R. E. Norwood

Effects of Bending Stiffness in Magnetic Tape

Abstract: The elasticity of magnetic tape is an important factor in determining the shape of a tape loaded over a magnetic head Over a single apex the radius of curvature of the tape is inversely proportional to the wrap angle and to the square root of the ratio of tape tension to bending stiffness. If a constant-pressure support is assumed instead of a knife edge, the radius of curvature increases considerably. A chart is provided for use in calculating the radius of curvature under different loading conditions.

Over a double apex the results are the same as for a single apex except when the distance between apexes becomes small. By increasing the radius of curvature of the head to conform to the radius of curvature of the tape, the rate of head wear can be greatly reduced.

Introduction

The purpose of this communication is to describe some of the effects of tape elasticity or bending stiffness on the conformity between a magnetic recording head and the tape passing over it. Only the static tape shape is considered; that is, actual air bearing effects are approximated by assuming that air pressure is constant throughout the region between the tape and the head. Dynamic effects have been ignored.

Most previous analyses of tape shape have been based on the assumption that the tape is perfectly flexible. On visual examination it certainly appears flexible; in application, however, the radii over which the tape is loaded are so small that stiffness takes on primary importance.

In this communication, the basic equations are derived and then applied to loading over a double-apex representation of a real head contour. The effect of spacing between apexes is illustrated and two special cases are shown to correspond to two different single-apex configurations.

Although this note in itself will not enable one to design an optimum head, it should enable a designer to determine the upper and lower bounds of the radius of curvature of the tape at the gap in an actual application.

Basic equations

We begin by considering the differential equation that describes the shape w of the tape when loaded by an arbitrary pressure distribution $P(x)$:

$$D \frac{d^4 w}{dx^4} - T \frac{d^2 w}{dx^2} = P(x), \tag{1}$$

where D is the bending stiffness of the tape and T is the tension in the tape. We consider the problem in the x direction only, ignoring effects across the width of the tape such as anticlastic curvature. Also, we are considering relatively small displacements from the x axis, so that the curvature of the tape can be represented by the second derivative. In practice, most tape wrap angles are small enough to meet this criterion.

Dividing by D and introducing the parameter λ for $\sqrt{T/D}$, we have

$$\frac{d^4 w}{dx^4} - \lambda^2 \frac{d^2 w}{dx^4} = P(x)/D. \tag{2}$$

For this equation the homogeneous solution, or the case when the pressure is zero, is

$$w(x) = Ae^{-\lambda x} + Be^{\lambda x} + Cx + G, \tag{3}$$

where A, B, C and G are constants to be determined by the boundary conditions. To find the complete solution for an arbitrary pressure distribution, $P(x)$, we apply the method of variation of parameters[1] and obtain the desired result:

$$\begin{aligned} w(x) = {} & Ae^{-\lambda x} + Be^{\lambda x} + Cx + G \\ & - \frac{e^{-\lambda x}}{2D\lambda^3} \int_0^x P(x) e^{\lambda x}\, dx \\ & + \frac{e^{\lambda x}}{2D\lambda^3} \int_0^x P(x) e^{-\lambda x}\, dx - \frac{x}{D\lambda^2} \int_0^x P(x)\, dx \\ & + \frac{1}{D\lambda^2} \int_0^x P(x) x\, dx. \end{aligned} \tag{4}$$

The author is located at the IBM Systems Development Division Laboratory in Boulder, Colorado.

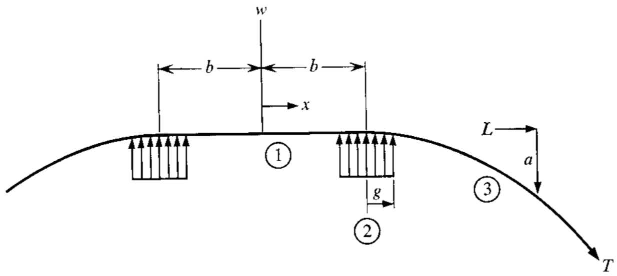


Figure 1 Schematic representation of tape configuration over a double-apex head.

Analysis

Most actual magnetic heads consist of two apexes, one containing the read gap and the other the write gap. In practice there is an air bearing between the tape and the apex, which creates the pressure supporting the tape. To simulate this condition we will assume that pressure is uniform over a certain distance $2g$. Of course pressure is not uniform in practice, but this assumption enables us to obtain a reasonable estimate of the tape shape without detailed consideration of the effects of the air bearing.

It is of interest to determine how the contour of the tape at the apex being considered is affected by the proximity of the other apex and the pressure supporting the tape. The problem is sketched in Fig. 1. We will consider the shape of the tape over the right-hand constant-pressure region.

First we consider region 1, between the coordinate axis and the beginning of the constant-pressure region. The shape of the tape in this region is given by

$$w(x) = Ae^{-\lambda x} + Be^{\lambda x} + Cx + G. \tag{5}$$

From symmetry conditions we know that the curvature at $x = -(b - g)$ is equal to the curvature at $x = b - g$. Therefore

$$Ae^{-\lambda(b-g)} + Be^{\lambda(b-g)} = Ae^{-\lambda(-b+g)} + Be^{\lambda(-b+g)}. \tag{6}$$

Combining and rearranging terms, we have

$$Ae^{\lambda g}e^{\lambda b}(e^{-2\lambda b} - e^{-2\lambda g}) = Be^{\lambda g}e^{\lambda b}(e^{-2\lambda b} - e^{-2\lambda g}). \tag{7}$$

Thus we see that A must be equal to B.

By applying a similar symmetry condition on the shape at $-b + g$ and $b - g$, we can write

$$\begin{aligned} &Ae^{-\lambda(b-g)} + Be^{\lambda(b-g)} + C(b - g) \\ &\quad = Ae^{-\lambda(-b+g)} + Be^{\lambda(-b+g)} + C(-b + g). \end{aligned} \tag{8}$$

We know that $A = B$; hence C must be zero.

We could also evaluate G, by arbitrarily setting $w(x)$ equal to zero at $x = 0$; but since we are interested mainly in the first and second derivatives rather than in the exact shape, we will leave G undetermined.

In region 3, to the right of the constant-pressure region, the expression for the tape shape is

$$\begin{aligned} w(x) = {} & A(e^{-\lambda x} + e^{\lambda x}) + G \frac{-e^{-\lambda x}P}{2D\lambda^4}(e^{\lambda(g+b)} - e^{\lambda(b-g)}) \\ & - \frac{e^{\lambda x}}{2D\lambda^4}(e^{-\lambda(b+g)} - e^{-\lambda(b-g)}) \\ & - \frac{2gxP}{D\lambda^2} + \frac{2gP(b+g)}{D\lambda^2}. \end{aligned} \tag{9}$$

The second derivative is given by

$$\begin{aligned} w''(x) = {} & \lambda^2 A(e^{-\lambda x} + e^{\lambda x}) \\ & - G\frac{\lambda^2 e^{-\lambda x}P}{2D\lambda^4}(e^{\lambda(b+g)} - e^{\lambda(b-g)}) \\ & - \frac{e^{\lambda x}\lambda^2 P}{2D\lambda^4}(e^{-\lambda(b+g)} - e^{-\lambda(b-g)}). \end{aligned} \tag{10}$$

At the end of the tape we assume that the applied torque is zero, so that $w''(L)$ is zero. Next we let L approach infinity, and obtain

$$0 = Ae^{\lambda L} - \frac{e^{\lambda L}P}{2D\lambda^4}(e^{-\lambda(b+g)} - e^{-\lambda(b-g)}). \tag{11}$$

Thus we can find A as

$$A = \frac{P}{2D\lambda^4}e^{-\lambda b}(e^{-\lambda g} - e^{\lambda g}). \tag{12}$$

If we substitute for A and set the shape at L equal to $-a$, then as L approaches infinity we obtain

$$a/L = 2Pg/D\lambda^2. \tag{13}$$

As L approaches infinity, $w'(L)$ approaches $-a/L$; hence the above equation confirms the overall vertical force balance.

We can now write the complete equations for the shape of the tape within the constant-pressure region:

$$\begin{aligned} w(x) = {} & \frac{P}{2D\lambda^4}\Big[e^{-\lambda b}(e^{-\lambda g} - e^{\lambda g})(e^{-\lambda x} + e^{\lambda x}) + \frac{2DG\lambda^4}{P} \\ & - 2 + e^{-\lambda x}e^{\lambda(b-g)} \\ & + e^{\lambda x}e^{-\lambda(b-g)} - (x - b + g)^2\lambda^2\Big]; \\ w'(x) = {} & \frac{P}{2D\lambda^4}[e^{-\lambda b}(e^{-\lambda g} - e^{\lambda g})(-\lambda e^{-\lambda x} + \lambda e^{\lambda x}) \\ & - \lambda e^{-\lambda x}e^{\lambda(b-g)} + \lambda e^{\lambda x}e^{-\lambda(b-g)} \\ & - 2(x - b + g)\lambda^2]; \\ w''(x) = {} & \frac{P}{2D\lambda^4}[e^{-\lambda b}(e^{-\lambda g} - e^{\lambda g})(\lambda^2 e^{-\lambda x} + \lambda^2 e^{\lambda x}) \\ & + \lambda^2 e^{-\lambda x}e^{\lambda(b-g)} + \lambda^2 e^{\lambda x}e^{-\lambda(b-g)} - 2\lambda^2]. \end{aligned} \tag{14}$$

Since we are primarily interested in the curvature at $x = b$, we substitute and obtain

$$w''(b) = \frac{P}{2D\lambda^2}\left[2(e^{-\lambda g} - 1) + e^{-2\lambda b}(e^{-\lambda g} - e^{\lambda g})\right]. \quad (15)$$

Let us now consider some special cases. As b becomes very large,

$$w''(b) = \frac{2P}{2D\lambda^2}(e^{-\lambda g} - 1) = \left(\frac{-a\lambda}{2L}\right)\left(\frac{1 - e^{-\lambda g}}{\lambda g}\right), \quad (16)$$

which represents the case in which the tape is wrapped over a single apex with a total angle of wrap of $\tan^{-1}(a/L)$. We next take the radius of curvature R as the inverse of the second derivative, and θ as the total angle of wrap. Then R at the center of the constant-pressure region is

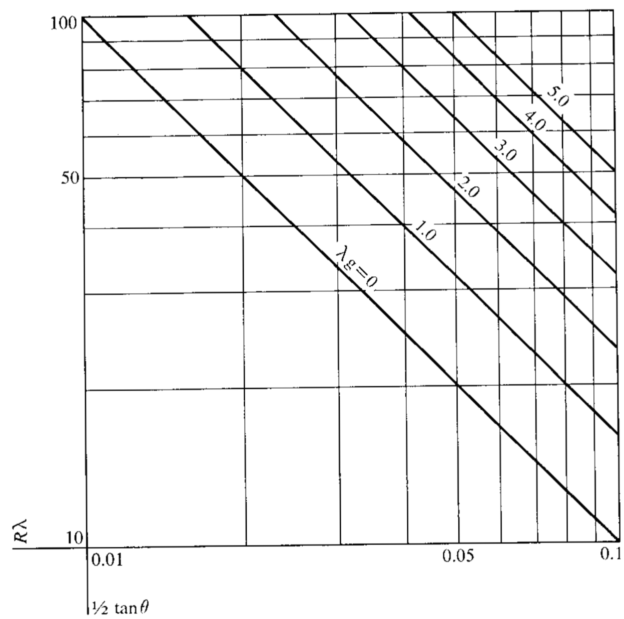
$$R(b) = \frac{-2}{\lambda \tan\theta}\left(\frac{\lambda g}{1 - e^{-\lambda g}}\right). \quad (17)$$

We see that the radius of curvature is negative as expected, and that it is inversely proportional to $\tan\theta$ and λ. When g approaches zero, as would occur over a knife edge, the modifying factor $\lambda g/(1 - e^{-\lambda g})$ approaches unity. When λg becomes large, about 5 or more, we can express R as

$$R(b) = -2g/\tan\theta. \quad (18)$$

Figure 2 shows the dependence of R on λ, g, and θ. The parameters are presented in nondimensional form for simplification.

Figure 2 Dimensionless plot of tape radius of curvature vs. angle of wrap, for various loading conditions.



Let us consider a typical case. We first assume a total wrap angle of 7°, obtaining a value for $(\tan\theta)/2$ of 0.061. For wrap over a knife edge, where $g = 0$, we employ the $\lambda g = 0$ curve in Fig. 2 and obtain a value for $R\lambda$ of 16.5. We will assume that the tension in the tape is 0.7 lb/inch and that the bending stiffness is 5×10^{-4} lb/inch. We then obtain a value for λ of 37.4. Now we can calculate R, obtaining a value of 0.44 inch.

If we assume that the pressure is distributed over a length of 0.060 inch, we have a value of 0.030 inch for g and a value of 1.12 for λg. For this value of λg, Fig. 2 gives a value of 27.5 for $R\lambda$ and a value of 0.73 inch for R.

Instead of letting b become large, let us now allow λg to approach zero. After substituting for P, we have

$$w''(b) = \frac{-a\lambda}{2L}(1 + e^{-2\lambda b}). \quad (19)$$

The above expression illustrates the effect of the second apex upon the first when both apexes are knife edges. As b becomes small, we approach the case of a single apex with total wrap of $2a/L$; as b becomes large, we approach the case of a single apex with a total wrap of just a/L. Since λb occurs as a negative exponent of e, λb need not be very large before its effect can be neglected. For example, if λ is taken as 40 in.$^{-1}$ and b as 0.075 inch, $e^{-\lambda b}$ equals 0.00249 and the effect of b can be neglected.

To determine the effect of b with a finite constant-pressure region, we factor out the expression for $w''(b)$, when b is large, to obtain

$$w''(b) = \frac{2P}{2D\lambda^2}(e^{-\lambda g} - 1) \cdot \left[1 + \frac{e^{-2\lambda b}(e^{-\lambda g} - e^{\lambda g})}{2(e^{-\lambda g} - 1)}\right]. \quad (20)$$

Taking g as 0.030 inch, and λ and b as before, we have

$$w''(b) = \frac{P}{D\lambda^2}(0.301 - 1.0) \cdot \left[1.0 + \frac{0.00249(0.301 - 3.32)}{2(0.301 - 1.0)}\right];$$

hence the effect of b can again be neglected.

Another interesting point can be brought to light by considering the slope of the tape at $x = b$:

$$w'(b) = \frac{P}{2D\lambda^3}\left[-2g\lambda - e^{-2\lambda b}(e^{-\lambda g} - e^{\lambda g})\right]; \quad \text{or}$$

$$w'(b) = \frac{-a}{2L}\left[1 + \frac{e^{-2\lambda b}}{2}\frac{(e^{-\lambda g} - e^{\lambda g})}{g\lambda}\right]. \quad (21)$$

We see that the slope of the tape at the apex is not necessarily equal to half of $-a/L$, as one might expect. A practical effect is that the static point of tangency of the tape with the recording head contour may not occur at the read and write gaps. However, so long as λb is relatively large, the slope will be nearly half of $-a/L$.

Conclusions

The major result obtained from these calculations is that tape stiffness is an important factor in determining the shape taken by a tape passing over a recording head. The radius of curvature of the tape at the apex of a typical head, for example, can be calculated to be of the order of 0.5 inch. Since the head is often ground to a radius of 0.1 inch or less, the tape and the head are nearly in line contact. This condition produces good signal output, but it also produces a high wear rate because of the small clearance and the high unit pressure between the tape and the head. As the head wears, the head radius of curvature increases and the head begins to conform to the tape shape. At the same time the air bearing is developed over a larger area, thus increasing the radius of curvature of the tape. As these two effects proceed, the unit pressure eventually becomes so low, and the head-to-tape separation so large, that the wear rate is very much reduced.

One criterion for good head design, then, is that the radius of curvature of the head at the gap should be nearly equal to the radius of curvature of the tape. Though this condition results in a low wear rate, it may also produce too much head-to-tape separation, because it makes the air bearing more efficient. In this case it may be necessary to destroy or diminish the air bearing effectiveness by means of intertrack, longitudinal slots in the head. Thus it is possible to obtain a low wear rate, by virtue of the good conformity between head and tape and the large wear area and associated low unit pressure, while at the same time achieving a small head-to-tape separation for good signal output and resolution.

Another criterion is that the head should be symmetric. That is, if an observer could "stand" at one gap, he should "see" the same condition up the tape path as down it; the adjacent apexes should be located at the same distance from the observer, and the angle of wrap over each of these apexes should be the same. The static tape shape would then be tangent to the head at the gap, and the forward and backward behavior would be the same.

Acknowledgments

Thanks are due to J. H. Tiao and R. A. Riley for their support of this work.

Reference

1. F. B. Hildebrand, *Advanced Calculus for Engineers*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1949.

Received July 26, 1968