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Off-axis Acoustoelectric Domains in CdS*

Abstract: In CdS crystals oriented with the electric drift field parallel to the c axis, acoustoelectric domains consist of off-axis shear waves. This is because there is no acoustoelectric gain for shear waves traveling along the hexagonal axis, while the gain may be large in an off-axis direction. The particular angle at which the gain is a maximum depends on the angular dependence of the electromechanical coupling coefficient and the component of the electron drift velocity along that angle. These factors combine to make the angle of maximum gain a function of drift velocity along the c axis. Using a stroboscopic strain-birefringent method, we observed the off-axis domains directly. The domain tilt angle has been found to depend on drift velocity in roughly the same way as predicted from the small-signal angular dependence theory. Discrepancies may be the result of large-signal effects or of angular dispersion.

Introduction

Many experiments have been performed with semiconducting piezoelectric crystals demonstrating the spontaneous growth of acoustic flux and the formation of acoustoelectric domains when the electron drift velocity is greater than the sound velocity. In hexagonal CdS crystals one of two principal crystallographic directions is usually chosen for study. Usually the sample is oriented so that the longitudinal dimension of the sample and the drift velocity are in the basal plane (perpendicular to the c axis). This is the direction most conducive to shear-wave interaction. The other principal direction takes the drift velocity parallel to the c axis, which favors pure longitudinal mode interaction. In the latter case, many workers have reported that the acoustoelectric domains consist mainly of off-axis shear waves rather than longitudinal waves.[1] Shear waves arise essentially because the shear-wave velocity is less than half the longitudinal-wave velocity, and are therefore more easily pumped for $v_d > v_s$. They are off-axis because the domain is thought to begin from thermal- and/or shock-excited acoustic waves initially spread over a wide frequency and angular band and which grow in that frequency range and in that angular direction with maximum gain. Recently, the development of a technique for direct visual observation of acoustoelectric domains in two dimensions[2] has opened the possibility of testing some of these ideas.

* Research supported in part by Naval Air Systems Command under contract N00019-69-C-0139
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Theory

The Hutson and White small-signal gain equation[1] does not strictly apply to the case of shear waves propagating in other than principal crystallographic directions. The problem has recently been treated in detail by Kikuchi, Chubachi, and Sasaki[3], and by Klein[4] in the same general context of small-signal, linear theory. The gain equations have been generalized to include propagation in an arbitrary direction in an anisotropic piezoelectric semiconductor. Although the complete process of domain growth and formation surely involves highly nonlinear interactions, it is likely that the linear theory applies to the early growth stage of the acoustic flux and hence leaves its imprint on the final domain shape. According to this linear theory, modified to apply to a general propagation direction in an hexagonal crystal,

$$\alpha = \frac{K^2(\theta)}{2v_s(\theta)} \frac{\omega_c \gamma}{\gamma^2 + [(\omega/\omega_D) + (\omega_c/\omega)]^2}. \qquad (1)$$

This gain equation is cast into the same form as the one-dimensional Hutson–White expression with the usual meanings: $K(\theta)$ is the equivalent of the electromechanical coupling coefficient, but is now a function of θ, the angle between the c axis and the wave normal; $v_s(\theta)$ is the phase velocity of sound, again a function of θ as is $\omega_D = v_s^2(\theta)/D$; and $\gamma = 1 - [v_d/v_s(\theta)] \cos\theta$ takes into account the component of v_d along the direction of propagation. For our present purposes we may immediately specialize to the high-conductivity limit applicable to semiconducting material, $\omega_c^2 \gg \omega_D^2$, and at the frequency of maximum

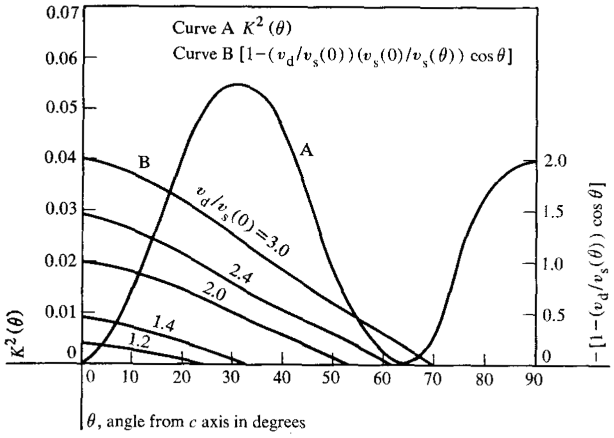


Figure 1(A) K^2 (θ) vs. θ for the T_2 (mixed shear) mode, (B) Drift velocity function γ vs. θ with $v_d/v_s(0)$ as parameter.

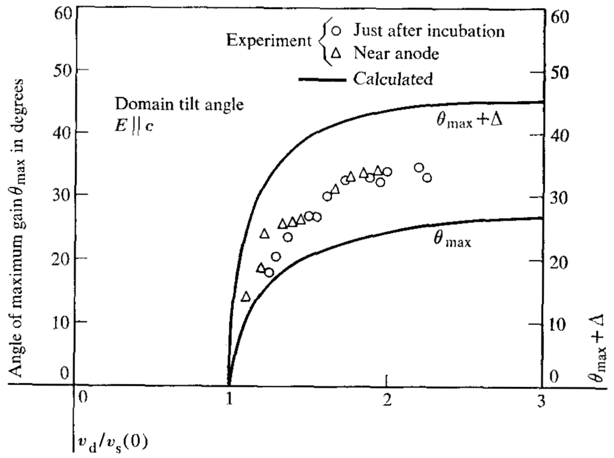
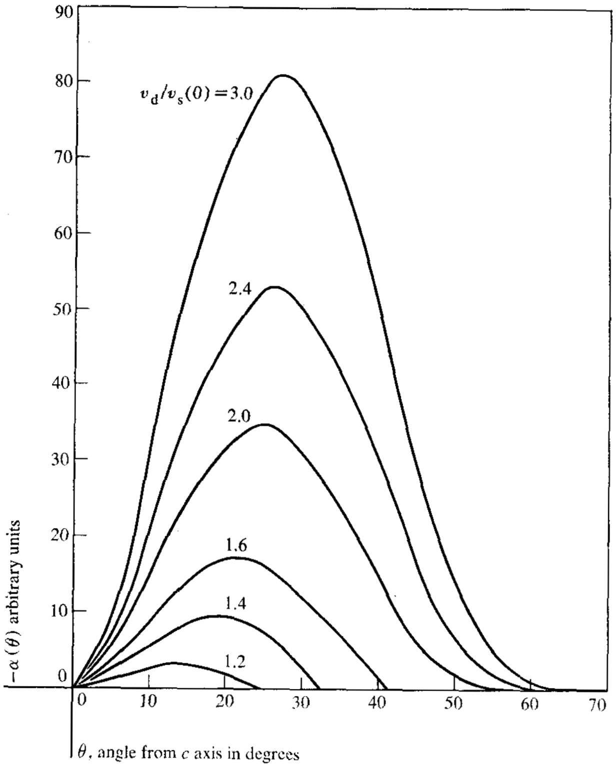


Figure 3 Lower curve: angle of maximum gain from Fig. 2 vs. $v_d/v_s(0)$. Upper curve: angle of maximum gain corrected by the deviation angle Δ, plotted vs. $v_d/v_s(0)$. Experimental points: measured domain tilt angle vs. $v_d/v_s(0)$.

Figure 2 $\alpha(\theta)$ vs. θ, the dependence of small-signal gain on propagation direction relative to the c axis.



gain $\omega_0^2(\Theta) = \omega_c\omega_D$, including all the angular dependent factors:

$$\alpha(\theta) \propto \left(\frac{v_s(\theta)}{v_s(0)}\right) K^2(\theta)\left[1 - \left(\frac{v_d}{v_s(0)}\right)\left(\frac{v_s(0)}{v_s(\theta)}\right)\cos\theta\right] \quad (2)$$

The function $K^2(\theta)$ for the T_2 (mixed shear) mode in CdS has been computed[3,4] and is shown in Fig. 1(A). It has a maximum at approximately 30°. The drift velocity function in square brackets is plotted in Fig. 1(B) for various values of $v_d/v_s(0)$. The final $\alpha(\theta)$ in arbitrary units is shown in Fig. 2. Note that the angle of maximum gain decreases as $v_d/v_s(0) \to 1$ because of the cutoff in Fig. 1(B); i.e., $[v_d/v_s(\theta)]\cos\theta$ must always be larger than one to get any gain at all. This angle of maximum gain is shown as a function of $v_d/v_s(0)$ in the lower curve of Fig. 3.

Results

Figure 4 shows some representative photographs of off-axis domains taken in a semiconducting CdS sample oriented with the drift field parallel to the c axis. The drift velocity ratios refer to the incubation period. The domain angle is easily obtained by direct measurement on an enlarged projection of the original negative. On the assumption that the domain angle is a measure of the angle of maximum gain, a comparison can be made with the calculation of Fig. 3. The data plotted were taken on two crystals of similar characteristics. Since the theory applies strictly only to the earliest stages of domain formation, an attempt was made to make domain photographs as close to the end of the incubation time as possible. For comparison, another set was taken with the domain near the anode. Both sets are plotted. There appears to be no great difference between them, indicating that the domain angle does not change appreciably over that part of the transit interval during which it is visible. This observation was confirmed by separate photographs.

The data appear to follow the general shape of the curve computed from the angular dependence theory, but lie distinctly above it at all angles. Now we must ask what

Table 1. Off-axis domain transit time (Electric field, E, parallel to c, sample length 4.25 mm).

$v_d/v_s(0)$	Domain Angle θ (degrees)	$v_s(\theta)$ 10^5 (cm/sec)	$v_s(\theta)/\cos\theta$ 10^5 (cm/sec)	T_{calc} (μsec)	T_{obs} (μsec)
1.25	18	1.88	1.97_5	2.15	2.20
1.33	22	1.93	2.08	2.04	2.05
1.47	25	1.98	2.18	1.95	1.95

the photographs actually show. The angularly dependent velocity appearing in the theory is the phase velocity because gain depends on phase-matching. But the photographs show the local concentration of strain in some mean square sense, and thus measure the local strain energy or the disturbance. It has long been known from calculations by Musgrave and collaborators[5] that because of the acoustic anisotropy of hexagonal crystals a large angular deviation exists between the phase and group velocity. This is a form of spacial dispersion which causes the phase front to propagate at an angle with respect to the ray or energy flow direction. The deviation Δ can be computed from a knowledge of the angular dependence of the phase velocity.[6] Using this relationship between Δ and θ, we can plot the total off-axis angle for energy propagation vs. the drift velocity ratio, to obtain the upper curve of Fig. 3. Note that this curve lies everywhere above the measured points.

Conclusions

It is not possible to say with certainty why the observed domain angles do not agree exactly with the calculated energy flow. Probably this discrepancy is due to our inability to observe domains in the linear region where the theory is properly applicable. Our values of $v_d/v_s(0)$ were obtained during the incubation time. At the end of that time, v_d begins to fall toward v_s. Somewhere along this path the linear theory fails. All of the photographs used to make Fig. 3 were taken at times past this point; but a few pictures have been obtained, for different crystals and under somewhat different circumstances, which show a bending of the domain. These suggest that the domain angle may "track down" as v_d falls, but the system is still almost linear. When the domain is fully formed, however, it remains stable and maintains its angle during transit as already noted. The point is that the effective v_d may actually be somewhat less than estimated from the ohmic current. This would at least shift the points in the direction of the calculated curve.

An internal check on the validity of the photographic evidence of Fig. 3 can be made by independent electrical measurement of the domain transit time for a c axis sample. Because the domain consists of off-axis waves traveling at the angle θ with respect to the c axis and the

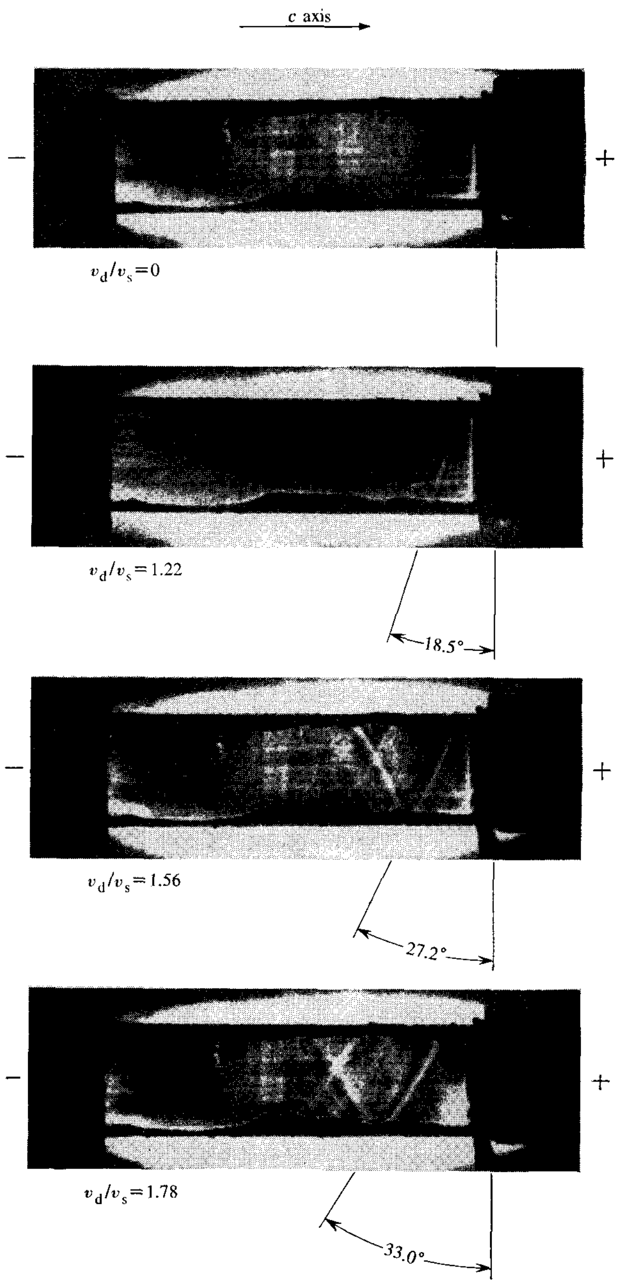


Figure 4 Photographs taken by the stroboscopic strain-optic technique, showing the dependence of domain tilt angle on initial drift velocity. The picture at the top was taken with no voltage applied to the crystal to show the background against which the domain appears as illustrated in the other three photographs.

domain itself travels *along* the c axis, the longitudinal axis of the sample, the domain velocity is $v_s(\Theta)/\cos\Theta$. Thus, the transit time from cathode to anode depends on Θ which, in turn, depends on $v_d/v_s(0)$. Table 1 assembles data taken electrically and photographically on the same sample. The measured transit time is in good agreement with that calculated from the domain angle. Thus, we are led to believe that what we see in the photographs is no artifact.

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6. Data supplied to R. W. Smith by J. deKlerk (see also Ref. 3).

Received April 14, 1969