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Power Conversion in Nonlinear Resistive Elements Related to Interference Phenomena*

Abstract: This paper derives the power conversion in a nonlinear resistive element that is in series with linear resistors. The derivation of the power conversion is analogous to the Manley and Rowe analysis. The power at the fundamental and intermodulation frequencies is defined as the product of the Fourier coefficients of the current and voltage components at these frequencies. In the analysis, the Fourier coefficients are derived as functions of the generator voltage in a lumped-element circuit. To evaluate the Fourier coefficients, the relationship between current and generator voltage is expressed by a power series. The coefficients of the power series are given in the form of polynomials that are valid for nonlinear resistive elements with exponential characteristics.

For the investigation of interference phenomena, the equivalence between the lumped-element circuit and a microwave circuit is derived and the generator voltage is related to the power flux density carried by the incident waves intercepted by support structures. The transfer of power at the intermodulation frequencies to radiating structures is then described. Finally, the power at the intermodulation frequencies is evaluated numerically for different power levels in the incident waves. The computed values were verified experimentally.

Introduction

In recent years, interference phenomena on ships and other vehicles were observed. These interferences were related to corroded joints on support structures. It was found that such joints exhibit frequency conversion properties similar to the properties of nonlinear resistive elements.[1]

To describe the interference phenomena, it was assumed that support structures of a vehicle intercept electromagnetic waves from onboard transmitters and guide the waves to the corroded joints. Intermodulation frequencies in the corroded joints are generated, guided and reradiated by the support structures. Since the frequencies of some of the newly generated signals can fall within the pass band of receivers on board the vehicle, serious interference problems can result from the intermodulation signals generated by the corroded joints.

To investigate interference signals resulting from the above situation, an analysis will be performed on the power conversion capability in a nonlinear resistive element that is resistively terminated. From the analysis we will derive the characteristics of the power at the intermodulation frequencies as a function of the power carried by the incident waves. In particular, we will analytically verify an experimental observation that showed that power at some of the intermodulation frequencies can decrease while the power carried by the larger of the incident waves increases.

For the analysis we assume an equivalent circuit where the corroded joint is represented by a nonlinear resistor with exponential current-voltage characteristic. The structures that intercept the incoming waves and reradiate the frequency converted waves are represented by linear resistors. The generator voltage in the equivalent circuit is related to the power carried by the incident waves. There is no restriction on the power relations among the incident waves.

The functional relationship between the current and the generator voltage is expressed as a power series. The coefficients of the power series are given in the form of polynomials. Since a large number of terms in the power series are required to approximate the current-generator/voltage relationship, the polynomials are given in recursive form.

Expressions are presented for the absorbed power at the fundamental frequencies and for the reradiated power at the intermodulation frequencies. The power conversion can be computed numerically from these expressions for typical nonlinear resistive elements as a function of the power in the incident waves.

Investigations on nonlinear resistors have been published previously. The power conversion in a nonlinear resistive element was treated analytically in Refs. 2–4 but in these

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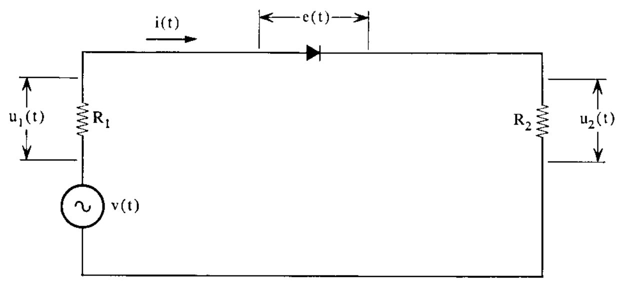


Figure 1 Lumped-element circuit, where a nonlinear resistor is in series with two linear resistors.

analyses the linear series resistors were not considered. L. M. Orloff[5] and L. D. Neidleman,[6] in their analyses, have included linear resistors in series with the nonlinear element. However, Orloff has computed only the first six coefficients of the power series representing the current-voltage relationship, and Neidleman, in his application of FORMAC, has evaluated only the first eight coefficients.

None of the previous analyses lends itself to a verification of the experimentally observed decrease in power at some intermodulation frequencies with increasing power in the incident waves. The decrease in power is related to the linear resistors in the circuit. Their effect, however, became apparent only after we had evaluated a large number of terms in the power series representing the current-voltage relationship.

Power relations

Power conversion in nonlinear resistive elements was treated in general form in several publications.[2–4] In the present paper, we derive the power conversion in a nonlinear resistor that is placed in an actual circuit. In the circuit the nonlinear resistor is in series with linear resistors. The derivation of the power conversion in the nonlinear resistor is analogous to Manley and Rowe's analysis of the energy relations in nonlinear reactances.[7]

In our derivation we will assume that the generator voltage in the circuit is composed of three sine waves (i.e., at the frequencies f_1, f_2 and f_3) that are not harmonically related. The current can then be expressed as a function of the generator voltage and can be represented by the Fourier series

$$i = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} I_{m,n,p} \times \exp j(m\omega_1 + n\omega_2 + p\omega_3)t \,, \tag{1}$$

where $\omega = 2\pi f$.

Since i is real,

$$I_{m,n,p} = I^*_{-m,-n,-p} \quad \text{and} \quad I_{-m,-n,-p} = I^*_{m,n,p} \,.$$

The voltage across the circuit elements as a function of the generator voltage, expressed by a Fourier series, is

$$v = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} V_{m,n,p} \times \exp j(m\omega_1 + n\omega_2 + p\omega_3)t \,. \tag{2}$$

Since v is real,

$$V_{m,n,p} = V^*_{-m,-n,-p} \quad \text{and} \quad V_{-m,-n,-p} = V^*_{m,n,p} \,.$$

The product of current and voltage in Eqs. (1) and (2) gives the power at the fundamental frequencies, at the harmonics, and at the sum and difference frequencies. Expressions for the power, in general form, follow directly from Manley and Rowe's analysis.[7] The real power at the frequency $mf_1 + nf_2 + pf_3$ is

$$P_{m,n,p} = V_{m,n,p} I^*_{m,n,p} + V^*_{m,n,p} I_{m,n,p} \,. \tag{3}$$

Since in our analysis we will not consider any reactances in the circuit, the real power is

$$P_{m,n,p} = 2 V_{m,n,p} I_{m,n,p} \,. \tag{4}$$

Fourier coefficients of current and voltage components

The analysis considers the nonlinear resistor operated in the circuit shown in Fig. 1, where the nonlinear resistor is in series with linear resistors. The circuit does not contain any frequency selective elements.

To investigate the power relations in the nonlinear resistor, we must evaluate the Fourier coefficients in Eqs. (1) and (2). To do so, we will express the time varying current in the circuit and the time varying voltages across the circuit elements as functions of the time varying generator voltage. We assume that the generator voltage $v(t)$ is composed of three sine waves:

$$v(t) = a \cos \omega_1 t + b \cos \omega_2 t + c \cos \omega_3 t \,. \tag{5}$$

In general, the current-voltage characteristic of a nonlinear resistive element can be approximated by an exponential function:

$$i(t) = i_0[e^{\alpha e(t)} - 1] \,, \tag{6}$$

where $i(t)$ is the current through the nonlinear resistor, $e(t)$ is the voltage across the nonlinear resistor, and i_0 and α are constant values that are typical for a specific nonlinear resistor.

The voltage across the nonlinear resistor in the circuit in Fig. 1 is

$$e(t) = v(t) - u(t) = v(t) - i(t)R \,, \tag{7}$$

where $u(t) = u_1(t) + u_2(t)$, and $R = R_1 + R_2$.

The relationship between current and generator voltage follows from Eqs. (6) and (7):

$$i(t) = i_0\{e^{\alpha[v(t) - i(t)R]} - 1\} \,. \tag{8}$$

The functional relationship in Eq. (8) is given in transcendental form. An explicit relation between current and generator voltage in the form of a power series is given by

$$i(t) = \sum_{k=1}^{\infty} \frac{1}{k!} H_k(z) v^k(t) , \tag{9}$$

where $H_k(z)$ are polynomials in z and

$$z = \frac{di}{dv} = \frac{\alpha i_0}{1 + i_0 \alpha R} \tag{10}$$

evaluated at $v(t) = 0$.

The polynomials $H_k(z)$ in recursive form were derived by H. D. Mills[8] (this issue):

$$H_k(z) = \frac{\alpha^{k-1}}{R} \sum_{j=0}^{2k-1} a_{k,j} (zR)^j \tag{11}$$

and

$$a_{k,j} = j a_{k-1,j} - 2(j-1) a_{k-1,j-1} + (j-2) a_{k-1,j-2} \, . \tag{12}$$

The relation between current and generator voltage in a circuit where a nonlinear resistor is in series with linear resistors is expressed in Eq. (9) in the form of a power series. The expression is valid for a nonlinear resistor with an exponential current-voltage characteristic. In the relation between current and generator voltage, the linear resistors introduce a deviation from the exponential dependence of current upon voltage. We have to realize that the functional relation in Eq. (8) cannot be approximated with sufficient accuracy by a power series unless a very large number of terms in the power series is considered.

The relation between the voltage across the nonlinear resistor and the generator voltage, and between the voltage across the linear resistors and the generator voltage in the form of power series follow from Eqs. (7) and (9).

To evaluate the Fourier coefficients of the current and the voltages across the nonlinear and across linear resistors as functions of the amplitudes of the three sine waves from Eqs. (5), (7) and (9), we use an expression that was derived previously by C. A. A. Wass.[9] From this expression the Fourier coefficients of a time varying function can be computed, that is, represented by a power series. The Fourier coefficients of the current and voltage components at the fundamental frequencies and at the intermodulation frequencies are represented in the form of series. A few terms of the Fourier coefficients $I_{1,0,0}$; $I_{0,1,0}$ $I_{0,0,1}$; $I_{1,1,0}$ and $I_{1,0,2}$ are given in the Appendix.

Power transfer

Radiating systems on ships and on space vehicles operate in general at ultrahigh frequencies or at microwaves. At these frequencies, we measure the power carried by the incident and reflected waves and power in waves at newly generated frequencies. A general equation is presented, Eq. (4), for the power at the fundamental frequencies and at the intermodulation frequencies in a nonlinear resistive element. The Fourier coefficients in this equation are derived from the current and voltage relations in the lumped-element circuit in Fig. 1. It is required to relate the lumped-element circuit to an equivalent microwave circuit. In a lumped-element circuit, the circuit elements are small compared to a wavelength and no impedance transformation occurs along the interconnecting transmission lines.

The investigation in this paper considers a corroded joint on an open metal structure. At microwaves the nonlinear junction in a corroded joint will still be small compared to a wavelength. The transmission path on the open metal structure, however, can be many wavelengths long. But, on the metal structures waves can propagate only in the form of hybrid TE-TM mode surface waves.[10] Surface waves of this type are highly attenuated. On transmission paths that are highly attenuated, impedance transformation is minimized. Although the preceding analytical results do not take into consideration any impedance transformation, they can be assumed to be valid for a corroded joint on an open metal structure.

The equivalent microwave circuit is given in Fig. 2, which shows two antennas, two attenuators and a nonlinear resistor. They represent the support structures that intercept, guide and radiate the waves together with the corroded joint. The function of the linear resistors in Fig. 1 is equivalent to the function of the antennas and attenuators in Fig. 2 when we assume that the equivalent characteristic impedances of the support structures are frequency independent. This assumption can be made for operation over a limited frequency range. Since we are concerned with interference signals that are within the frequency range of the radiating systems on the vehicle—within the UHF and microwave range—the equivalence between Fig. 1 and Fig. 2 can be assumed.

The incoming waves are intercepted by a support structure on the vehicle that functions as a receiving antenna. The power incident on the corroded joint is

$$P_{\text{in}\, m,n,p} = S_{m,n,p} A_e f_\alpha \, ,^{*} \tag{13}$$

where $S_{m,n,p}$ is the power flux density in the incoming waves, A_e is the effective absorption cross section of the support structure, and f_α is a factor representing the attenuation over the transmission path on the metal structure.

The amplitudes of the sine waves in Eq. (5) are related to the power in the incident waves by

$$a_{m,n,p} = \sqrt{8 P_{\text{in}m,n,p} Z_0} \, ,^{*,\dagger} \tag{14}$$

where Z_0 is the equivalent characteristic impedance of the open metal structure that guides the incoming waves to the

* $m, n, p = 1, 0, 0; 0, 1, 0; 0, 0, 1$; respectively.
† $a_{1,0,0} = a; a_{0,1,0} = b; a_{0,0,1} = c$

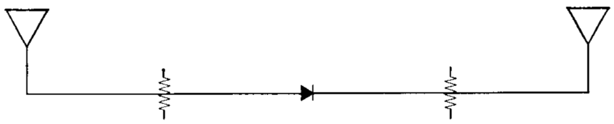


Figure 2 Schematic presentation of corroded joint that functions as a nonlinear resistive element, and of support structures that function as attenuators and antennas (microwave equivalent circuit).

corroded joint. (In Eq. (14) it is assumed that only half of the power that is intercepted by the support structure is guided to the corroded joint. This is consistent with the definition of the effective absorption cross section[10] of the support structure that relates the power flux density of the intercepted waves to the power that is guided to the corroded joint.)

At the corroded joint one part of the incident waves is absorbed, one part is reflected and one part is transferred to the output guiding structure. The part of the incoming waves that is absorbed by the corroded joint can be converted to waves at the intermodulation frequencies. The waves at the intermodulation frequencies are guided and radiated by the two support structures.

The power at the fundamental frequencies that is absorbed by the nonlinear resistor follows from Eq. (4) for $V_{m,n,p} = E_{m,n,p}$; it is

$$P_{\text{absorbed }m,n,p} = 2\ E_{m,n,p} I_{m,n,p}\ , \tag{15}$$

where $E_{m,n,p}$ and $I_{m,n,p}$ can be computed from Eqs. (5), (7) and (9) for $R = 2Z_0$ and from C. A. A. Wass' expression.[9]

The power at the intermodulation frequencies $mf_1 \pm nf_2 \pm pf_3$ that is guided and radiated by the support structures follows from Eq. (4) for $V_{m,n,p} = U_{m,n,p}$. It is

$$P_{m\,n,p} = 2\ U_{m,n,p} I_{m,n,p}\ , \tag{16}$$

where $U_{m,n,p}$ is the Fourier coefficient of the voltage component across the linear resistors in the lumped element circuit at the frequencies $mf_1 \pm nf_2 \pm pf_3$. (The equivalence of the linear resistors in the lumped element circuit and the characteristic impedance of the guiding and radiating structures in the microwave circuit were outlined before.)

For the microwave circuit, where $R = 2Z_0$, we obtain from Eq. (7)

$$U_{m,n,p} = 2\, I_{m,n,p} Z_0\ . \tag{17}$$

Then the power at the intermodulation frequencies in Eq. (16) is

$$P_{m,n,p} = 4\, I^2_{m,n,p} Z_0\ . \tag{18}$$

Because we have assumed the same equivalent characteristic impedance for the input and output transmission paths in the microwave circuit, half the power in Eq. (18) is guided and radiated by the structure that had intercepted the incoming waves. The other half power is guided and radiated by the output radiating structure.

Characteristics of Fourier coefficients of current components

The general power relations in the foregoing sections do not yield any quantitative results. The numerical evaluation of the power at the intermodulation frequencies from Eq. (18) was restricted by limitations of our computer program to comparatively low power levels. To learn more about the characteristics of the power at the intermodulation frequencies we investigated the characteristics of the Fourier coefficients of the current components $I_{m,n,p}$.

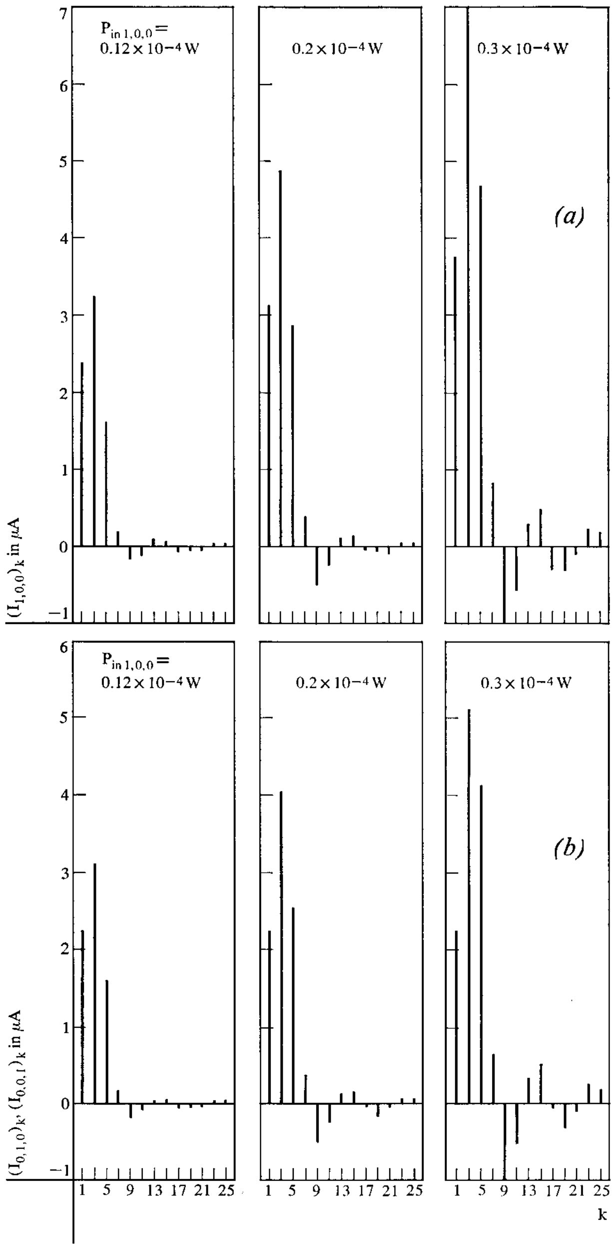
For the investigation of interferences on ships and other vehicles, we assumed that the physical and chemical structure of a corroded joint corresponds to a metal-metal compound rectifier. Then, for a certain asymmetry of the junction of the corroded joint, its nonlinear current-voltage relation can be described by the exponential function in Eq. (6).

For a nonlinear resistor characterized by the exponential function in Eq. (6), the relationship between the Fourier coefficient $I_{m,n,p}$ and the power in the incident waves is determined by the coefficients in the power series in Eq. (9). The coefficients are weighted by the contribution of the amplitudes of the incident waves to the current component at the frequencies $mf_1 \pm nf_2 \pm pf_3$. Thus, the characteristics of the Fourier coefficients are closely related to the characteristics of the polynomials $H_k(z)$ in the power series.

We numerically evaluated the polynomials $H_k(z)$ for typical values of the constants α and i_0 of a nonlinear resistor and for typical values of the linear resistor R in the circuit. We found that the magnitude of the polynomials $H_k(z)$ is determined by all three constant parameters α, i_0 and R. The polynomials are not all positive but alternate between positive and negative values. In particular, the polynomials for small values of k are positive and very small. They become larger as k increases, reach a maximum value, decrease and become negative. Then, when k is further increased, the polynomials $H_k(z)$ go through negative and positive cycles of increasing amplitudes. (When the linear resistor R is zero, all the coefficients in the power series in Eq. (9) are positive.)

In the series representation of the Fourier coefficients of the current components, where the polynomials $H_k(z)$ are weighted by the contributions of the amplitudes of the incident waves to the current components, the weighting factors have the effect of damping the alternating cycles of positive and negative values of the polynomials.

We examined the contribution of positive and negative terms in the series presentation of the Fourier coefficients. To do so, we computed the contribution of the first 25



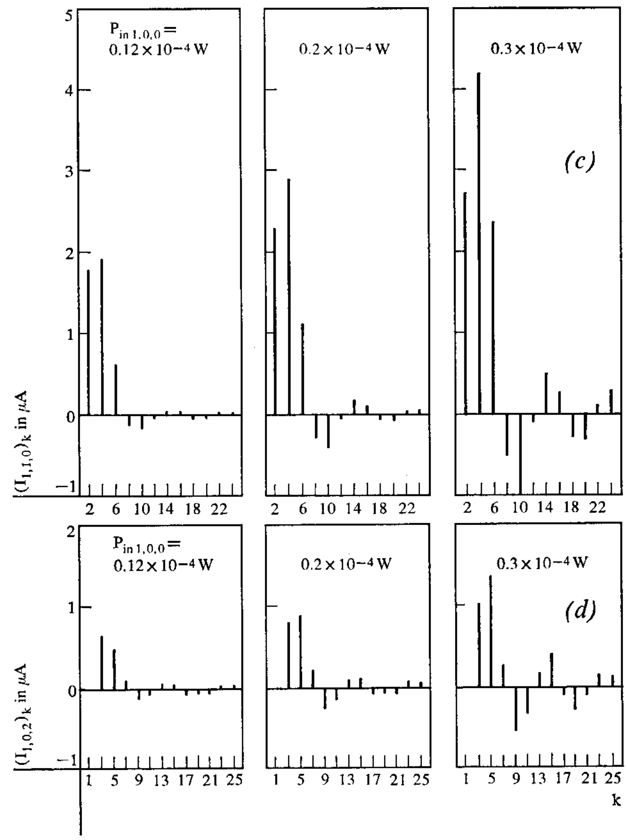


Figure 3 Contributions of the first 25 terms in the series representation of the Fourier coefficients: (a) $I_{1,0,0}$, (b) $I_{0,1,0}$ and $I_{0,0,1}$, (c) $I_{1,1,0}$, and (d) $I_{1,0,2}$. In all computations $P_{\text{in }0,1,0} = P_{\text{in }0,0,1} = 0.105 \times 10^{-4}$W, $\alpha = 23\text{V}^{-1}$, $i_0 = 3 \times 10^{-6}$A, and $R = 100\Omega$.

terms of the power series to the Fourier coefficients of the current components at the fundamental frequencies f_1, f_2, and f_3, and at the intermodulation frequencies $f_1 \pm f_2$ and $f_1 \pm 2f_3$. In particular, we computed the change of the contribution to the Fourier coefficients that occurs when we increase the power carried by the incident wave at the frequency f_1, while the power carried by the incident waves at the frequencies f_2 and f_3 remained constant. (The signal at the frequency f_1 is the largest of the three signals.) The numerical evaluation was performed for power levels in the incident waves that were considered typical for interferences between radiating systems on ships and space vehicles.*

The results are shown in Fig. 3. Apparently, the main contribution of the Fourier coefficients is contained within the first terms in the series where the polynomials are positive. The first positive terms are largest for the fundamental frequencies and decrease with increasing order of the intermodulation frequencies. The higher order terms in the series where the polynomials go through cycles of negative and positive values constitute a correction factor that is negative. The contribution from the terms that alternate between negative and positive values is almost the same at the different frequencies.

When the power carried by the incident waves is small, the negative correction factor is quite insignificant. This means the alternating cycles of negative and positive values of the polynomials are damped quite strongly by the weighting factor. When the power carried by the incident waves becomes larger, the contribution of the negative correction terms becomes more effective. When the incident power is further increased, the alternating cycles of negative and

* The polynomials $H_k(z)$ and the contribution of the k terms of the series representing the Fourier coefficients $I_{m,n,p}$ were evaluated by an IBM 1410.

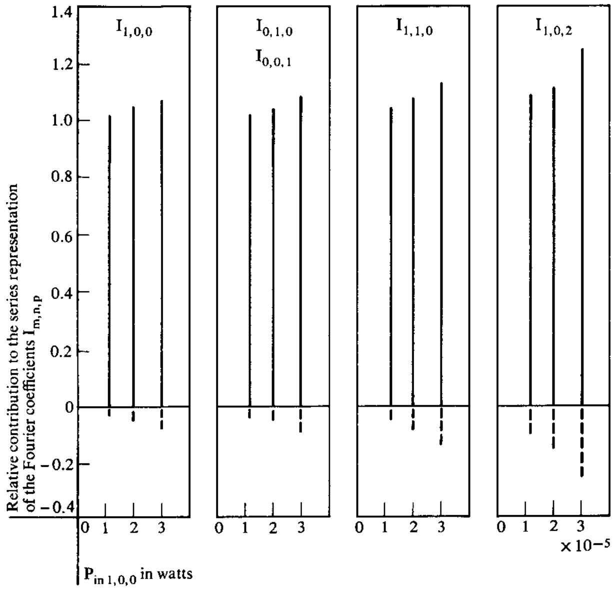


Figure 4 Relative contribution of the sum of the first positive terms, $k = 1$ to 7, (shown as solid lines), and of the negative correction terms, $k = 8$ to 25, (shown as dashed lines), to the series representation of the Fourier coefficients, $I_{m,n,p}$. Same circuit parameter values as for Fig. 3.

positive values of the polynomials are damped by the weighting factor for very large values of k only. For smaller values of k, the alternating cycles of negative and positive values increase in amplitude.

The relation between the first positive terms in the series representation of the Fourier coefficients and of the negative correction factor becomes more apparent in Fig. 4, where we give the relative contribution to the Fourier coefficients $I_{m,n,p}$ from the sum of the first positive terms in the series representation, to the sum of the alternating negative and positive terms. This relation is given as a function of the power in the incident wave at the frequency f_1. It is apparent that the negative correction terms become more effective as the power carried by the incident wave at the frequency f_1 becomes larger. Furthermore, the contribution from the negative correction terms is more significant at the higher order intermodulation frequencies, where the contribution from the first positive terms is smaller.

From the data in Fig. 4 we can extrapolate the characteristics of the current components for higher power levels. The current components at the lower order intermodulation frequencies will always be larger than the current components at the higher order intermodulation frequencies. When the power carried by the incident wave at f_1 further increases, the contribution of the negative correction term to the current components will become even more effective.

It is apparent that at the higher power levels, the current components will not increase continuously. Each of the current components will reach a maximum value as the rate of increase of the first positive terms in the series representation of the Fourier coefficient becomes equal to the rate of increase of the negative correction term. When the power in the incident wave at f_1 is further increased, the rate of increase of the negative correction term will exceed the rate of increase of the first positive terms and the current component will become smaller.

The contribution from the negative correction terms will be more effective at the higher order intermodulation frequencies than at the lower order intermodulation frequencies. Therefore, the maximal values of the current components at the lower order intermodulation frequencies will be obtained at a higher power level of the incident waves than the maximal values of the current components of the higher order intermodulation frequencies.

Power at intermodulation frequencies

The power at the intermodulation frequencies $mf_1 \pm nf_2 \pm pf_3$ defined in Eq. (18) is proportional to the square of the Fourier coefficient of current component $I_{m,n,p}$. Consequently, the characteristics of the power at the intermodulation frequencies follow directly from the characteristics of the current components in the preceding section. We conclude that for given power level of the incident waves, the power at the lower order intermodulation frequencies is higher than the power at the higher order intermodulation frequencies. (This is in accordance with the limitations on the power at the intermodulation frequencies that are given in general form by R. H. Pantell.[2]) Furthermore, when the power carried by the incident waves is small and then increases, the power at the lower order intermodulation frequencies increases faster than the power at the higher order intermodulation frequencies.

We computed the power at the intermodulation frequencies $f_1 \pm f_2$ and $f_1 \pm 2f_3$ as a function of the power carried by the incident wave at the frequency f_1 from the Fourier coefficients in the last Section. The results are shown in Fig. 8. The power at the frequencies $f_1 \pm f_2$ is higher and increases very fast when the power in the incident wave at the frequency f_1 becomes larger. The power at the frequencies $f_1 \pm 2f_3$ is lower and increases considerably slower.

The power carried by the incident waves in our computation is approximately one order of magnitude lower than the power level for maximum transfer of power from the incident waves to the nonlinear resistor where $E_{m,n,p} = U_{m,n,p}$.* Although we have not computed the current components for the higher power level, we have extrapolated their characteristics from data computed at the lower power level. We conclude from the analysis that the power at each of the intermodulation frequencies will increase, reach a maximum value and then decrease when the power in the larger of the incident waves increases. The maximum power

* $m, n, p = 1, 0, 0; 0, 1, 0; 0, 0, 1$; respectively.

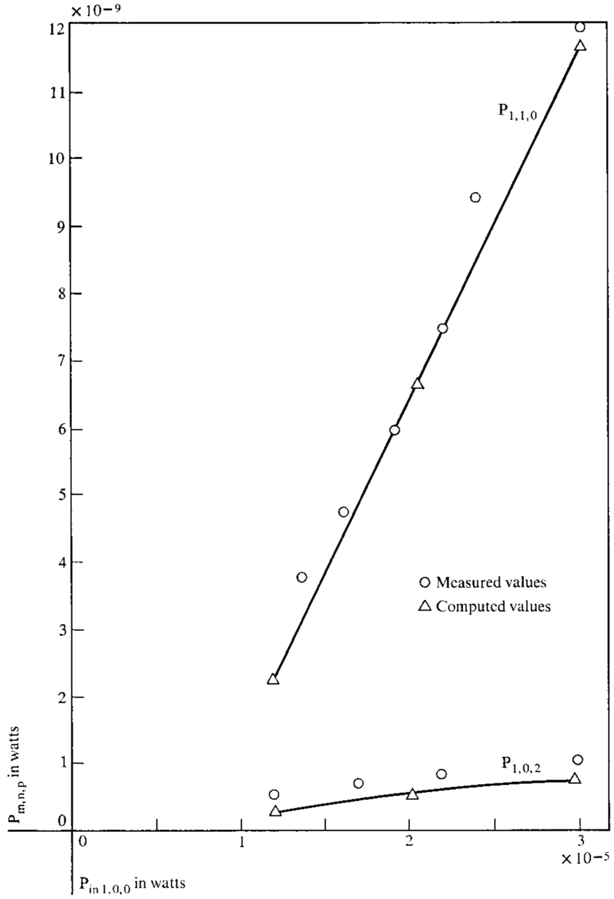


Figure 5 Power at frequencies $f_1 - f_2$ and $f_1 + 2f_3$ for low values of incident power at f_1. Same circuit parameter values as for Fig. 3.

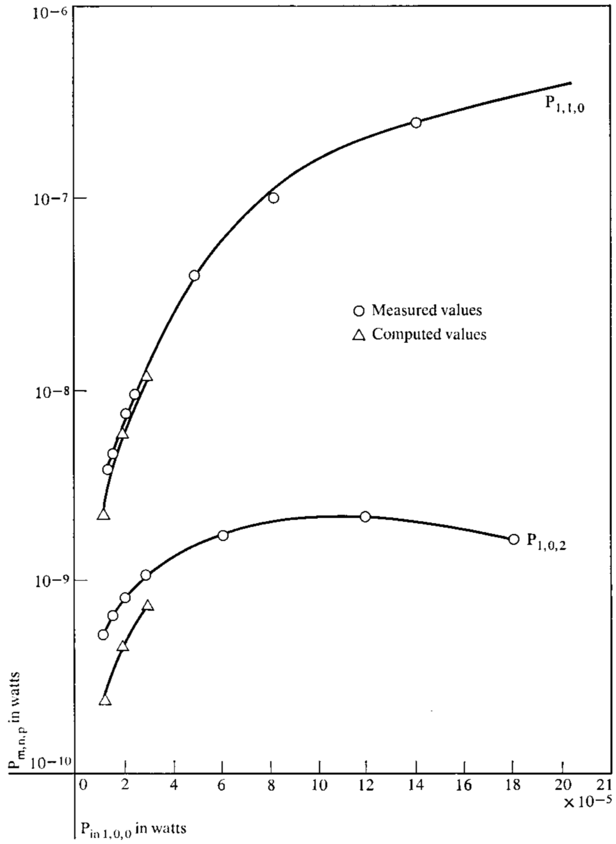


Figure 6 Power at frequencies $f_1 - f_2$ and $f_1 + 2f_3$ for intermediate values of incident power at f_1. Same circuit parameter values as for Fig. 3.

at the low order intermodulation frequencies will be obtained at a higher power level of the incident waves than the maximum of power at the higher order intermodulation frequencies. These characteristics were observed experimentally.

Experimental verification

We built a model of the microwave circuit in Fig. 2 where a microwave nonlinear resistor diode of the type 1 N21E is in series with the inner conductor of a 50-ohm strip transmission line. Microwave attenuators were placed at the input and output of the diode. Three microwave signals were directed to the nonlinear resistor.

We measured the power at the intermodulation frequencies that were within the microwave band, and investigated the change of power at the intermodulation frequencies that occurred when we increased the power in the largest of the signals while the power in the two smaller signals remained constant. The results of our measurements are shown in Figs. 5, 6 and 7. (We measured the sum of the power in the waves travelling towards the load and towards the generator.)

In Fig. 5 we compare the measured values of the power at the intermodulation frequencies $f_1 - f_2$ and $f_1 + 2f_3$ to the computed values. To correlate the measured and computed values, we evaluated the Fourier coefficients for the parameters α and i_0 of a microwave diode of the type 1N21E and for the characteristic impedance of the transmission path of 50 ohms.

There is very close agreement between the power measured at the intermodulation frequencies in the microwave band and the power that was computed from equations derived for a low-frequency equivalent circuit. The close agreement confirms the validity of our assumption that reactances need not be considered in the analysis. Even the shunt capacitance of the nonlinear resistor can be neglected.

We had assumed that the shunt capacitance may be disregarded for the following reason. The power at the inter-

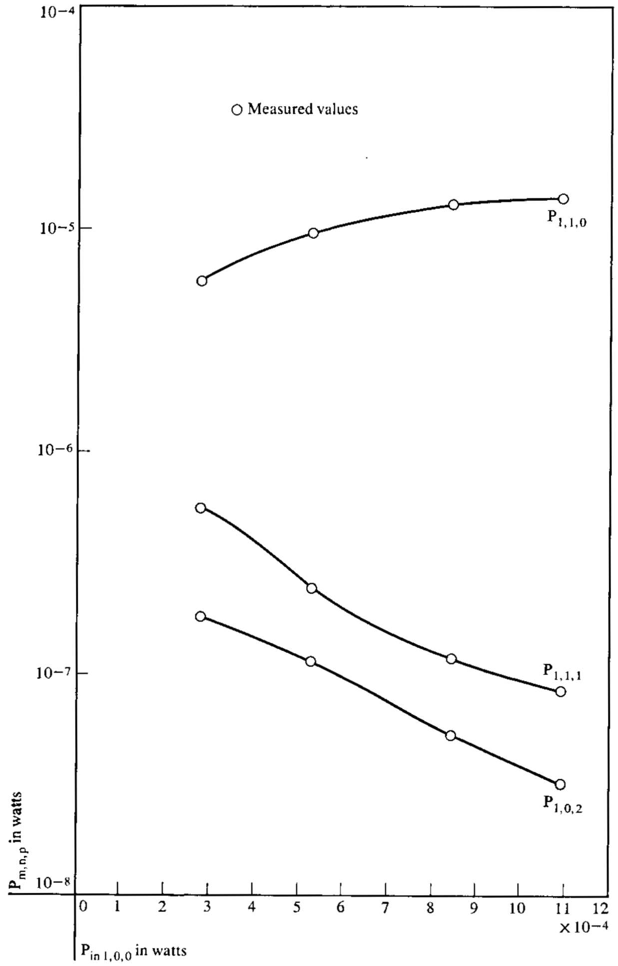


Figure 7 Power at frequencies $f_1 - f_2$, $f_1 - f_2 + f_3$, and $f_1 - 2f_3$ for high values of incident power at f_1. Same circuit parameter values as for Fig. 3.

modulation frequencies in Eq. (16) is directly related to the distortions in the voltage waveform across the nonlinear resistor in Eq. (7). The distortions occur in the forward conduction region, where the instantaneous impedance of the nonlinear resistor is small. In this region the shunt capacitance is actually least effective.

Figures 6 and 7 give the power at the intermodulation frequencies measured at higher power levels in the incident waves. These measurements were performed to confirm the characteristics of the power at the intermodulation frequencies that were extrapolated from the computed data.

In Fig. 6 we show an extension of the measured values in Fig. 5 to higher power in the incident wave at the frequency f_1. In Fig. 7 we present measured values of the power at the intermodulation frequencies $f_1 - f_2$, $f_1 - f_2 + f_3$ and $f_1 - 2f_3$. The power level of the incident waves is approximately one order of magnitude above the power level in Fig. 6.

We can observe that the power at the low order intermodulation frequencies is higher than the power at the higher order intermodulation frequencies. Furthermore, power at frequencies $f_1 - f_2$ continues to increase as power in the incident wave at f_1 becomes larger, while the power at $f_1 - f_2 + f_3$ and at $f_1 \pm 2f_3$ reaches maximum values and then decreases. Thus, the measured characteristics are in agreement with the characteristics of the power at the intermodulation frequencies that were extrapolated from the analytical results for lower power levels.

Conclusions

From the analysis presented in this paper we derived the characteristics of the power at the intermodulation frequencies that were generated in a nonlinear resistive element in series with linear resistors. We found that the power at the lower order intermodulation frequencies is always higher than the power at the higher order intermodulation frequencies. When the power in the incident waves is small, the power at the intermodulation frequencies is small. When the power carried by the larger of the incident waves increases, the powers at the intermodulation frequencies increase, reach maximal values, and decrease. The maximum power at the low order intermodulation frequencies is obtained at a higher power level for the incident waves than the maximum power at the higher order intermodulation frequencies.

These characteristics are significant when we evaluate interferences between radiating systems on ships and other vehicles. Obviously, the power at the intermodulation frequencies, generated in a nonlinear resistive element in an actual circuit, can become smaller although the power in some of the incident waves increases. Furthermore, the power at the low order intermodulation frequencies can increase, while the power at the higher order intermodulation frequencies is decreasing.

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Appendix

Fourier coefficients of the current components $I_{1,0,0}$; $I_{0,1,0}$; $I_{0,0,1}$; $I_{1,1,0}$ and $I_{1,0,2}$ when the generator voltage is given by Eq. (5). (First terms only.)

$$I_{1,0,0} = \frac{1}{2}\left[H_1(z)a + \frac{1}{3!}H_3(z)\left(\frac{3}{4}a^3 + \frac{3}{2}ab^2 + \frac{3}{2}ac^2\right) + \cdots\right]$$

$$I_{0,1,0} = \frac{1}{2}\left[H_1(z)b + \frac{1}{3!}H_3(z)\left(\frac{3}{4}b^3 + \frac{3}{2}a^2b + \frac{3}{2}bc^2\right) + \cdots\right]$$

$$I_{0,0,1} = \frac{1}{2}\left[H_1(z)c + \frac{1}{3!}H_3(z)\left(\frac{3}{4}c^3 + \frac{3}{2}a^2c + \frac{3}{2}b^2c\right) + \cdots\right]$$

$$I_{1,1,0} = \frac{1}{2}\left[\frac{1}{2!}H_2(z)ab + \frac{1}{4!}H_4(z)\left(\frac{3}{2}a^3b + \frac{3}{2}ab^3 + 3abc^2\right) + \cdots\right]$$

$$I_{1,0,2} = \frac{1}{2}\left[\frac{1}{3!}H_3(z)\frac{3}{4}ac^2 + \frac{1}{5!}H_5(z)\left(\frac{15}{4}ab^2c^2 + \frac{15}{8}a^3c^2 + \frac{5}{4}ac^4\right) + \cdots\right].$$

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