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Parametric Amplification and Frequency Shifts in the Acoustoelectric Effect

Abstract: Mechanisms for the downshift in the frequency of maximum acoustic intensity f_{mi} for high flux domains in piezoelectric semiconductors are reviewed. For the simple case where an externally introduced acoustic wave (pump) produces a single-frequency domain in photoconducting CdS, clear evidence is given that the downshift in f_{mi} is due to parametric amplification of thermal acoustic noise. For a pump of 990 MHz, after some initial growth ($v_d = 1.14\, v_s$), the pump is found to be depleted In the pump depletion region, signals in a 200 MHz band about the even subharmonic (445 MHz) are found to grow. At pump strains of about 10^{-6} the signals propagated at angles to the pump equal to those that give phase matching according to the dispersion of linear theory. For higher pump strains, however, the collinear process is dominant. The signal domain is narrower than the pump domain, as expected, because the parametric growth is exponentially dependent on pump strain. The downshifting of f_{mi} in the region where deviations from linear theory are still small is discussed in terms of a parametric interaction model, with the initial acoustic strain distribution considered as an incoherent pump.

Introduction

The linear theory of acoustic amplification in piezoelectric semiconductors[1] is in satisfactory agreement with experiment[2] only for small signals. When acoustic domains are propagated, in which the acoustic flux levels are high, important departures from the predictions of this theory have been found experimentally.[3,4] In particular, it has been observed that the frequencies of maximum net acoustic gain and intensity decrease, as the acoustic intensity increases, to a value as much as an order of magnitude lower than that predicted by linear theory.[3,4] In this paper we consider mechanisms which may explain these downshifts.

Traveling-wave parametric amplification of acoustic waves provides us with one such mechanism. Since we have unambiguous evidence for strong parametric amplification due to externally introduced power in piezoelectric semiconductors,[5–7] let us consider this process first.

Parametric amplification

Parametric amplification may be thought of as a stimulated emission process[8] in which a pump phonon of frequency ω_p emits signal and idler phonons of frequency ω_s and ω_i, respectively, where for conservation of energy

$$\omega_p = \omega_s + \omega_i. \quad (1)$$

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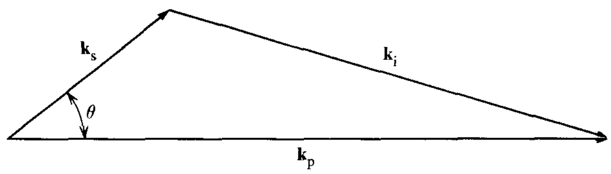


Figure 1 Phase matching for parametric amplification where $\mathbf{k}_p$, $\mathbf{k}_s$, $\mathbf{k}_i$ are the wave vectors of the pump, signal and idler respectively.

In this paper we consider only parametric amplification of the thermal noise. The identification of "signal" and "idler" frequencies, therefore, is arbitrary. The phase matching, conservation of momentum condition is shown in Fig. 1 in which $\mathbf{k}_p$, $\mathbf{k}_s$, and $\mathbf{k}_i$ represent the pump, signal and idler wave vectors, respectively. Since we have found strong transfer of pump energy to signal energy, we assume that the phase matching condition holds. The three waves are shown propagating at angles to one another, indicating the presence of dispersion. If there were no dispersion, conservation of energy and momentum would be satisfied by collinear waves.

Keeping terms in the electric field up to second order in the displacement, Conwell and Ganguly have set up coupled wave equations for pump, signal and idler shear waves and solved them for the initial rate of growth of the signal and idler.[6,9] This theory predicts, for conditions generally met in the experiments, that the gain will be greatest for $\omega_s \simeq \omega_i$,[10] as is experimentally observed.[5-7] Consider then the growing solution for the signal amplitude u_s where $\omega_s = \omega_i$ and all waves are confined to the basal plane. For drift velocity v_d greater than sound velocity v_s, the solution may be written approximately[6]

$$u_s(x_s) \cong u_s(0)e^{(|\eta_0|+|\alpha_e|-\alpha_l)x_s}, \tag{2}$$

where $u_s(0)$ is the signal amplitude at $x_s = 0$, $|\alpha_e|$ is the linear acoustoelectric gain under amplifying conditions,[1] α_l is the nonelectronic loss and $|\eta_0|$ the parametric gain coefficient defined in Ref. 6. Parametric gain would be observable in the presence of linear gain only when $|\eta_0|/|\alpha_e|$ is of order unity. For small θ (defined in Fig. 1) we have

$$\frac{|\eta_0|}{|\alpha_e|} = \frac{e_{113}}{2\epsilon v_s}\mu S_p \frac{\left[f\left(\gamma, \frac{\omega_p}{\omega_D}, \frac{\omega_s}{\omega_D}, \frac{\omega_c}{\omega_p}\right)\right]^{\frac{1}{2}}}{|\gamma|}, \tag{3}$$

where e_{113} is the piezoelectric coefficient, ϵ is the dielectric constant, μ is the mobility, ω_c is the conductivity divided by the dielectric constant, ω_D is the velocity of sound squared divided by the electron-diffusion constant, S_p is the pump strain, $\gamma = 1 - \mu E/v_s$, E is the electric field and f is a function given in Ref. 6. Note that this ratio is proportional to μ and S_p. The factor $f^{\frac{1}{2}}$ (which comes from $|\eta_0|$) is only weakly dependent on γ and is of order unity.[6]

Single frequency domain

Let us next discuss the experimental evidence for parametric amplification. First we consider an unambiguous situation in which the initial spectrum of the acoustic domain consists of a single frequency. Figure 2 shows how we generate and detect the single frequency domain (SFD). A photoconducting CdS bar of dimensions $1 \times 1 \times 4$ mm^3 is placed with one polished end inserted into the high-electric-field region of a coaxial cavity with a loaded Q of about 1000. The cavity is subjected to a 0.5 μsec pulse with nominal peak microwave power of 20 watts. Since the c axis of the sample is perpendicular to the long direction, a 0.5 μsec shear wave acoustic pulse is transduced into the sample. The sample is grounded about 10 mils from the polished end in order to reduce the microwave fields in its body. However, similar results were obtained when the entire sample was placed within the microwave cavity.[6] The trailing edge of the microwave pulse is made coincident with the leading edge of a voltage pulse of sufficient amplitude V to cause $v_d \gtrsim v_s$ ($\gamma = 0.14$). This value of V is chosen in order that the SFD be slightly amplified, but the amplification of the thermal noise be negligible.

The acoustic waves were detected by means of Brillouin scattering of a 10 mW 6328 Å He–Ne laser beam which was chopped to keep the laser-induced photoconductivity below 1% of the total photoconductivity.[6] It was focused

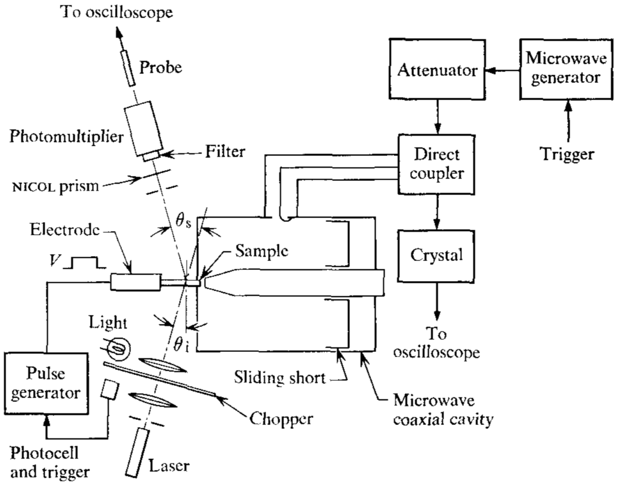


Figure 2 Apparatus for generating and detecting a single frequency domain. The sample is illuminated from above (not shown).

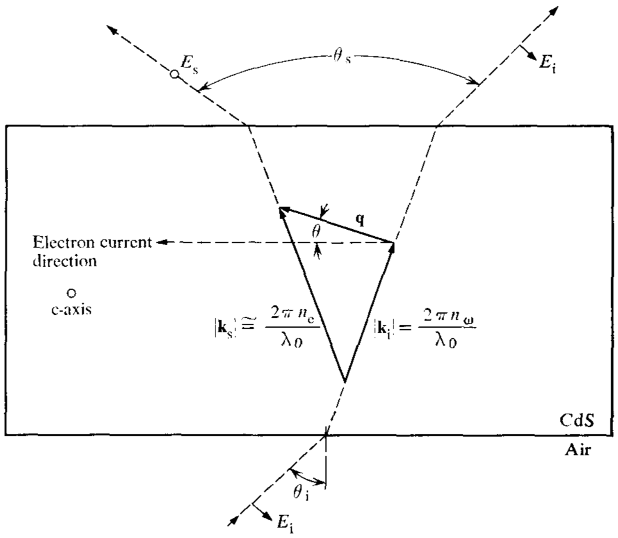


Figure 3 Brillouin scattering from a shear wave in CdS with particle displacement in the c direction.

to a spot about 0.2 mm in diameter on the sample. Figure 3 illustrates the Brillouin scattering process in more detail. A photon of wave number k_i absorbs or emits a phonon of wave number q, energy being conserved. Since the phonon energy is much less than the photon energy, the change in energy of the scattered photon may be neglected. The conservation of momentum condition is shown in Fig. 3. $\mathbf{k}_s > \mathbf{k}_i$ because scattering by a shear wave rotates the plane of polarization of the light by 90° and CdS is birefringent. A measurement of θ_i and θ_s uniquely determines q and θ. The resolution in acoustic frequency is about 50 MHz and the resolution in θ is about $\frac{1}{2}$°.

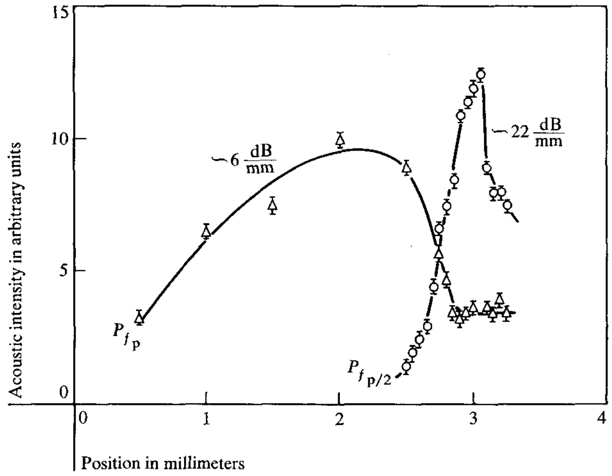


Figure 4 P_{f_p} and $P_{f_p/2}$ vs x, where the power is traveling in the basal plane at the angles shown in Fig. 6. A photoconducting 715 ohm-cm CdS sample is used.

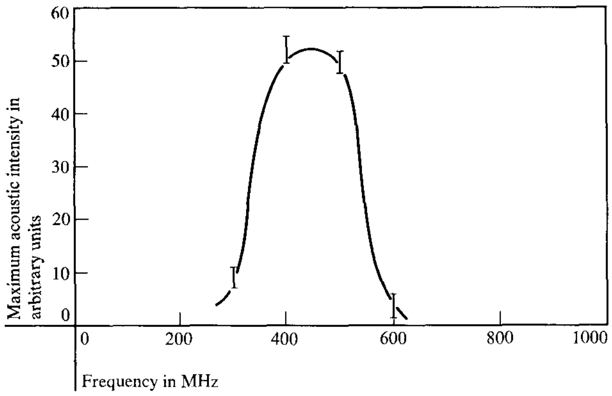


Figure 5 Maximum intensity of the signals parametrically amplified from noise vs frequency.

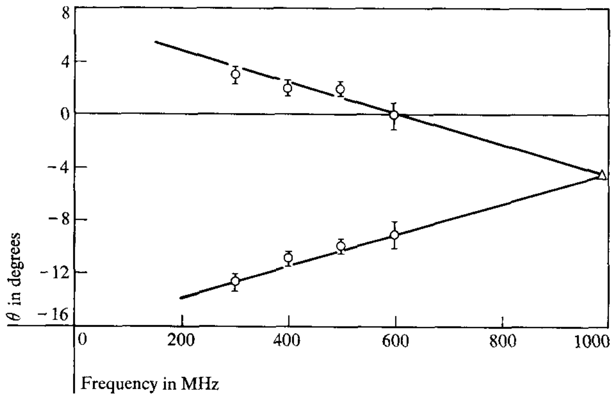


Figure 6 The angle of acoustic propagation vs frequency. The solid line represents the phase matching condition using linear theory dispersion for $\gamma = 0.1$.

For the data to be discussed, the SFD was transduced in at $f_p = 990$ MHz. Similar results were obtained at other frequencies. The sample resistivity was 715 ohm-cm, which corresponds to a frequency of maximum net gain f_{mn} of about 450 MHz according to linear theory.[1,11] The frequency spectrum of the acoustic domain changes as the domain propagates down the sample, as shown in Fig. 4. The power at f_p, P_{f_p}, is at first amplified by the linear gain at about 6 dB/mm to a maximum value, after which it falls rapidly to a saturated value. In the region in which P_{f_p} drops, we observe the growth of acoustic waves in the frequency range around the even subharmonic at $f_p/2$. Such behavior is characteristic of parametric amplification where the pump is the SFD and the signal and idler are amplified from the thermal noise. In Fig. 4 we see that, for example, $P_{f_p/2}$ grows at about 22 dB/mm, reaches a peak and drops quickly. Such attenuation often implies the generation of waves around $f_p/4$.[6] The frequency distribution of the parametrically amplified noise at 3 mm (see Fig. 4) is plotted in Fig. 5. It peaks around $f_p/2$ and has a half-width of about 200 MHz.* The waves shown in Fig. 5 are not collinear with the pump but propagate at the angles indicated by the points in Fig. 6. Note that the pump propagation direction is at an angle of $-4\frac{1}{2}°$ to the current direction, possibly because of a misalignment of the sample in the cavity. At each signal frequency, the intensity peaks symmetrically on either side of the pump direction, as one would expect. The solid lines represent the theoretical predictions using conservation of momentum (illustrated in Fig. 1) and the dispersion according to linear theory,[1] assuming $\gamma = 0.1$. (The curve is not very senitive to γ). Clearly, there is good agreement between experiment and second-order theory, at least for pump strains equal to or below those present in this experiment i.e., $S_p \leq 10^{-6}$. In previous experiments where $S_p \sim 10^{-5}$, the signals have been found to be collinear with the pump.

It is interesting to look at the domain shapes of the pump and signals in the region of pump depletion. Figure 7 shows an oscilloscope trace of the photomultiplier output vs time due to Brillouin scattering from P_{f_p} and $P_{f_p/2}$ in the pump depletion region. Note that the pump is depleted and the signal is strong only in the center core of the domain. Thus the signal domain is narrower than the pump domain. This narrower signal domain is to be expected for a nonlinear process which, in the present case, depends exponentially on the pump strain.

Downshifting in domains

Now that we have considered the simple case of a SFD where the mechanism for the downshifting of the fre-

* When the voltage is increased, one begins to detect in addition the mixing terms $f_p + f_s$, $2f_s$ and $2f_p$. These are initially at least an order of magnitude below $f_p/2$.

quency of maximum intensity f_{mi} is clearly parametric, let us consider the case of the conventional acoustoelectric domain amplified from the noise by electrons with supersonic drift velocities. As stated previously, the frequency of maximum gain and intensity in propagating domains can decrease to a value as much as an order of magnitude below the predictions of linear theory[3,4] and the acoustic intensity can continue to grow several orders of magnitude even though the current and electric field remain relatively constant.[12,13] In our samples the domains arise from acoustic waves continuously amplified at a high-gain region near the upstream contact. Potential probing showed the electric field to be high in this region. The domains narrow as they propagate downstream.

The significant results of this experiment are shown in Fig. 8 in which f_{mi} is plotted versus distance from the cathode x and the current is plotted vs time in units of x/v_s. The initial γ as determined from the ohmic field was 0.65 and the frequency of maximum net gain,[1,11] f_{mn}, was 2.5 GHz for a 10.4 ohm-cm CdS sample calculated on the basis of a mobility of 300 cm^2/V-sec. When the domain is first observed the current is still approximately ohmic and $f_{mi} \simeq$ 2.7 GHz, in reasonable agreement with the above prediction[17,18] in view of the fact that K^2 and the nonelectronic loss are not known accurately. As the current falls to its saturated value, f_{mi} drops to about 1.2 GHz, while the total acoustic intensity increases by a factor of about 85 (relative intensities are given in the diagram). Some researchers in the field have attempted to describe the nonlinear regime by assuming that linear theory holds with $\gamma \equiv 1 - v_d/v_s$. Such a model predicts that, as v_d drops in time, the gain drops, causing the frequency of maximum net gain to drop as well.[19] Putting the relevant numbers into the gain theory[1] and subtracting the expected loss[11] data for our sample gives a downshift which is much too small to explain the experimentally observed downshifts. For longer samples or higher applied voltages, f_{mi} and the relative intensity approach a limiting value which, in the case of f_{mi}, can be more than an order of magnitude less than the maximum net gain according to linear theory. The high-frequency acoustic waves saturate first and peak at the same θ as do the low-frequency waves.

Let us attempt to understand the downshifting in f_{mi} by considering the situation in the region where deviations from linear theory just begin to occur. In this case a second order calculation should be valid. Using a high-intensity single-frequency acoustic wave as an initial spectrum, Butcher and Ogg[20] have concluded that, due to trapping of the carriers, the apparent ω_c would decrease as the acoustic intensity increased, thus causing a downshifting in the frequency of maximum gain. However, Ganguly and Conwell[21] have done a calculation in which terms in the electric field up to third order in the displacement are kept. This calculation predicts that mixing of the

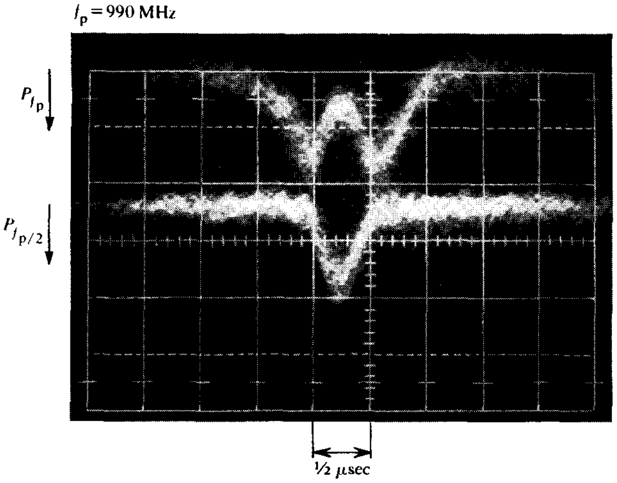
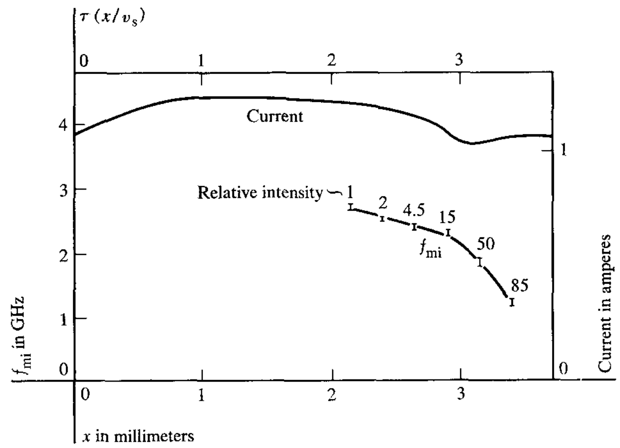


Figure 7 Brillouin scattering from P_{f_p} and $P_{f_{p/2}}$ vs time for the region of pump depletion. f_p = 990 MHz.

Figure 8 Frequency of maximum intensity f_{mi} vs distance x in mm and current vs time in units of x/v_s where v_s is the shear wave velocity for a 10.4 ohm-cm CdS sample. The approximate relative intensity of the domain is noted above the points.



different frequencies will cause, in effect, an increase in the apparent ω_c. Therefore, for small deviations from linear theory, only parametric amplification would seem to be available to generate downshifts in f_{mi}.

An approximate calculation of the effect of parametric amplification on the acoustic spectrum in a domain has been made using the second order calculation mentioned earlier. Up-conversion has been neglected since in the SFD it is found that down-conversion dominates, at least in the early stages of growth. The initial spectrum consists of an acoustic domain formed through amplification from the thermal noise, the amplification characteristic being given by linear theory. Each frequency is assumed to be pumped by all higher frequencies in the domain. The resulting net parametric gain at any given frequency is obtained by summing over all contributions, taking into account the randomness of the phases. At $\gamma = 0$ the parametric gain is found to increase monotonically from zero at the high-frequency cutoff of the domain (where linear gain equals lattice loss) to a peak at half the frequency of maximum net gain $f_{mn}/2$ predicted by linear theory.[22] One might then speculate that, for $v_d > v_s$, where the gain characteristic is determined by a combination of the linear and parametric gain, and for sufficiently large parametric gain, the frequency of maximum net gain would smoothly shift downward (as observed in CdS,[12,15–17] ZnO[4] and GaAs[18]) and a subsidiary peak would appear at $f_{mn}/2$. Such a subsidiary peak has been reported for GaAs in the region of small deviations from linear theory.[18,23] One might expect to observe stronger parametric effects in GaAs from Eq. (3) because the important parameter in $|\eta_0|/|\alpha_e|$, $(e/2\epsilon v_s)\mu$, is six times greater for GaAs than for CdS, principally because the mobility of GaAs is about 25 times that of CdS.

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