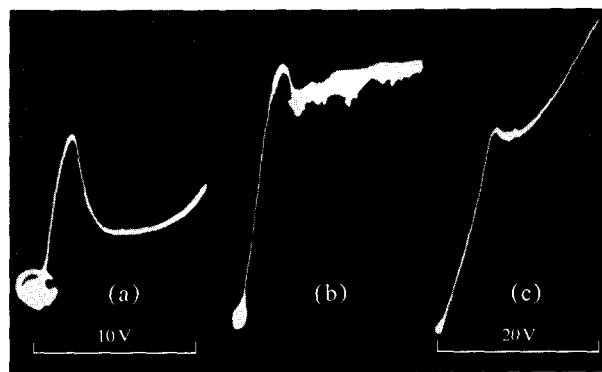


Figure 5 Current-voltage characteristics of n-Ge with shallow-donor density $3 \times 10^{12} \text{ cm}^{-3}$ at 4.2°K (a), 20°K (b), and 30°K (c). Current is in a $\langle 111 \rangle$ direction. (From reference 4).

length chosen for the experiments. At higher illumination levels, domains were formed, and current oscillations were observed with frequency corresponding to the sample transit time. At higher temperatures the I - V curve shows noise in the region of the BNDC, due to the presence of high-frequency oscillations.

Kastal'skii and Ryvkin cited the results of experiments where uniaxial stress was applied in the $\langle 111 \rangle$ direction parallel to the current as further evidence in favor of their analysis. As discussed above, $\langle 111 \rangle$ uniaxial compression lowers the energy of the valley along the stress direction relative to that of the other three $\langle 111 \rangle$ valleys. The relative population of the heavy valley increases in zero field, and the explanation of the effect predicts a smaller BNDC effect. Their experiments show that at 4.2°K stress does decrease the depth of the negative conductivity, and it vanishes completely with a compression of the order of 200 to 300 kg/cm². Here the valleys are separated by 6 to 10 kT_i , so that in zero field the population of the light valleys is negligible, and intervalley transfer tends only to increase the conductivity. Finally they noted that when the field was applied along a $\langle 100 \rangle$ direction, with respect to which all four $\langle 111 \rangle$ valleys are equivalent, no instability or BNDC was observed.

Of the instabilities discussed in these papers, this one is understood in the greatest detail. The experiments of Kastal'skii and Ryvkin are in quantitative agreement with the predictions of Kastal'skii. The electron energies involved in this effect are low enough that only electron states in the $\langle 111 \rangle$ valleys of the conduction band need be considered, explaining in part the success of simple theory in treating the phenomenon.

Conclusion

In the two years since the publication of Conwell's review¹⁶ of high field transport, our understanding of the behavior

Table A Tabulation of materials in which BNDC effects have been observed and some of their properties.

Material	Minimum threshold field (V/cm)	Drift velocity at threshold (cm/sec)	Special conditions	Ref.
n-GaAs	3200	2×10^7		7, 18
n-InP	7200	—		7, 17
n-GaAs _{1-x} P _x	2300		$x \geq 0.6$	19
n-CdTe	13000	1.3×10^7		20
n-InAs	1400	$> 2 \times 10^7$	Uniaxial stress ($> 1.4 \times 10^4$ kg/cm ²) along $\langle 111 \rangle$	21
n-ZnSe	38000	1.5×10^7		22
p-Ge	8(4.2°K) 300(77°K)	$\sim 5 \times 10^6$ (4.2°K)	$T \leq 160^\circ\text{K}$ Uniaxial stress and current along $\langle 111 \rangle$ or $\langle 100 \rangle$	23, 24
$\langle 100 \rangle$ or $\langle 110 \rangle$ n-Ge	1800(27°K) 2300(77°K)	1.4×10^7 (77°K)	$T \leq 120^\circ\text{K}$	25
Stressed n-Ge	270(27°K) 1900(300°K)	$\sim 2 \times 10^7$ (27°K)	Uniaxial stress to split $\langle 111 \rangle$ minima, current perpendicular to stress	3
$\langle 111 \rangle$ n-Ge	2.0(4.2°K)	$3 - 5 \times 10^6$	$T \leq 30^\circ\text{K}$ $n \lesssim 5 \times 10^{13}$ cm ⁻³ current along $\langle 111 \rangle$ 77°K, hydrostatic pressure $\geq 10^4$ bars ^a	4, 26
n-InSb	200	$\sim 3 \times 10^7$		27
n-Ga _{1-x} In _x Sb	600	$\sim 1 \times 10^7$	$x \geq 0.3$	28

^a Note added in proof: The Gunn effect has recently been observed at atmospheric pressure at 77°K in n-InSb, although the presence of a large amount of impact ionization makes it impossible to observe the effect for longer than the order of 10^{-9} sec. [J. E. Smith, Jr., J. C. McGroddy, M. I. Nathan, S. A. Porowski, and W. Paul, to be published.]

of germanium in high electric fields has increased significantly. The experiments described in these papers have emphasized the necessity of including intervalley scattering to the higher energy $\langle 100 \rangle$ valleys in calculations of the high field properties in n-Ge. Recent theoretical work has demonstrated the importance of the non-parabolicity of the energy-band structure and the non-classical nature of the phonon distribution. The experiments described in these papers are at least qualitatively understood at this point. Monte Carlo and similar numerical techniques have had some success at accounting for the gross features of the BNDC found in cooled, unstrained $\langle 100 \rangle$ and $\langle 110 \rangle$ n-Ge, described in I. Because of the complexity of the problem, such techniques appear to be necessary for a proper treatment of any of the high-field phenomena described in these papers, with the possible exception of the BNDC in $\langle 111 \rangle$ unstrained n-Ge at very low temperatures. The average carrier energies are rather small here, and simple theory has been successful in accounting for the experimental results. The success of the numerical techniques in treating the high-field transport problem in GaAs as well as in $\langle 100 \rangle$ n-Ge suggests that such techniques have great potential for treating the effects described in these papers, as well as a much broader range of high-field transport problems.

Appendix

For the sake of completeness we list in Table A all the materials which have been reported to date to exhibit the Gunn effect and related intrinsic longitudinal BNDC effects. The threshold field, the electron drift velocity at the threshold field, and special conditions (e.g. low

temperatures or the application of stress) required for observation of the effect are also indicated. It is interesting to note that except for Kastal'skii and Ryvkin's low temperature instability in Ge that saturated drift velocity is in all cases within a factor of two of 10^7 cm/sec. In addition to the materials listed in the table, unsuccessful attempts at observing these effects have been reported in uniaxially strained n-type silicon and p-type InSb.¹⁰

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