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An Application of the Automatic Dynamic Response Analyzer

Abstract: An experiment to obtain the coefficients of a circuit delay equation was designed and implemented on the ADRA/M44 system in order to gain experience in the system's use. The process involves automatic calibration and diagnostic procedures, data collection under computer control and multivariate regression analysis. Some conclusions are drawn about the planning and use of computer controlled instruments such as ADRA.

Introduction

As integrated circuit packages become more complex, the problem of testing them becomes more difficult. Equipment capable of testing the time-dependent behavior of scores of intricately connected logic circuits will be required in the near future, and such a system is described in the companion paper by Stuckert and Bland.[1] The development of methods to use testers effectively must accompany the hardware effort.

A natural first step is the characterization of delay in unit logic. The discussion in this paper is focussed on the single circuit, although neither equipment nor method is so limited. In particular, we are interested in the classic problem of predicting the delay of a pulse through a logic circuit.

The concept of pulse delay is easy to grasp intuitively but hard to make precise because the measurement technique must apply to pulses of differing shapes. The definition of delay used here is the time interval between the instant at which the input pulse first attains a certain voltage and the instant at which the output pulse first attains the same voltage on the corresponding part of the output pulse. This definition is incomplete in that a criterion is not given for selecting the voltage, but it is a definition which can be implemented as a computer program.

A delay equation is the conventional predictive model that the circuit designer provides for the logic designer. The coefficients of the delay equation for a circuit type are obtained by measurements in the laboratory, often backed up by computer simulations. This paper describes how the ADRA/M44 system[1,2] has been used to collect the data from which one obtained first the form of the delay equation and then the magnitude of the coefficients by the use of regression analysis.[3]

The nature of the experiment

The purpose of this experiment was to gain experience in methodology, not to provide logic designers with a useful equation. The variables which are controlled (the independent variables) are six in number and not necessarily the most important ones. The choice was dictated by their availability in ADRA. The terms which are included in the delay equation do not necessarily have physical significance, but are selected only because they provide a best fit to the data. The delay equation so obtained applies only to unit (single-level) logic.

The circuit is a Motorola MECL 351 current switch emitter follower with five inputs (Fig. 1). Computer controlled variables are:

1. Input pulse rise (or fall) time.
2. Input pulse amplitude.
3. Input dc offset.
4. Output load.
5. Emitter power supply voltage, V_{ee}.
6. Collector power supply voltage, V_{cc}.

Each variable can be controlled at three levels. Two of the levels are at the extremes of the circuit's working range for the variable; the third is approximately in the middle.

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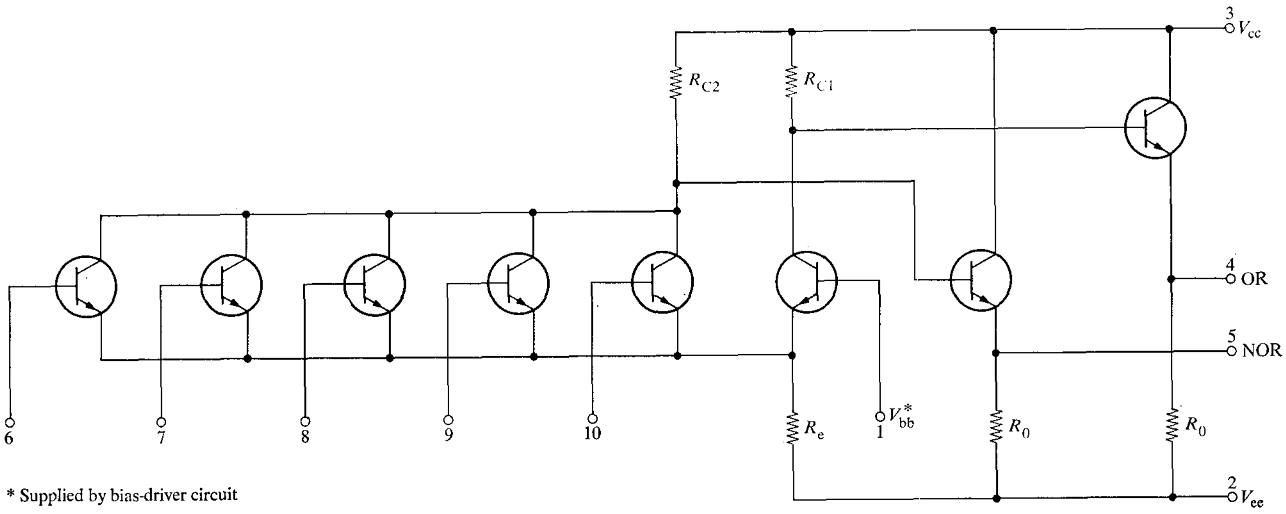

Figure 1 Motorola MECL 351 circuit.

The selected experimental design is called "balanced and central composite." The experiment consists of 57 runs, far less than the maximum number of combinations possible for six variables at three levels ($3^6 = 729$).[6]

The definition of delay was given in the preceding section. The choice of voltage level at which to measure delay is V_{bb} (the base bias voltage of the transistor), which is derived from V_{ee} and V_{cc} by means of a special MECL circuit. V_{bb} is preferred to a constant voltage level because it is the center of symmetry of the gain curve (V_{out} versus V_{in}) and the level at which the edges of the input pulse have maximum slope.

Planning the experiment

The planning of the experiment involved five distinct parts:

(a) The selection of values for each level of the controlled variables and the appropriate adjustments to the hardware.
(b) A statistical design for the experiment.
(c) Various programs to aid in setting up and calibrating the hardware.
(d) A procedure to determine which terms shall be included in the delay equation.
(e) The regression analysis to provide the coefficients of the terms.

Some of these parts are now described briefly. The design of the experiment, (b), is described in order to give a feeling for a representative design without detailed justification. Prior tests establish the measurement error[5] and suggest the amount of replication which is required. Since all the variables are available at three levels, only second-order interactions are considered. The maximum number of terms consists of:

The variables alone:	6
The variables squared:	6
All interactions C_2^6:	15
The constant term:	1
	Total: 28 terms.

The variable values are represented in the X matrix (Fig. 2) by $+1$, 0, -1. First, all combinations of the extreme values ($+1$, -1) of five of the variables are used and the value of the sixth variable is obtained by confounding. They provide 32 measurements. Next, each variable is set at each extreme value while all other variables are set at the nominal (0) value. These twelve combinations are replicated to provide 24 measurements. One more measurement is taken with all variables at their nominal value. The total number of measurements is then 57 and the dimension of the X matrix is 57 by 28 (Fig. 2). The first six columns correspond to the linear terms, the seventh to the constant term and the remaining twenty-one columns correspond to the quadratic and interaction terms.

The programs used in system setup and calibration, (c), are an essential part of the ADRA system and necessary regardless of the nature of the experiment. They include a test for the stability of reference levels and amplifier gains as well as tests for equipment malfunction and measurement error.[5]

The program to control the hardware is written in FORTRAN, apart from three machine-language subroutines. The only unusual aspect stems from certain timing characteristics of ADRA due to power supply settling times and

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	1
3	-1	-1	-1	1	-1	1	1	1	1	1	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1
4	-1	-1	-1	1	1	-1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	1	-1	1
5	-1	-1	1	-1	-1	1	1	1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1	1	1	1	-1	1	-1	1
6	-1	-1	1	-1	1	-1	1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	-1	1	1	-1	1
7	-1	-1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	1	1	1
8	-1	-1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
9	-1	1	-1	-1	-1	1	1	1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	1	-1	1	1	-1	1	-1	1
10	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
11	-1	1	-1	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1	-1	1	1	1
12	-1	1	-1	1	1	1	1	1	-1	1	-1	-1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1	1	1	1	1
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14	-1	1	1	-1	1	1	1	1	-1	-1	1	-1	-1	1	1	-1	1	1	1	-1	1	1	1	-1	-1	1	1	1
15	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	1	1	1	-1	1	1	-1	1	1	-1	1
16	-1	1	1	1	1	-1	1	1	-1	-1	-1	-1	1	1	1	1	1	-1	1	1	1	-1	1	1	-1	1	-1	1
17	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	-1	1	1	1	-1	1	1	-1	1	-1	1
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20	1	-1	-1	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	1	1	1
21	1	-1	1	-1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1	1	1	1	1
22	1	-1	1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1	-1	1	1	1
23	1	-1	1	1	-1	1	1	1	-1	1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
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28	1	1	-1	1	1	-1	1	1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1	1	1	1	-1	1	-1	1
29	1	1	1	-1	-1	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	1	-1	1
30	1	1	1	-1	1	-1	1	1	1	1	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1
31	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
33	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
35	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
36	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
37	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
38	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
39	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
40	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
41	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
42	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
43	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
44	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
45	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
46	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
47	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
48	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
49	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
50	0	-1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	-1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 2 The X matrix.

relay closure times. The appropriate delays have been written into the basic processes so that the programmer need no longer concern himself with these characteristics. The control program is described in greater detail in subsequent sections of this paper.

The choice of significant terms, (d), is carried out by comparing the adequacy of different models.[4] The most general model is

$$Y = b_0 + b_1X_1 + \cdots + b_6X_6 + b_{11}X_1^2 + \cdots + b_{66}X_6^2 + b_{12}X_1X_2 + \cdots + b_{56}X_5X_6\,. \qquad (1)$$

The relative importance of the terms in Eq. (1) is assessed by computing $s\{b_i\}$—the standard deviation of the ith estimated coefficient, $F_i = (b_i/s\{b_i\})^2$—a measure of the relative importance of the ith term, and R^2—the multiple correlation coefficient. R^2 is the proportion of the total variation of the measured performance explained by the empirical model. $s\{b_i\}$ is determined by the experimental design.

The procedure for simplifying the model consists of dropping those statistically insignificant terms for which F is less than some critical value, as found in tables corresponding to a given level of confidence.

Many library programs for multivariate regression analysis are available. The subject is widely treated in texts on statistics.[4]

The data collection procedure

In planning the computer program that controls data gathering, an experimenter may be inclined to model his manual method. This is not always most appropriate, even if it were adequate. For example, he would tend to calibrate his equipment at the beginning of the experiment, possibly plot a correction curve and correct his data after the experiment is completed. Calibration time is short when it is done by the computer so that one can conveniently recalibrate every time the environment combination is changed.

It is not necessary in this experiment to calibrate the ADRA system over the entire voltage range. Only a narrow band on either side of V_{bb} is of interest because the purpose is to determine the instant at which the magnitude of a pulse at the leading or trailing edge is equal to V_{bb}. Calibration then consists of selecting two accurately known voltages, V_1 and V_2, which straddle V_{bb}, the correct voltage level. V_1', V_2' and V_{bb}' are the voltages as read by the computer, including calibration error.

Assuming linearity,

$$\frac{V_2 - V_{BB}}{V_2 - V_1} = \frac{V_2' - V_{BB}'}{V_2' - V_1'} \qquad (2)$$

$$V_{BB}' = V_2' - \frac{(V_2 - V_{BB})}{V_2 - V_1}\cdot(V_2' - V_1')\,. \qquad (3)$$

The voltage at the circuit must be V_{bb} whenever the digitizer output indicates V_{bb}'. All voltages on the right hand side of Eq. (3) are known.

The time t_{bb} at which the pulse reaches V_{bb} is found by a binary search of sample times. It is, of course, very unlikely that V_{bb}' will be read at a discrete sample time. For linear interpolation, the two sample times t_2 and t_1 which give the nearest larger and smaller voltages are found, and an estimate of the crossing time is obtained by interpolating again. Fig. 3 shows both the uncorrected and the corrected values of V_{bb}.

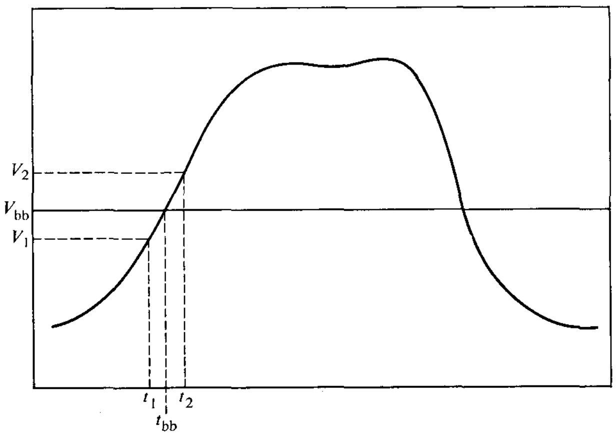
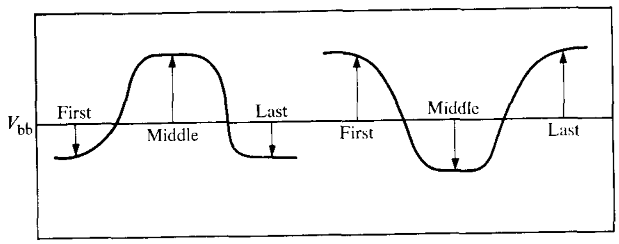


Figure 3 Pulse measurement points.

Figure 4 Test for pulse presence.



Two other techniques are used to give control over accuracy. Near V_{bb} the signal is an almost linear function of time, so that interpolating between large time steps still provides good accuracy. Time step size is a program parameter which can be adjusted for a good balance between speed and precision.

The standard deviation of error in the instrumentation is reduced by averaging over repeated readings of the same voltage. If n is the number of repetitions, the standard deviation is reduced as $1/\sqrt{n}$ when the distribution is normal, as it was found to be in ADRA. 'n' is another program parameter.

To assure good working order of the equipment one must incorporate into the program procedures which detect any gross equipment malfunction, which retry automatically and which give an indication of solid failure.

Most system malfunctions cause a total loss of pulse, while circuit malfunctions only cause loss of the output pulse. The test in the program for pulse presence is that the first and last samples must have values both lower or both higher than V_{bb} and that the middle sample must be on the opposite side of V_{bb}. (See Fig. 4.)

A double retry before the program forces premature termination is very effective against transient failures. The computer controlled display is also an excellent diagnostic tool when programmed to display input and output pulses to the circuit, as well as various reference lines. (See Fig. 5.)

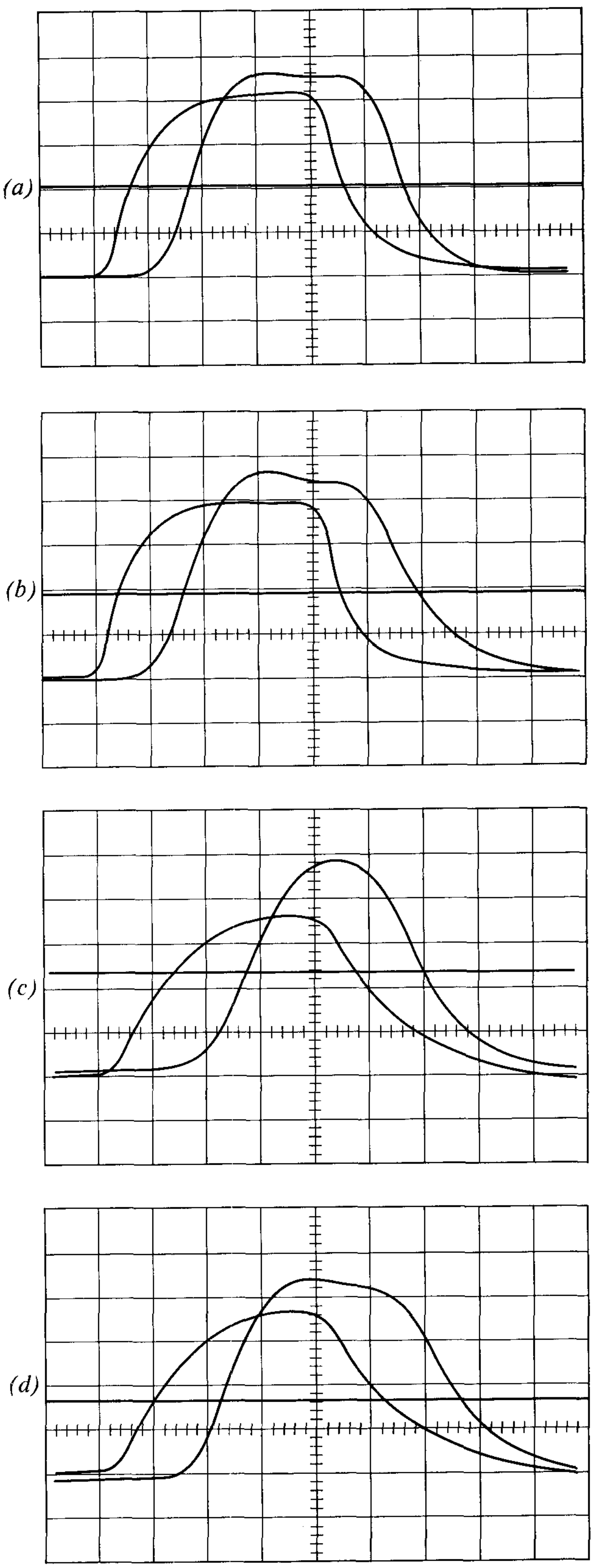


Figure 5 Displays of input and output pulses. (a) all variables nominal; (b) V_{ee}, V_{cc} high, all else low; (c) V_{ee}, load high, all else low; (d) slow rise time, V_{cc} high, all else low.

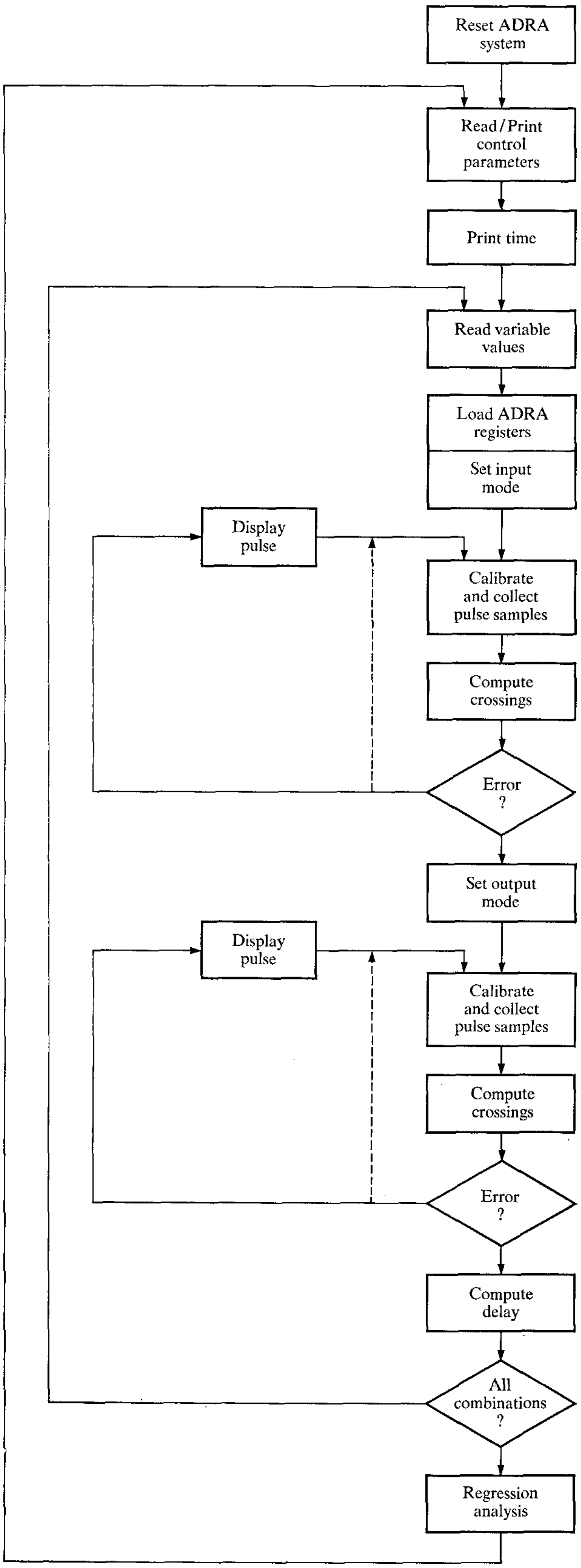


Figure 6 Data collection program.

The program

The organization of the data collection program is shown in Fig. 6. Program subroutines may be divided into (a) basic machine processes and (b) basic problem processes. Basic machine processes include setting the sense lines (PSL), reading from the Direct Data Channel (which includes unpacking and converting complement notation to signed integer) and reading the time clock. Basic problem processes include

PULSE (m, n)—collects 2^n samples for the first m channels,
INTERP (y, i, j)—interpolates between the i^{th} and j^{th} sample at level y,
CROSS (y, k, t_1, t_2)—computes the times t_1 and t_2 at which the k^{th} channel pulse crosses level y, and
DISPLAY (x, y, k, n)—displays vertical and horizontal reference line at x and y, pulse from the k^{th} channel, 2^n points per trace.

There is one 1024 $\times$ 15 array in storage for the voltage values at the maximum number of 1024 sample times in all 15 channels.

The following parameters are under the experimenter's control:

1. Number of bursts[1]: 1–32 (a control of the sampling units).
2. Number of samples/pulse (power of 2): 1–10 (range from 2 to 1024).
3. Number of averaged samples (power of 2): 0–5 (range from 1 to 32).
4. Number of tests: 1–90.
5. Display options: input pulse, output pulse, both, neither.
6. Number of channels: up to 4.

The control of variables is accomplished by placing the X matrix (57 by 28) on file and reading one row from the file for each test. The same X matrix is available for the analysis which follows in order to reduce the possibility of clerical error. The X matrix is generated by a separate program. A sample of computer output is shown in Fig. 7.

Analysis of results

The result of the regression analysis consists of 28 coefficients b_{ij} and 28 values F_{ij} from the significance tests. They are best set out in array form to clarify the interactions of variables (Table 1). The constant term is at the (0, 0) position. R^2 = (the sum of squares due to the regression)/(total sum of squares, corrected for the mean) = 99.95%.

Inspection of the F array lets one determine the relative significance of terms. The seven most significant terms in order of decreasing significance are:

$$Y = b_0 + b_2x_2 + b_4x_4 + b_5x_5 + b_1x_1 + b_{22}x_2^2 + b_6x_6.$$

POINTS 64, TESTS 57, REPEATS 10, METHOD 1, AVERAGED 4, PROCESS 0, DISPLAY 0 OUTPUTS 1

	Amplitude	DC Offset	Load	Rise time	V_{cc}	V_{ee}	V_{bb}	Leading Delay	Trailing Delay	V_{bb}'
1	−1	−1	−1	−1	−1	−1	137.5	11.67	13.98	134.3
2	−1	−1	−1	−1	1	1	181.5	12.31	13.21	181.0
3	−1	−1	−1	1	−1	1	113.5	12.83	15.01	110.8
4	−1	−1	−1	1	1	−1	205.5	13.91	13.24	205.8
5	−1	−1	1	−1	−1	1	113.5	11.50	14.81	111.8
6	−1	−1	1	−1	1	−1	205.5	12.71	12.56	204.2
7	−1	−1	1	1	−1	−1	137.5	13.04	14.25	135.4
8	−1	−1	1	1	1	1	181.5	13.71	13.75	180.5
9	−1	1	−1	−1	−1	1	113.5	12.36	17.67	111.1
10	−1	1	−1	−1	1	−1	205.5	13.64	14.80	206.4
11	−1	1	−1	1	−1	−1	137.5	14.02	16.49	136.2
12	−1	1	−1	1	1	1	181.5	14.66	16.03	181.7
13	−1	1	1	−1	−1	−1	137.5	12.69	16.47	135.9
14	−1	1	1	−1	1	1	181.5	13.41	15.66	181.2
15	−1	1	1	1	−1	1	113.5	13.87	17.35	111.4
16	−1	1	1	1	1	−1	205.5	15.10	15.38	205.0
17	1	−1	−1	−1	−1	1	113.5	10.98	14.71	110.5
18	1	−1	−1	−1	1	−1	205.5	11.78	12.71	203.2
19	1	−1	−1	1	−1	−1	137.5	12.37	14.39	133.3
20	1	−1	−1	1	1	1	181.5	12.88	13.87	179.5
21	1	−1	1	−1	−1	−1	137.5	11.13	13.93	133.3
22	1	−1	1	−1	1	1	181.5	11.64	13.38	179.7
23	1	−1	1	1	−1	1	113.5	12.34	15.01	109.4
24	1	−1	1	1	1	−1	205.5	13.17	13.10	202.5
25	1	1	−1	−1	−1	−1	137.5	12.08	16.46	133.4
26	1	1	−1	−1	1	1	181.5	12.66	15.54	180.2
27	1	1	−1	1	−1	1	113.5	13.26	17.34	109.3
28	1	1	−1	1	1	−1	205.5	14.11	15.11	203.1
29	1	1	1	−1	−1	1	113.5	11.95	17.59	109.1
30	1	1	1	−1	1	−1	205.5	12.86	14.67	204.7
31	1	1	1	1	−1	−1	137.5	13.49	16.50	133.0
32	1	1	1	1	1	1	181.5	14.01	15.77	178.7
33	0	0	0	0	0	0	158.0	10.90	11.48	154.9
34	0	0	0	0	0	−1	170.0	10.98	11.25	166.9
35	0	0	0	0	0	−1	170.0	10.99	11.23	166.7
36	0	0	0	0	0	1	146.0	10.83	11.76	142.1
37	0	0	0	0	0	1	146.0	10.83	11.80	142.4
38	0	0	0	0	−1	0	125.5	10.61	12.06	121.8
39	0	0	0	0	−1	0	125.5	10.60	12.06	121.5
40	0	0	0	0	1	0	193.5	11.37	11.08	189.8
41	0	0	0	0	1	0	193.5	11.31	11.06	191.4
42	0	0	0	−1	0	0	158.0	10.48	11.22	154.1
43	0	0	0	−1	0	0	158.0	10.48	11.22	154.6
44	0	0	0	1	0	0	158.0	11.92	11.90	155.4
45	0	0	0	1	0	0	158.0	11.91	11.92	153.5
46	0	0	−1	0	0	0	158.0	10.88	11.43	154.7
47	0	0	−1	0	0	0	158.0	10.88	11.56	154.7
48	0	0	1	0	0	0	158.0	10.90	11.49	155.3
49	0	0	1	0	0	0	158.0	10.91	11.50	155.6
50	0	−1	0	0	0	0	158.0	12.06	13.81	155.4
51	0	−1	0	0	0	0	158.0	12.05	13.85	155.1
52	0	1	0	0	0	0	158.0	13.06	16.20	154.5
53	0	1	0	0	0	0	158.0	13.04	16.23	155.3
54	−1	0	0	0	0	0	158.0	11.38	11.50	153.7
55	−1	0	0	0	0	0	158.0	11.37	11.49	155.0
56	1	0	0	0	0	0	158.0	10.59	11.54	155.0
57	1	0	0	0	0	0	158.0	10.65	11.50	155.6

Figure 7 Example of data collection.

The type and levels of the variables are:

x_1	Amplitude, mV	480	520	560
x_2	Load time constant, nsec	7.5	16.5	24
x_3	DC offset, mV	−70	−90	−110
x_4	Rise time, nsec	8.5	19	30
x_5	V_{cc}, Volts	1.25	1.29	1.33
x_6	V_{ee}, Volts	−3.4	−3.5	−3.6

Table 1 Regression analysis results. First pass; $R^2 = 99.95\%$.

j \ i	b_{ij} 0	1	2	3	4	5	6
0	14.328	−0.292	1.955	0.114	0.821	0.468	−0.146
1		0.072	0.008	−0.038	−0.030	−0.054	0.018
2			−0.450	0.008	−0.019	0.063	−0.043
3				0.017	0.006	0.046	−0.021
4					−0.145	0.049	0.003
5						−0.013	−0.012
6							−0.053

j \ i	F_{ij} 0	1	2	3	4	5	6
0	1.19×10^6	1028	46190	158	8151	2644	258
1		8.3	0.7	15.1	9.7	31.8	3.5
2			322	0.6	0.5	42.8	20.0
3				0.5	0.3	23.0	4.8
4					33.5	25.5	0.1
5						0.3	1.5
6							4.4

Table 2 Regression analysis results. Second pass; $R^2 = 99.29\%$.

j \ i	b_{ij} 0	1	2	3	4	5	6
0	14.305	−0.292	1.955		0.821	0.468	−0.146
1							
2			−0.535				
.							
.							
6							

j \ i	F_{ij} 0	1	2	3	4	5	6
0	170×10^3	121	5451		962	312	30.4
1							
2			150				
.							
.							
6							

A new X matrix (57 by 7) is constructed by deleting the columns which represent the 21 least significant terms and the regression calculation is repeated (Table 2). The b coefficients do not change very much because the columns of X are nearly orthogonal.

The dc offset does not have a significant effect on the circuit delay. The effect of the emitter voltage is quite small.

A useful form of the equation should enable one to substitute for the variables the values in their practical units; millivolts, volts or nanoseconds. The b coefficients can be obtained in the properly scaled form by using the assigned values in practical units in the X matrix.

Conclusions

Because the purpose of the experiment was to gain experience in the methodology of computer controlled experiments and not to develop a delay equation, the conclusions concern that which is implicit rather than explicit in the previous discussion.

To perform an experiment through a computer is instructive in a number of ways. The desirability of planning the entire experimental technique in detail and in advance from the diagnostics during data gathering to the derivation of final results goes beyond the common requirements of laboratory procedure. The use of conventional but powerful statistical tools to test the limits of accuracy and precision of the equipment and then to obtain data at those levels is natural when the data is collected by computer. Confidence in the final results is enhanced by the statistical design of an economical experiment, by automatic failure detection and by self-calibrating facilities.

The effect is to permit an experimenter to define his experiment at a fairly abstract level in terms of the X Matrix, or even in terms of the parameters of the program which generates the X Matrix.

There are concomitant disadvantages. The ADRA system cannot be used spontaneously in the sense, say, of running a quick check on a novel circuit. Often it is as hard to do a simple thing as it is to do a complex one. But having once developed the software tools and having acquired familiarity with statistical methods, subsequent experiments are of course more readily accomplished.

Finally, one must state that the ideal time to plan the experiment is before the hardware design is frozen. It is to the credit of the hardware planners that the equipment and the task matched so well, when the task was not defined in any detail at construction time. The amount of computation required to support ADRA's data gathering rate was not foreseen. Consequently, ADRA is somewhat underpowered in the M44 processor.[2]

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