

Special Section on Difference Equations

Foreword

The swift and dramatic rise of the computer as a new mathematical aid in every sort of human endeavor has, as one might expect, been reflected in mathematics itself. The field of numerical analysis, once a rather narrow province, has become a major region of mathematical activity. One of the most prophetically stimulating developments in numerical analysis occurred some years before the appearance of electronic digital computers. This was the paper "On the Partial Difference Equations of Mathematical Physics" by Richard Courant, Hans Lewy, and Kurt Friedrichs, published in *Mathematische Annalen* in 1928. This paper has formed the basis of modern investigations into the numerical analysis of partial differential equations. The ideas exposed still prevail.

Professor Courant, his colleagues and his students have continued to help provide the numerical theory that digital computation requires. For a number of years, Professor Courant has been closely associated with mathematical developments in IBM and has played an important role in stimulating interaction between the mathematics community and workers in computer development.

We have thought it appropriate, in recognition of his continuous role in numerical analysis and his close ties with IBM, to dedicate the three following papers to Professor Courant. We have asked three mathematicians prominent in the field of the numerical analysis of partial differential equations to survey the developments since the 1928 publication. Professors Lax, Parter, and Widlund have all, during the course of their careers, come under the direct or indirect influence of Professor Courant.

We have also decided to reprint an English translation of the original *Mathematische Annalen* paper because English versions have had a somewhat limited distribution. The one presented here was prepared by Professor Phyllis Fox, and we are greatly indebted to her and to the U. S. Atomic Energy Commission for its use here.

In addition we are most grateful to Drs. Hirsh Cohen, Herman H. Goldstine, and Ralph E. Gomory for their help in planning this group of papers and particularly to Dr. Willard Miranker for his efforts in acquiring the appropriate manuscripts.

Authors of the Papers on Difference Equations

Richard Courant

Courant Institute of Mathematical Sciences, New York University

Studied at universities at Breslau and Zurich; Ph.D., 1910, University of Göttingen. Honorary degrees include E.D., 1955, Technische Hochschule, Darmstadt; D.E., 1958, Technische Hochschule, Aachen; Sc.D., 1958, New York University; Sc.D., 1958, Case Institute of Technology; Sc.D., 1965, Adelphi University. Was Assistant and Instructor of Mathematics, 1910-1914 and Professor of Mathematics, 1919-1920, University of Münster, Professor of Mathematics and Director, 1920-1933, Mathematics Institute, Göttingen; Lecturer, 1933-1934, Cambridge; Professor of Mathematics and Head of Mathematics Department, New York University, 1934-1958 and Director of the Institute of Mathematical Sciences, NYU, 1936-1958; is now Professor Emeritus of Mathematics and Science Advisor to the University, at Courant Institute of Mathematical Sciences, NYU. Professor Courant is a consultant to the Research Division, IBM and to the General Atomic Division, General Dynamics Corporation. He is a member of Accademia Nazionale dei Lincei (Rome), Akademie der Wissenschaften (Göttingen), American Mathematical Society, American Philosophical Society, American Physical Society, Cosmos Club, Mathematical Association of America, National Academy of Sciences, New York Academy of Sciences, Royal Danish Academy of Sciences and Letters, and Royal Netherlands Academy of Sciences and Letters; he is an honorary member of the Accademia dei Lincei and of the Mathematical Society of Moscow; and a Foreign Member of the Academy of Sciences of the USSR. Awards include the Knight-Commander's Cross and Star of the Order of Merit of the Federal Republic of Germany, 1958, the Navy Distinguished Public Service Award, 1958, and the Award for Distinguished Service to Mathematics, Mathematical Association of America, 1965. Among his many books a few of the distinguished contributions include *Funktionentheorie* (with A. Hurwitz), *Methods of Mathematical Physics* (with D. Hilbert) Vols. I and II, and *Differential and Integral Calculus*, Vols. I and II. Professor Courant's broad fields of interest and endeavor have included the theory of functions, mathematical analysis and physics, and the calculus of variations.

Kurt O. Friedrichs

Courant Institute of Mathematical Sciences, New York University, New York

Ph.D. in Mathematics, University of Göttingen, Germany, 1925. Professor, Technische Hochschule, Braunschweig, Germany, 1930-1937. Came to USA in 1937 and joined New York University. At present is Distinguished Professor of Applied Mathematics and Director of the Courant Institute of Mathematical Sciences. Member, National Academy of Sciences; Fellow, American Academy of Arts and Sciences; Member, American Physical Society, American Mathematical Society, Mathematical Association of America. Professor Friedrichs' current research interests are in methods of mathematical physics, differential- and pseudo-differential equations.

Peter D. Lax

Courant Institute of Mathematical Sciences, New York University, New York

Ph.D. in Mathematics, 1949, New York University. He has been a staff member at the Los Alamos Scientific Laboratory and is now Professor of Mathematics and Director of the A.E.C. Computing and Applied Mathematics Center at the Courant Institute. His current research interests are in partial differential equations, numerical analysis, harmonic analysis, scattering theory, and applied mathematics. Professor Lax received the Ford award in 1966. Member, American Mathematical Society and the Mathematical Association of America.

Hans Lewy

University of California, Berkeley

Ph.D. in Mathematics, 1926, University of Göttingen. Privatdocent at Göttingen from 1927 to 1933. Taught at Brown University from 1933 to 1935 and in 1935 joined the faculty of the Mathematics Department at the University of California at Berkeley, where he is currently a professor. Research interests are the partial differential equations, calculus of variations, and hydrodynamics.

Seymour V. Parter

University of Wisconsin, Madison, Wisconsin

B.S. and M.S. in Mathematics, Illinois Institute of Technology; Ph.D. in Mathematics, New York University. Has taught at MIT, Indiana University, Cornell University, and Stanford University. Also, has been a staff member at the Los Alamos Scientific Laboratory, a visiting mathematician at the Brookhaven National Laboratory, and a consultant to the Lawrence Radiation Laboratory and the Midwest Research Institute. His current research interests, as a Professor of Computer Sciences and Mathematics at Wisconsin, include numerical methods for differential equations and Toeplitz forms. Member, American Mathematical Society, Mathematical Association of America, Association for Computing Machinery, Society for Industrial and Applied Mathematics, and Sigma Xi.

Olof B. Widlund

University of Uppsala, Uppsala, Sweden

M.S. in Mathematics, 1960; Ph.D. in Mathematics, 1964, Royal Institute of Technology, Stockholm; Ph.D., Uppsala University. Instructor at Royal Institute of Technology, 1958-1964. Research assistant, Swedish Technical Research Council 1964-1966, doing research on numerical treatment of partial differential equations. Visitor at Courant Institute of Mathematical Sciences and University of California at Berkeley, 1955 and 1966. Dr. Widlund is now assistant professor in the Department of Computer Sciences at Uppsala University. He is currently interested in theoretical numerical analysis and partial differential equations. Member, American Mathematical Society and Swedish Mathematical Society.