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A Two-Dimensional Mathematical Analysis of the Diffused Semiconductor Resistor*

Abstract: A two-dimensional mathematical analysis is presented of the electrical properties of the diffused semiconductor resistor. An important conclusion is that substantially more electric current crowding exists within this semiconductor device than heretofore suspected, particularly in the vicinity of the ohmic contacts. Considered in this analysis is the influence on the electrical characteristics of various impurity atom distributions arising from a two-step diffusion process. The results of this investigation are presented graphically.

Introduction

Diffused silicon resistors are used extensively in integrated circuits. Despite the extensive use of this semiconductor component, little theoretical information is available concerning its electrical properties. For this reason, this paper presents the results of a two-dimensional mathematical investigation of the electrical properties of diffused resistors in silicon. Included in this analysis are the influence of impurity atom scattering and that of transport velocity upon the mobility of holes and electrons in silicon; separate calculations are therefore presented for n-type and p-type structures.

The diffused resistor is a simple structure. Diffusion techniques are used to form in a slice of silicon a planar p–n junction of stripe geometry. Thereafter, ohmic contacts are located on the semiconductor surface at the two ends of this diffused region. Electrical conduction between these ohmic contacts (which provides the electrical resistance of the structure) takes place through semiconductor material of inhomogeneous impurity atom density. The p–n junction produces electrical isolation for a diffused resistor; the biasing voltage upon this junction is everywhere maintained in the reverse direction.

Throughout this investigation, diffused resistor operation is approximated by a type of boundary value problem seldom considered in the technical literature: electrical conduction within semiconductor material containing an inhomogeneous impurity atom distribution. Complications arise in this analysis because the main body of a diffused resistor (the region far removed from the ohmic contacts) contains an impurity atom gradient that is directed at right angles to the electric field. This situation implies the necessity of mathematically approximating the operation of a diffused resistor by a boundary value problem containing a minimum of two spatial variables.

The material presented here results from numerical solutions of boundary value problems that mathematically approximate the operation of a diffused semiconductor resistor. Finite difference methods have been used. Although these numerical methods do not provide explicit equations describing the electrical properties of a diffused resistor, such information is obtained indirectly. From a series of computer calculations, parameters normally described by mathematical formulae are presented in graphical form; thereby, information derived from the present investigation is readily available for engineering purposes.

Mathematical methods

This analysis involves the solution of two substantially different boundary value problems. The first problem is associated with the fabrication of a diffused resistor, and the solution of this problem yields the two-dimensional impurity atom distribution in a completed device. The second

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problem involves the operation of a diffused resistor, and the solution of this problem yields the potential and electric current distributions arising from an applied voltage between two ohmic contacts. The solution of this second boundary value problem is subject to constraints imposed by the boundary conditions of a diffused resistor, and also to constraints imposed by the previously calculated impurity atom distribution. The following discussion outlines the methods by which these two boundary value problems are solved and describes the limitations imposed on the applicability of these solutions because of the use of mathematical simplifications.

• *Impurity atom distribution*

A two-step diffusion process is frequently used in the fabrication of diffused resistors. A rectangular opening is first made in an oxide diffusion mask located on the surface of a slice of silicon. Thereafter, a constant C_0 impurity atom source is used to deposit impurity atoms on the exposed silicon surface (Step I); during this deposition process, impurity atoms diffuse only a short distance into the semiconductor material. Next the impurity atom source is removed, and impurity atoms are diffused into the semiconductor material. During this diffusion process (Step II), oxide growth takes place within the diffusion mask opening.

A one-dimensional analysis of the two-step diffusion process[1] shows that the resulting impurity atom distribution lies in between the distributions produced by the constant C_0 diffusion process and an instantaneous source diffusion process. After completing the initial impurity atom deposition (Step I), the resulting impurity atom distribution can be approximated by a complementary error function. The second step in this two-step diffusion process (diffusion after removing the impurity atom source) changes this impurity atom distribution from a complementary error-function distribution to something approaching a Gaussian type of distribution. The degree of this change is determined by the depth to which impurity atoms are permitted to diffuse during Step II.

Throughout the present investigation, these two impurity atom distributions (constant source and instantaneous source) are used to bound the various impurity atom distributions obtained from a two-step diffusion process. This analytical technique has obvious advantages; the mathematical characterization of a two-step diffusion involves substantially more independent variables than either a constant C_0 diffusion or an instantaneous source diffusion. In this analysis, the use of two substantially different impurity atom distributions permits us to bound the electrical properties of a diffused resistor (fabricated by any arbitrary two-step diffusion process), and minimize the number of independent variables in the characterization of a device.

From the elementary theory of thermal diffusion,[2] the impurity atom distribution in a diffused resistor is assumed to be well approximated by solutions of the differential equation

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} = \frac{1}{D} \frac{\partial C}{\partial t}, \tag{1}$$

where $C(x, y, t)$ is the impurity atom distribution, and D is the diffusion constant for these impurity atoms in silicon. In addition to satisfying Eq. (1), the resulting impurity atom distribution must satisfy all boundary conditions imposed during device fabrication.

For this calculation of the impurity atom distribution, the semiconductor material (prior to impurity atom diffusion) is approximated by a matrix of several thousand spatial locations. This approximating matrix contains all necessary information concerning the location of the diffusion mask, the diffusion mask opening, and the boundary conditions imposed at the exposed semiconductor surface. Relaxation methods[3] are used to approximate the process of diffusion in semiconductor material; thereby we obtain the impurity atom distribution in a diffused resistor. Details of this computational method have been described in a previous publication[4] and need not be repeated here.

It is emphasized that these impurity atom distribution calculations are based on an assumption that impurity atom transport in a semiconductor is governed by elementary laws of thermal diffusion. This assumption has been questioned by many workers. It is therefore suggested that the analysis presented here may be subject to revision when more complete information is available concerning the mechanisms of impurity atom diffusion in a semiconductor.

• *Electrical properties of the diffused resistor*

A rigorous mathematical treatment of the diffused resistor problem differs little from an analysis of the high-low semiconductor junction.[5] An inhomogeneous impurity atom distribution exists throughout the semiconductor material comprising a diffused resistor; this material therefore contains a distribution of electrostatic charge. Furthermore, by applying a voltage to the diffused resistor, this electrostatic charge distribution can undergo a significant modification; hence, the voltage distribution within a diffused resistor is rigorously described by solutions of Poisson's equation. Although under some conditions this electrostatic charge density may become insignificant (if so, the voltage distribution would be well approximated by solutions of Laplace's equation), a condition of near charge neutrality cannot be taken for granted in an analysis of this type.

The present analysis makes use of mathematical simplifications previously used in connection with heat conduction in material of inhomogeneous thermal conductivity.[6] It can be shown that the temperature distribution in such material is representable as a sum of two different temperature distributions: first, the temperature distribution when the structure under consideration is assumed to have a homogeneous thermal conductivity, and second, the temperature distribution arising from a prescribed distribution of thermal sources and sinks within this same structure. As in a thermal conductor, it can be shown that the potential distribution in a diffused resistor is well approximated by the sum of two different potential distributions: the potential distribution within a resistor of homogeneous impurity atom distribution, and the potential distribution arising from a prescribed distribution of electrostatic charges within this resistor.

In a rigorous fashion, the electrical properties of a diffused resistor are given by solutions of the following set of differential equations:[7]

$$\operatorname{div}\operatorname{grad}\Psi = -\frac{q}{\mathcal{K}\epsilon_0}(C - n + p) \; ; \tag{2a}$$

$$\mathbf{J}_p = -qD_p \operatorname{grad} p - q\mu_p p \operatorname{grad}\Psi \; ; \tag{2b}$$

$$\mathbf{J}_n = qD_n \operatorname{grad} n - q\mu_n n \operatorname{grad}\Psi \; ; \tag{2c}$$

$$\operatorname{div}\mathbf{J}_p = q\mathcal{R}_p \; ; \tag{2d}$$

$$\operatorname{div}\mathbf{J}_n = q\mathcal{R}_n \; ; \tag{2e}$$

$$\mathbf{J}_T = \mathbf{J}_p + \mathbf{J}_n \, . \tag{2f}$$

By combining the set of equations above, three nonlinear differential equations are obtained, and these must be solved simultaneously. This solution must satisfy all constraints imposed by both the geometrical and the physical properties of a diffused resistor.

A rigorous solution of the diffused resistor problem would be an unwarranted expenditure of time and effort. The mechanisms characterized by Eqs. (2) are all rigorously correct, but many of these mechanisms have a trivial influence on the electrical properties of most diffused resistors. For example, by restricting this analysis to diffused resistors composed of extrinsic semiconductor material, the minority carrier mechanisms implied in Eqs. (2) have little significance in the applicability of this analysis to practical diffused resistors. These terms are therefore neglected, as well as the contribution of diffusion mechanisms to the electric current within a diffused resistor. In combination, these mathematical simplifications reduce Eqs. (2) to two differential equations.

$$\operatorname{div}\operatorname{grad}\Psi = -\frac{q}{\kappa\epsilon_0}(C + p) \, , \tag{3a}$$

$$\operatorname{div}\operatorname{grad}\Psi = -\,q \operatorname{grad}\Psi \cdot \operatorname{grad}(\mu_p p) \, , \tag{3b}$$

which are applicable to diffused resistors composed of p-type semiconductor material. Obvious modifications of these equations will render them applicable to diffused resistors composed of n-type material.

From (3) we have

$$\operatorname{grad}(\mu_p p) \cdot \operatorname{grad}\Psi = (C + p)/\kappa\epsilon_0 \, . \tag{4}$$

Equation (4) shows that Eqs. (3a) and (3b) are equivalent forms of Poisson's equation, the right side of (3b) being another mathematical formulation for the electrostatic charge within the semiconductor material.

Equation (3b) is the mathematical equation used in the solution of this boundary value problem. From (3b), when the material comprising a semiconductor resistor contains a homogeneous electrical conductivity [$0 = \operatorname{grad}(\mu_p p)$], the voltage distribution is characterized by solutions of Laplace's equation. If, instead, this material contains a large electrical conductivity gradient, yet a negligible electric field ($\mathbf{E} = -\operatorname{grad}\Psi$), Eq. (3b) shows that the voltage distribution remains a solution of Laplace's equation. If, instead, the product [$\operatorname{grad}(\mu_p p)\cdot\operatorname{grad}\Psi$] is non-zero (yet small when compared with the impurity atom density), Eq. (3b) shows that the region under consideration is no longer approximated by only Laplace's equation; instead the resulting voltage distribution becomes a sum of the voltage distributions derived from Laplace's equation and from the electrostatic charges existent within the semiconductor material (Poisson's equation). This mathematical view of the problem is equivalent to the previously mentioned formulation for the temperature distribution within material of inhomogeneous thermal conductivity.

The present solution of this diffused resistor problem is obtained from numerical solutions of Eq. (3b). Relaxation methods[3] are used, in two spatial dimensions. At each location within the relaxation matrix, the magnitude of [$\operatorname{grad}\Psi\cdot\operatorname{grad}(\mu_p p)$] is determined, relative to the impurity atom density at this same location; this test shows where the present mathematical simplifications are satisfactory. In regions where [$\operatorname{grad}\Psi\cdot\operatorname{grad}(\mu_p p)$] is small (relative to the impurity atom density), solutions of Eq. (3b) adequately characterize the resulting potential distribution. If, instead, the magnitude of [$\operatorname{grad}\Psi\cdot\operatorname{grad}(\mu_p p)$] approaches the impurity atom density at this same location, Eq. (3b) becomes an inadequate representation of the potential distribution; throughout such a region the more complete formulation of Eqs. (2) must be used.

In this analysis, only one region exists within the diffused resistor where Eq. (3b) is of questionable accuracy: in the immediate vicinity of the p–n junction. Because this region has little influence upon the overall electrical characteristics of a diffused resistor (this will be shown at a later time), the error arising from Eq. (3b) is of little consequence.

Consideration must be given, in the analysis of a diffused resistor, to mechanisms that influence charge carrier mobility.[8] Semiconductor materials exhibiting large values of impurity atom density are known to exhibit reduced hole and electron mobilities. Furthermore, it has been shown that the average drift velocity of conduction band electrons and valence band holes is not always proportional to an applied electric field. For this reason, throughout the present analysis published values are used for the small field

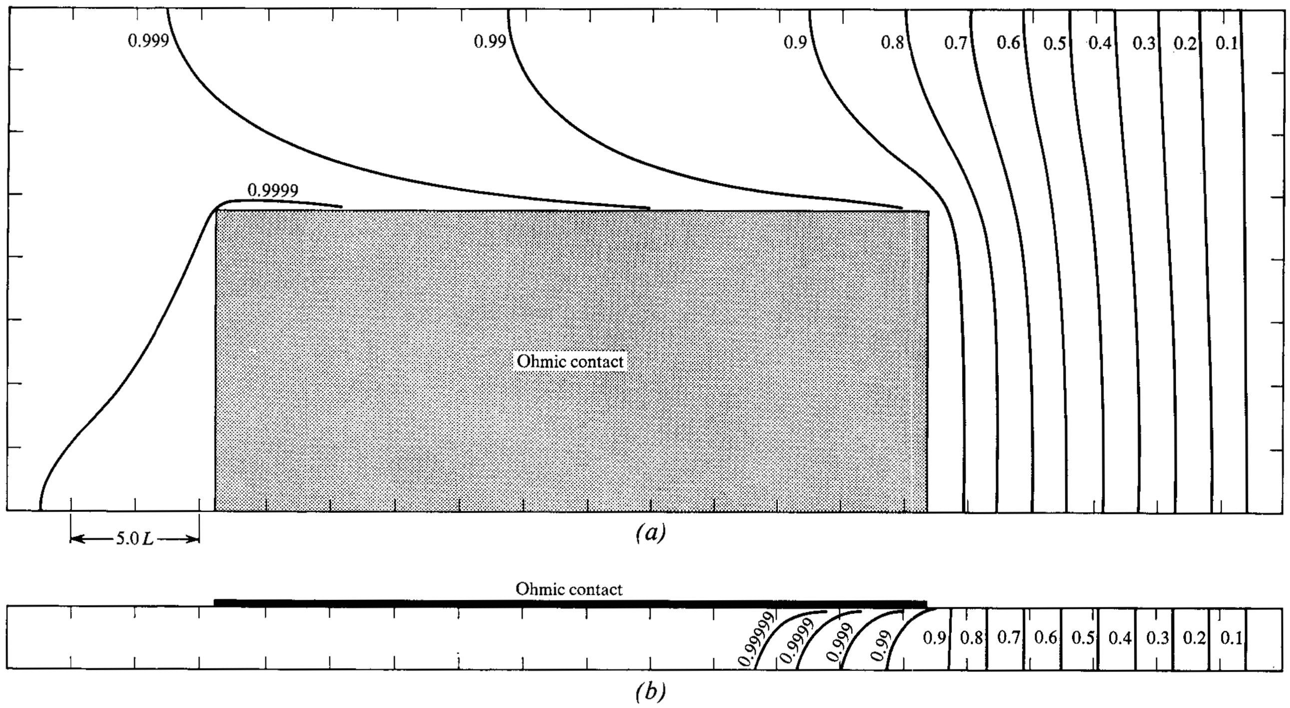


Figure 1 Calculated constant voltage contours in the ohmic contact region of a diffused silicon resistor: (a) top view; (b) side view. ($L = 2\sqrt{Dt}$.)

drift mobilities of holes and electrons.[8f] In addition, the present analysis utilizes the published experimental results of Ryder[8a] concerning the influence of large values of electric field upon charge carrier mobility.

Voltage distribution within a diffused resistor

Figure 1 illustrates the distribution of an applied voltage throughout the ohmic contact region of a typical diffused resistor. Because the present analysis is based on a two-dimensional approximation of this structure, the top and side views in Fig. 1 result from separate calculations of the problem. All dimensions are normalized in terms of the impurity atom diffusion length ($L = 2\sqrt{Dt}$); the structure shown has the geometrical dimensions of a typical device. The contours within this illustration show the calculated surfaces of constant voltage arising from an applied voltage.

In a diffused resistor, an electric current results from the drift of majority carriers in an electric field produced by the applied voltage. For this reason, the calculations illustrated in Fig. 1 provide important qualitative information concerning diffused resistor operation. In the top view of this structure (Fig. 1a), a negligible electric field exists in a direction perpendicular to the back and sides of the ohmic contact. In contrast, a substantial electric field exists near the end of this contact facing the main body of the diffused resistor; a variation of approximately factor two is present in the magnitude of this calculated electric field.

The side view of this ohmic contact region (Fig. 1b) shows little or no electric field in a direction perpendicular to almost all of the metal-semiconductor interface. A large part of the ohmic contact is therefore inactive and contributes little to the electrical properties of the semiconductor device. In fact, Fig. 1b shows that the only active portion of this ohmic contact is a small region (approximately 2.5 L in length) toward the main body of the resistor.

A slight disagreement can be observed in the location of some constant potential contours shown in Figs. 1a and 1b; this disagreement arises from the use of two spatial dimensions to approximate a three-dimensional structure. A consequence of this disagreement is a small uncertainty in the electric current distribution in the vicinity of an ohmic contact; this uncertainty, however, has little influence upon the applicability of the present analysis.

From Fig. 1, it is concluded that only a small portion of the ohmic contact actively contributes to the electrical properties of a diffused resistor. Because the electric current at a particular location is determined by the electric field at this same location, the potential distribution in a diffused resistor keeps almost the entire ohmic contact inactive.

One important conclusion can be derived from the calculations shown in Fig. 1. From these potential distributions, little advantage is gained by increasing the length of the ohmic contact; an increase in length has a negligible influence on the current density at the metal-semiconductor

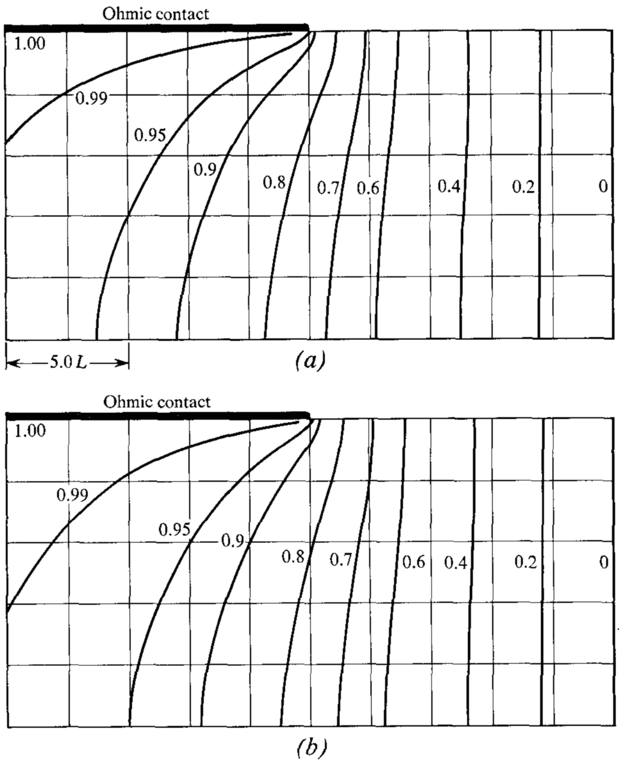


Figure 2 Calculated constant voltage contours at the edge of the ohmic contact: (a) instantaneous source diffusion; (b) constant C_0 diffusion. ($L = 2\sqrt{Dt}$.)

interface. In fact, the only geometrical dimension significantly influencing this current density is the ohmic contact width. For this reason, the remaining calculations are based on a structure in which this design modification has been introduced. The ohmic contact is assumed to extend across the entire width of this diffused region, and thereby the structure is well approximated by a two-dimensional analytical model.

There is little advantage in presenting further calculations of the entire ohmic contact region; only the right-hand end in Fig. 1 contributes to the electrical resistance of the structure. Therefore, the remainder of this investigation is directed toward the active region of an ohmic contact.

Throughout this series of potential calculations, little difference is observed between the voltage distribution within resistors fabricated by a constant C_0 diffusion process and that within instantaneous source-diffused structures. Figure 2 illustrates the calculated constant voltage contours for two such devices of identical geometrical dimensions. Although the calculations shown in Fig. 2 are for structures composed of p-type material, and at small values of applied voltage, little difference is observed when the material is assumed to be composed of n-type semiconductor material. Furthermore, only minor variations are observed in these potential distribution calculations when sufficient voltage is applied to attain the large-field mobility of holes and electrons in silicon.

Figure 2 provides a detailed view of the manner in which these contours of constant voltage are crowded near one end of the ohmic contact, and therefore shows that the electric current exhibits a substantial degree of crowding. At comparatively small values of total electric current, the current density at the ohmic contact can become exceedingly large as a result of the potential distribution shown in Fig. 2.

Electric current distribution in a diffused resistor

In regions far removed from the ohmic contacts, the electric current in a diffused resistor is crowded into a layer that is located near the semiconductor surface. This situation is a consequence of the impurity atom distribution introduced during device fabrication. Impurity atom diffusion from the semiconductor surface assures that the material residing near this surface has the largest electrical conductivity; this high conductivity material will therefore contain a large fraction of the total electric current.

Figure 3 shows the manner in which the electric flux becomes distributed within the main body of a diffused resistor (at a distance of at least 2.5 L from the ohmic contacts). The normalized electric flux in Fig. 3 is plotted against a normalized distance from the semiconductor surface (y/L). Figure 3 illustrates the calculated flux distribution in both n-type and p-type silicon, and in devices assumed to be fabricated by both a constant C_0 diffusion process and by an instantaneous source diffusion process (Gaussian). All of the calculations shown in Fig. 3 represent the operation of a diffused resistor at voltages not sufficient to produce an electric field dependence of charge carrier mobility.

Figure 4 gives the relative electric current distribution in the main body of a diffused resistor. This illustration shows the proportional distribution of electric current in a layer of material of arbitrary thickness (y/L) that is bounded on one side by the semiconductor surface. Figure 4 gives the calculations only for p-type semiconductor material; the difference between p-type and n-type material is not sufficient to require calculations for both.

In combination, Figs. 3 and 4 establish the degree of current crowding in the main body of a diffused resistor. For example, from Fig. 3, the current density at the semiconductor surface and at a depth of 2.0 L from this surface has ratios of approximately 20:1 and 40:1 respectively; these ratios depend upon the impurity atom distribution and upon the type of electrical conductivity (n-type or p-type). Figure 4 establishes the full consequence of this electric current distribution. From Fig. 4, we observe that about 99 per cent of the electric current in the main body

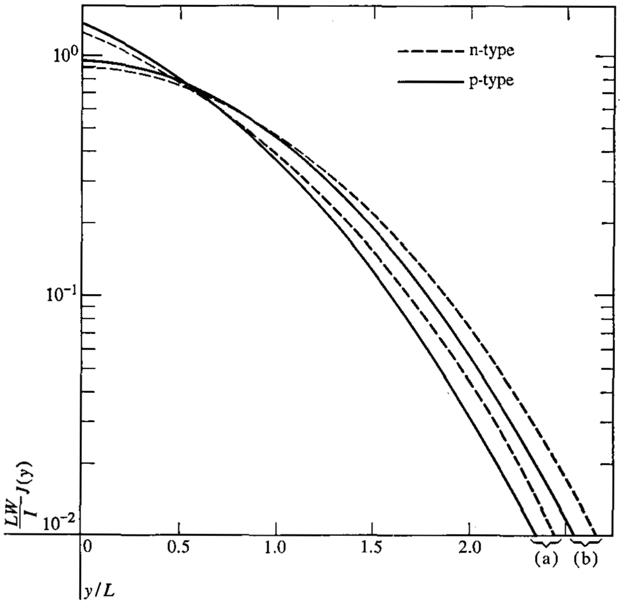


Figure 3 Calculated electric flux distribution in the main body of a diffused resistor: (a) constant C_0 diffusion; (b) instantaneous source diffusion. ($L = 2\sqrt{Dt}$.)

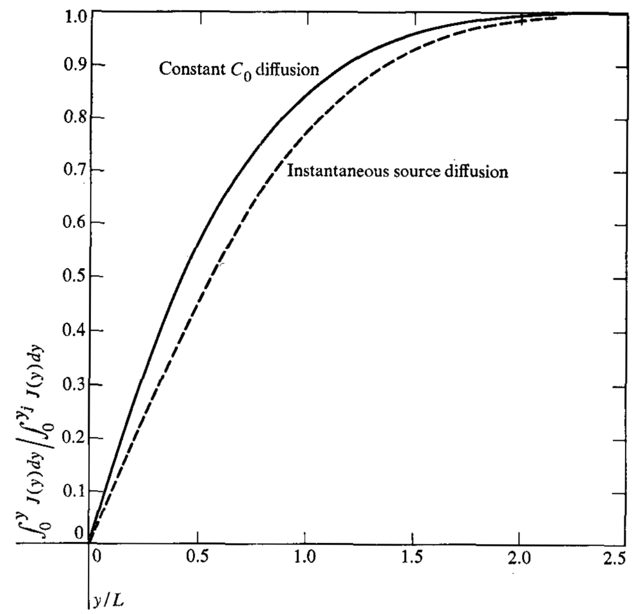


Figure 4 Relative electric current distribution in the main body of a diffused resistor. ($L = 2\sqrt{Dt}$.)

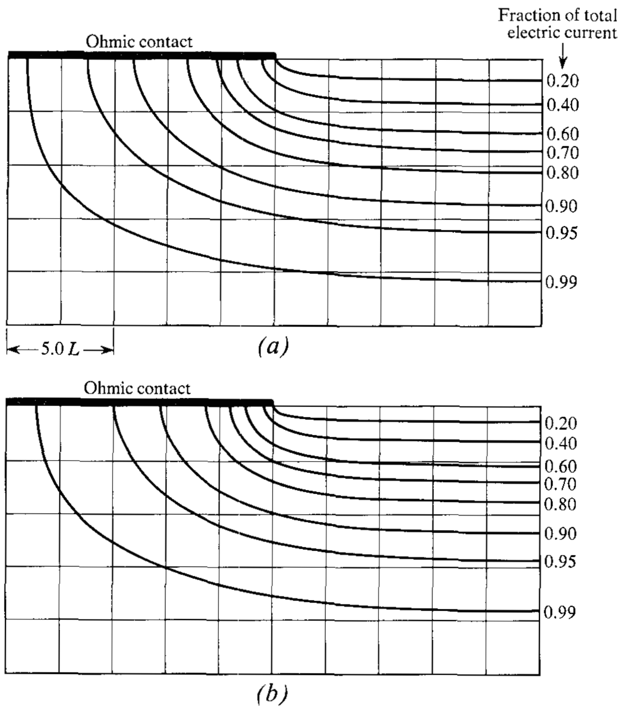
of a diffused resistor lies within a layer of thickness 2.0 L from the semiconductor surface.

The extensive current crowding in a diffused resistor implies that little error is introduced by the use of previously outlined mathematical simplifications. It has been shown that the equation used to approximate the potential distribution in a diffused resistor is not adequate in the vicinity of the p–n junction space-charge layer. However, because the p–n junction in most diffused resistors is located at a distance of at least 2.0 L from the semiconductor surface, Fig. 4 shows that this region contributes little to the electrical properties of the device. About 1 per cent of the total electric current in a diffused resistor is carried by material residing at a depth in excess of 2.0 L from the semiconductor surface.

The crowding shown in Figs. 3 and 4 becomes increasingly greater near an ohmic contact. Figure 5 illustrates the calculated contours of relative electric current distribution between the main body of a diffused resistor and its ohmic contact. The material residing between each contour and the bounding surface contains a specified fraction of the total electric current within the diffused resistor. Figure 5 is for a diffused resistor composed of p-type semiconductor material. In this illustration, the resulting electric field is maintained everywhere at a value insufficient to modify the mobility of holes.

At the ohmic contact, a substantial part of the total electric current exists within a short distance from the contact edge facing the main body of the resistor. In Fig. 6, the origin of the coordinate (x/L) is the edge of the ohmic contact. This figure illustrates the calculated electric flux distribution at this location, for n-type and p-type semiconductor material, and for the two types of impurity atom distributions. From this calculation, it is reasonable to assume that

Figure 5 Relative electric current distribution near the ohmic contact of a diffused resistor: (a) instantaneous source diffusion; (b) constant C_0 diffusion. ($L = 2\sqrt{Dt}$.)



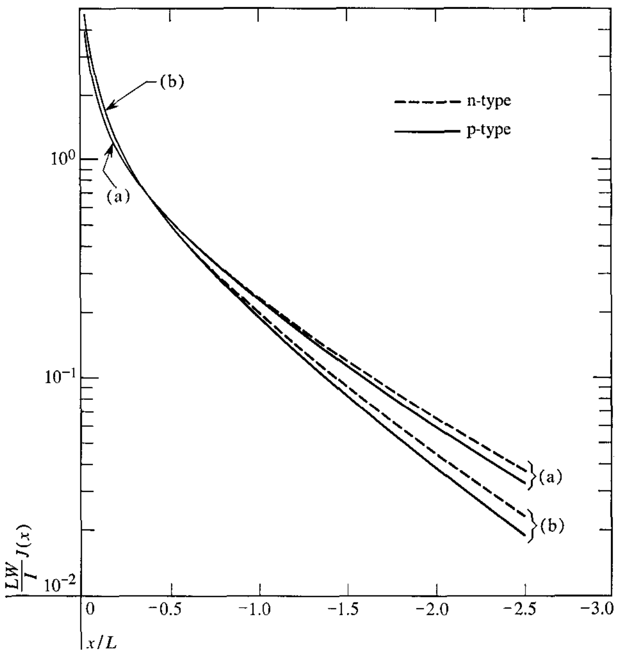


Figure 6 Calculated electric flux distribution at the ohmic contact of a diffused resistor: (a) instantaneous source diffusion; (b) constant C_0 diffusion. ($L = 2\sqrt{Dt}$.)

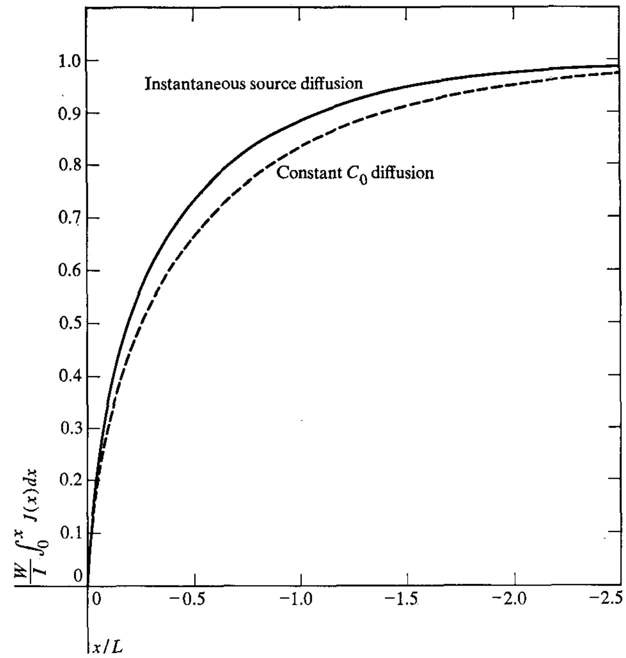


Figure 7 Relative electric current distribution at the ohmic contact of a diffused resistor. ($L = 2\sqrt{Dt}$.)

the electric flux density exhibits a variation of approximately two orders of magnitude within a distance of 2.5 L from the ohmic contact edge.

A consequence of this calculation is readily seen by applying Fig. 6 to a typical diffused resistor. From Fig. 6, at the edge of an ohmic contact ($x/L = 0$) we obtain $J \approx 5(I/LW)$. A typical diffused resistor has a width W of about 1.5×10^{-3} cm, and the impurity atom distribution can be characterized by a diffusion length ($L = 2\sqrt{Dt}$) of about 0.65×10^{-4} cm. From Fig. 6, at this location we obtain an electric current density J of about 5×10^4 amps/cm², for each milliampere of electric current I in the diffused resistor.

Regardless of the impurity atom distribution used during device fabrication (erfc, Gaussian, or any combination of the two-step diffusion), approximately 85 per cent of the total electric current in a diffused resistor enters the structure within a distance of about 1.0 L of its ohmic contact. This situation is illustrated in calculations shown in Fig. 7.

During this investigation, it was initially suspected that mobility variations (due to the large electric fields) would substantially reduce current crowding at the ohmic contact. For this reason, calculations were conducted for an n-type diffused resistor; electron mobility has been shown to be influenced by a smaller value of electric field than is the mobility of holes.[8a] Concluded from this calculation is that an unreasonable value of voltage is required to attain the terminal velocity for electrons, although this mechanism would substantially reduce current crowding. At practical operating levels, insufficient difference is observed in the mobility of electrons to obtain a significant reduction of electric flux crowding; this calculation is illustrated in Fig. 8.

Electrical resistance

For engineering purposes, the electrical resistance of a diffused resistor can be considered the sum of three resistive components: a main body and two end contacts. These three regions are separable at any location where the surfaces of constant potential (Fig. 1) are perpendicular to the semiconductor surface. From Fig. 1, it appears reasonable to assume that the end contacts are separable at a location that is 2.5 L (minimum) from the edge of the ohmic contact. In this fashion the electrical resistance of each region can be established, and the total resistance of a diffused resistor becomes the sum of these three terms.

The electrical resistance contributed by the main body is given by

$$R = \frac{l}{W}\rho_s , \qquad (5)$$

where ρ_s is the so-called sheet resistance. For convenience, Fig. 9 presents a simple relation between the C_0 and ρ_s; this illustration is based on an assumption that the p–n junction is a minimum of 2.0 L from the semiconductor surface. From either a measurement of ρ_s or from Fig. 9, Eq. (5) provides a means to establish the electrical resistance contribution from the main body of a diffused resistor.

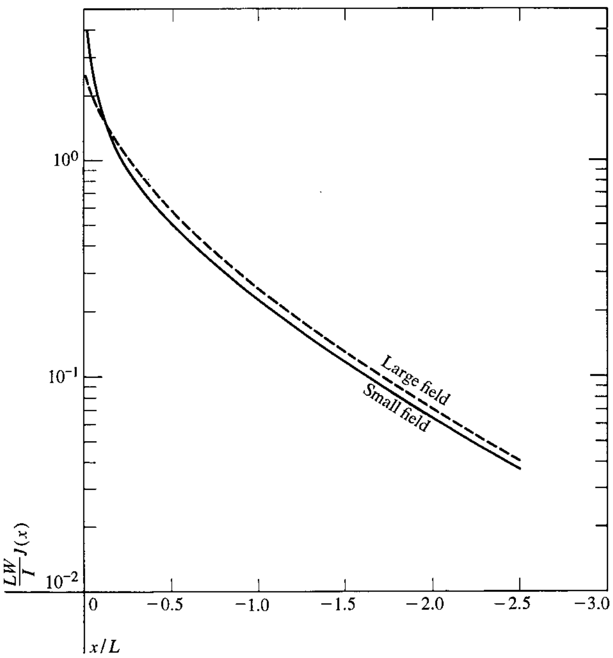


Figure 8 Calculated large field electric flux distribution near the ohmic contact of an n-type diffused resistor. ($L = 2\sqrt{Dt}$.)

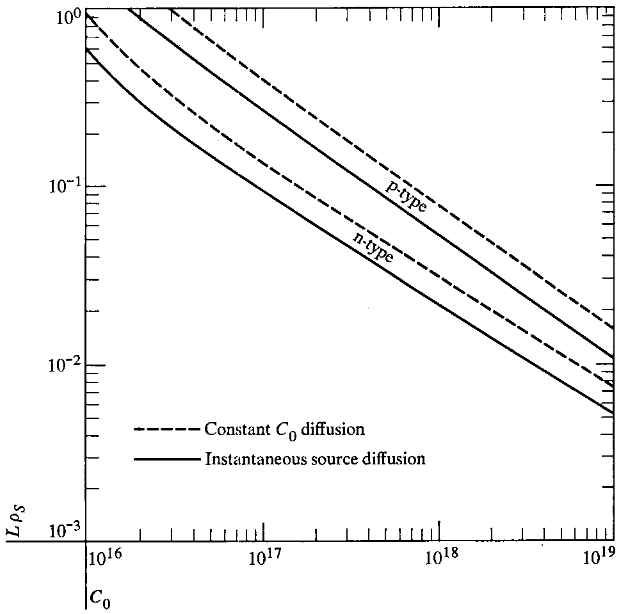


Figure 9 Calculated sheet resistance vs. impurity atom surface concentration. ($L = 2\sqrt{Dt}$.)

Separation of the end contact region was assumed to take place at a point 2.5 L from the edge of the ohmic contact. It can be shown that the calculated electrical resistance of a two-dimensional end contact region is within 5.0 per cent of the electrical resistance given by Eq. (5) when l is assumed to be 3.0 L (instead of 2.5 L). Because the electrical resistance of this end contact region is only a small part of the total resistance of the entire structure, this rule-of-thumb provides a simple means to calculate the electrical resistance of a diffused resistor with sufficient accuracy for engineering purposes.

Acknowledgments

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List of Definitions

C	= Impurity atom concentration.
C_0	= Impurity atom surface concentration.
D	= Impurity atom diffusion constant.
I	= Total electric current in diffused resistor.
$J(x)$, $J(y)$	= Electric current densities.
$L = 2\sqrt{Dt}$	= Impurity atom diffusion length.
W	= Width of diffused resistor.
l	= Length of main body of diffused resistor.
n	= Mobile electron density in semiconductor material.
p	= Mobile hole density in semiconductor material.
$\mathcal{R}_p$, $\mathcal{R}_n$	= Recombination rate for holes and electrons, respectively.
t	= Diffusion time.
x	= Spatial variable parallel to semiconductor surface.
y	= Spatial variable perpendicular to semiconductor surface.
y_j	= Junction depth.
μ_p, μ_n	= Mobility of mobile holes and electrons, respectively.
ρ_s	= Sheet resistance.

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