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Influence of Non-ideal Filters on the Transmission Characteristics of Resonant Transfer Switching Systems*

This communication analyzes the influence of non-ideal practical filters with finite cutoff slope on the transmission characteristics of general resonant transfer systems. Low-pass-to-lowpass resonant transfer transmission systems have been used in the past[1-5] as means for efficiently implementing time-division switching systems. A basic transmission path in these systems consists of a band-limiting lowpass filter at the input and an interpolation lowpass filter at the output, with two serial switches and a time-divided "highway" interconnecting the filters. The capacitive input impedances of the two filters (see Fig. 1) and the switching path impedances are tuned to resonance, so that each time the switch is closed all energy from the input is transferred to the output. This makes the resonant transfer method of time-division switching practically lossless and independent of the unavoidable distributed switching path inductances and capacitances.

More recent developments show how the concept of resonant transfer can be extended from the original lowpass-to-lowpass case to the lowpass-to-bandpass and bandpass-to-bandpass cases for implementation of systems that integrate time-division switching and frequency-division multiplexing.[6,7] Such an integrated switching and multiplexing system combines the advantages of efficient resonant transfer time-division switching with an unusual ease in generating single-sideband, amplitude-modulated versions of the original signal for further transmission on a frequency-divided highway. The single-sideband modulation products are obtained merely by using bandpass filters instead of lowpass filters at the output side of the resonant transfer switch, thus saving all the carrier supplies usually necessary in conventional frequency-division multiplex transmission systems. The remarkable advantages of integrating resonant transfer time-division switching with frequency-division multiplexing have to be bought with more complex channel filters, which will be the subject of the following discussion.

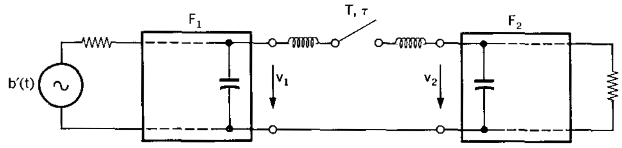

Figure 1 Basic resonant transfer circuit. (T = switching time interval, τ = switch closure time.)

In Ref. 7 it was shown that the sufficient condition for an ideal, general resonant transfer filter is that it have an impulse ring-off behavior (voltage response to an applied unit-current impulse) of the form:

$$r_B(t) = \cos\frac{(2m-1)\pi t}{2T}\;\frac{\sin\dfrac{\pi t}{2T}}{\dfrac{\pi t}{2T}}, \qquad t > 0, \tag{1}$$

where

T = switching interval
$m = 1, 2, 3, 4, \cdots$.

For the lowpass case, where $m = 1$, Eq. (1) reduces to the specific condition of Gibbs[5]

$$r_L(t) = \frac{\sin\dfrac{\pi t}{T}}{\dfrac{\pi t}{T}}, \qquad t > 0. \tag{2}$$

In a time-division switching system (lowpass-to-lowpass transmission) two identical lowpass filters of bandwidth π/T with ring-off behavior $r_L(t)$ should be selected. In an integrated switching and multiplexing system one would use ideal lowpass filters of bandwidth π/T and ideal bandpass filters with center frequencies at $\frac{1}{2}(2m-1)(\pi/T)$, which are also π/T wide (compare example in Ref. 7, $m = 16, 17, \cdots, 27$).

Unfortunately, ideal filters with infinite cutoff slopes, that have ring-off behavior according to Eq. (1), can only

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be approximated. Practical implementations of resonant transfer filters have been described by a number of investigators, such as Thomas,[3] Roehr et al.[7] and Gibbs.[5] The question is: how do these nonideal filters affect the over-all transmission of the resonant transfer system? Nonideal match between resonant transfer capacitors, nonideal filter slopes and nonideal out-of-band selectivity were discussed in Ref. 7. Here an analytic procedure will be outlined which enables one to find the nonideal transfer characteristic as a function of the filter ring-off behavior.

By elementary analysis of the basic resonant transfer circuit of Fig. 1 (see Ref. 3, for instance) it can be shown that the voltages v_1 and v_2 at the input and output sides, respectively, of the switch are interchanged at the switch sampling times nT, where $n = 0, 1, 2, 3, \ldots$, if the time τ during which the switch is closed is properly adjusted. Thus, $v_2(nT + \tau) = v_1(nT)$ and $v_1(nT + \tau) = v_2(nT)$. Due to this voltage interchange, the filters F_1 and F_2 at the input and output sides respectively of the switch receive voltage pulses of amplitude

$$\begin{aligned} p_2(nT) &= v_2(nT + \tau) - v_2(nT) \\ &= v_1(nT) - v_2(nT) = -p_1(nT). \end{aligned}$$

Using conventional z-transform notation

$$P_2(z) = V_1(z) - V_2(z), \tag{3}$$

where

$$V(z) = \sum_{n=0}^{\infty} v(nT)z^{-n} = V^*(s), \qquad z = e^{sT}, \qquad s = j\omega.$$

The voltage $v_2(t)$ at the input to filter F_2 depends on the sum of the products of past voltage pulses p_2 and filter ring-off behavior r_2:

$$\begin{aligned} v_2(nT) &= p_2(0)r_2(nT) + p_2(T)r_2[(n-1)T] + \cdots \\ &\qquad + p_2[(n-1)T]r_2(T) \\ &= \sum_{q=0}^{n-1} p_2(qT)r_2[(n-q)T]. \end{aligned} \tag{4}$$

Equation (4) can be understood as a digital filtering operation, where $r_2(t)$ is the pulse response of a filter whose pulse response gets longer with time. For most practical filters, especially those used in resonant transfer, $|r_2(nT)| < \epsilon$ for $n > N$, where ϵ is a small quantity and N is large enough. For $n > N$, Eq. (4) can be rewritten as

$$v_2(nT) = \sum_{q=0}^{N-1} p_2(qT)r_2[(n-q)T]. \tag{4a}$$

Taking the z-transform at both sides of Eq. (4a), after appropriate changes in summing variables and considering that $p_2(nT) = 0$ for $n < 0$, yields

$$V_2(z) = \left[\sum_{k=1}^{N} r_2(kT)z^{-k}\right]\left[\sum_{q=0}^{\infty} p_2(qT)z^{-q}\right]. \tag{5}$$

The second factor in Eq. (5) is the proper z-transform $P_2(z)$. Since $r_2(kT)$ is small for $n > N$, the upper limit of the first factor in Eq. (5) can be extended to infinity, giving

$$\begin{aligned} \sum_{k=1}^{\infty} r_2(kT)z^{-k} &= \sum_{k=0}^{\infty} r_2(kT)z^{-k} - r_2(0) \\ &= R_2(z) - r_2(0). \end{aligned} \tag{6}$$

Substituting Eq. (6) into Eq. (5) leads to

$$V_2(z) = [R_2(z) - r_2(0)]P_2(z). \tag{7}$$

Similarly, one can obtain for the voltage at the other side of the switch

$$V_1(z) = [R_1(z) - r_1(0)]P_1(z) + B(z), \tag{8}$$

where $B(z)$ is the z-transform of $b(t)$, the voltage at the output of the input filter F_1 due to the input generator. Substituting Eqs. (7) and (8) into Eq. (3) and using the fact that $P_1(z) = -P_2(z)$ generates the z-transfer function of the resonant transfer process as

$$\begin{aligned} H(z) &= \frac{P_2(z)}{B(z)} \\ &= [1 + R_1(z) + R_2(z) - r_1(0) - r_2(0)]^{-1} &(9) \\ &= \left[1 + \sum_{n=1}^{\infty} r_1(nT)z^{-n} + \sum_{n=1}^{\infty} r_2(nT)z^{-n}\right]^{-1}. &(9a) \end{aligned}$$

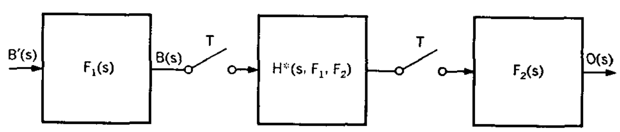
For resonant transfer between equal filters, $r_1(t) = r_2(t) = r(t)$,

$$H_e(z) = \left[1 + 2\sum_{n=1}^{\infty} r(nT)z^{-n}\right]^{-1}. \tag{9b}$$

$H(z) = H^*(s)$, Eq. (9), represents the transfer function of a sampled data process whose characteristic is determined by the ring-off behaviors of the filters F_1 and F_2. The loading effect between filters F_2 and F_1, an essential part in resonant transfer, has been replaced by a separate filter $H^*(s, F_1, F_2)$ which no longer influences F_1 or F_2 and can thus be represented in standard block diagram form (Fig. 2). The output of the over-all system is now in sampled data notation

$$\begin{aligned} O(s) &= B^*(s)H^*(s)F_2(s), \\ B(s) &= B'(s)F_1(s). \end{aligned} \tag{10}$$

Figure 2 Resonant transfer circuit in block diagram form.



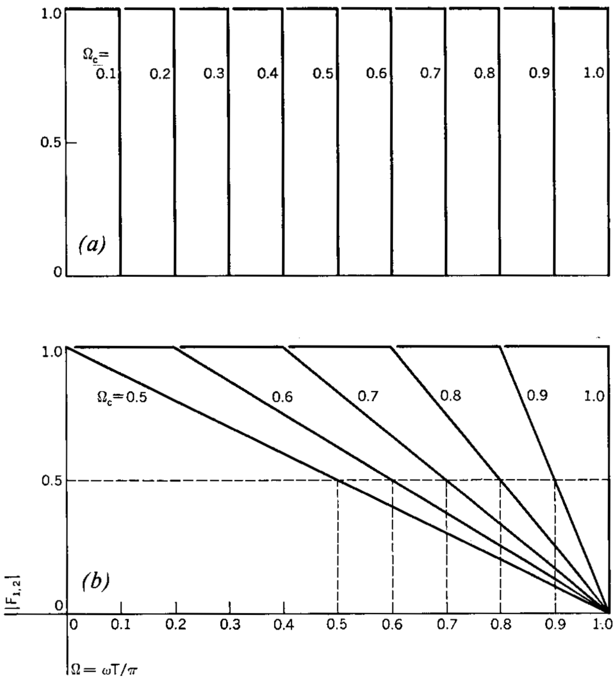


Figure 3 Filter transfer functions for identical input and output filters; i.e., $F_{1,2} = F_1(i\Omega) = F_2(i\Omega)$. (a) Ideal lowpass filters; (b) nonideal lowpass filters.

For a test signal with unity spectrum, $B'(s) = 1, b'(t) = \delta(t)$, the output transform is equal to the system transfer function

$$O_T(s) = F_1{}^*(s) H^*(s) F_2(s). \tag{10a}$$

Studying $H^*(s)$ for $s = j\omega$ gives a clear indication of how the system will be influenced by the nonideal ring-off behaviors of the filters. If $r_1(t)$ and $r_2(t)$ satisfy Eq. (1) or (2), ideal filter ring-off, $H^*(j\omega) = 1$. In a preliminary investigation $|H^*(s)|$ was computed for resonant transfer between 10 different rectangular lowpass filters (Fig. 3a) having the normalized ring-off behavior transforms

$$R_{1,2}{}^*(j\Omega) = \sum_{n=0}^{N} \frac{\sin n\pi\Omega_c}{n\pi\Omega_c} e^{-jn\pi\Omega}, \tag{11}$$

where

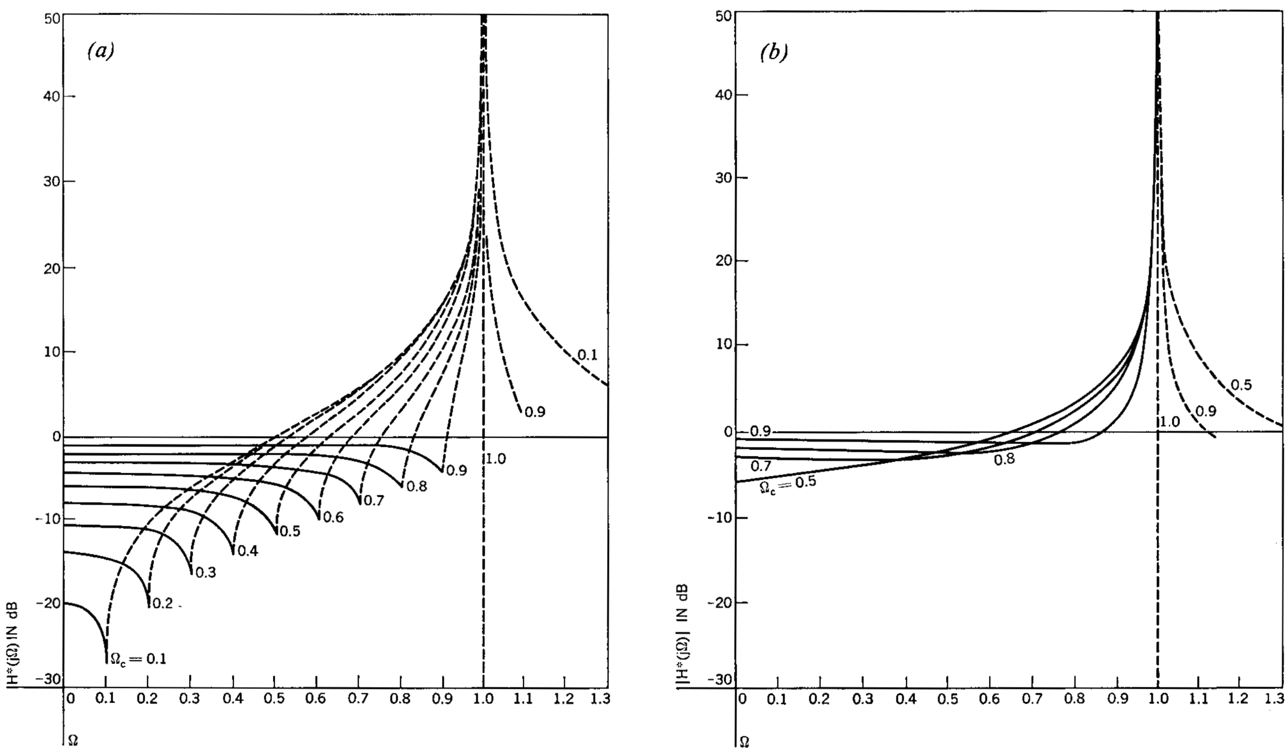
$$\Omega_c = 0.1,\ 0.2,\ \cdots,\ 1,$$

$$N = 50/\Omega_c,$$

$$\Omega = \omega T/\pi.$$

The transfer function, $|H^*(j\Omega)|$ is shown in Fig. 4a. This figure shows that the over-all transmission of a resonant transfer system can have a sizable flat loss at low frequencies and a distinct roll-off at the corners of the band,

Figure 4 Normalized resonant transfer function, $H^*(i\Omega)$. (a) Ideal filters; (b) nonideal filters. Dashed portions of the curves represent values outside the transmission range of the filters.



when the filters have a rectangular amplitude versus frequency characteristic less than π/T radians wide. In addition, approximations to nonideal filters, $F_1 = F_2$, were investigated (Fig. 3b) which have a normalized ring-off behavior transform of

$$R_{1,2}^*(j\Omega) = \sum_{n=0}^{N} \frac{\sin n\pi\Omega_c}{n\pi\Omega_c} \; \frac{\sin n\pi\Omega_d/2}{n\pi\Omega_d/2} \; e^{-jn\pi\Omega}, \quad (12)$$

where

$$\Omega_c = 0.5, 0.6, \cdots, 1.0,$$

$$\Omega_d = 2(1 - \Omega_c),$$

$$N = 50/\Omega_c,$$

$$\Omega = \omega T/\pi.$$

The resulting transfer functions $|H^*(j\Omega)|$ are shown in Fig. 4b.

The main result of this development is Eq. (9) which makes it possible to compute the influence of nonideal filters on the resonant transfer transmission process. Two classes of simple, theoretical filters were investigated to demonstrate the basic findings. More general, practical filters can be considered by using the z-transforms of their input impedances as $R_{1,2}^*(s)$. This will make it possible to compare the quality of resonant transfer filters reported in the literature.[2,3,5,7] In addition, it is planned to use Eqs. (9) and (10) for finding the best parameters of resonant transfer filters which produce an overall transfer function $O_T(s)$ approximating a desired amplitude characteristic in some optimum sense.

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