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Design Technique for High-Efficiency Frequency Doublers Based on the Manley and Rowe Energy Relations*

Abstract: This paper describes a design technique for high-efficiency microwave frequency doublers using a varactor diode. In this technique design parameters that can be measured in microwave circuits are derived from the Manley and Rowe energy relations in nonlinear reactances. These design parameters are: the characteristic impedances of the transmission lines of the microwave structure, the input and output powers of the frequency doubler, and the distances between the varactor diode and the filters that reject the fundamental frequency and the second harmonic.

The design technique for the microwave frequency doubler enables us to "match" the microwave structure to the varactor diode. Furthermore, the technique makes it possible to convert the maximum real fundamental power in the varactor diode to the second harmonic. The design procedure determines the operating region on the diode characteristic that, for a given diode, yields highest conversion efficiency. Our objective was to evaluate the design parameters of the frequency doubler for the region that gives this efficiency.

Two experimental models of a series type *L*-band frequency doubler were built to verify the design technique. The measured conversion efficiencies of the diodes operated in the doubler were in close agreement with the computed conversion efficiencies. We obtained 5 W output power from the varactor diode at 2.1 Gc/sec with 0.79 efficiency. (These values do not include circuit losses.)

Introduction

Several publications in the last few years have treated the analysis of high-efficiency harmonic multipliers[1-5] that operate with the varactor diode. In these publications, the characteristics of such a device are defined by the relationships among the diode's charge, current, and voltage. But in microwave structures, these electrical properties cannot be measured directly. Consequently, the development of high-efficiency microwave harmonic multipliers, when based on these properties, requires a comparatively large experimental effort.

This experimental effort can be reduced considerably when design parameters that can be defined and measured in microwave circuits are derived and evaluated. This paper describes a technique for designing a high-efficiency microwave frequency doubler, in which such parameters are derived. First, in this technique, the properties of the varactor diode in the doubler are defined by the maximum fundamental real power that can become converted to the second harmonic, by the power losses in the diode's series resistance, and by the equivalent impedances of the diode at the fundamental frequency and at the second harmonic. Secondly, the design parameters of the frequency doubler are related to these diode properties. These design parameters are: the characteristic impedances of the input and output transmission lines of the frequency doubler, which are made equal to the resistive components of the diode's equivalent impedances at the fundamental frequency and at the second harmonic; the input and output powers of the frequency doubler; and the spacings between the varactor diode and the filters that reject the waves at the fundamental frequency and the second harmonic.

In the design procedure, the diode type that can meet the specific requirement of the designer is first determined. The diode can be selected by using two sets of parametric curves which relate conversion efficiency and output power to the diode properties. The conversion efficiency of a varactor diode operating in a frequency doubler increases as the operating region is extended partly in the forward conduction region, until it reaches a maximum. The design technique enables us to compute the design parameters of the frequency doubler for the region that yields highest conversion efficiency.

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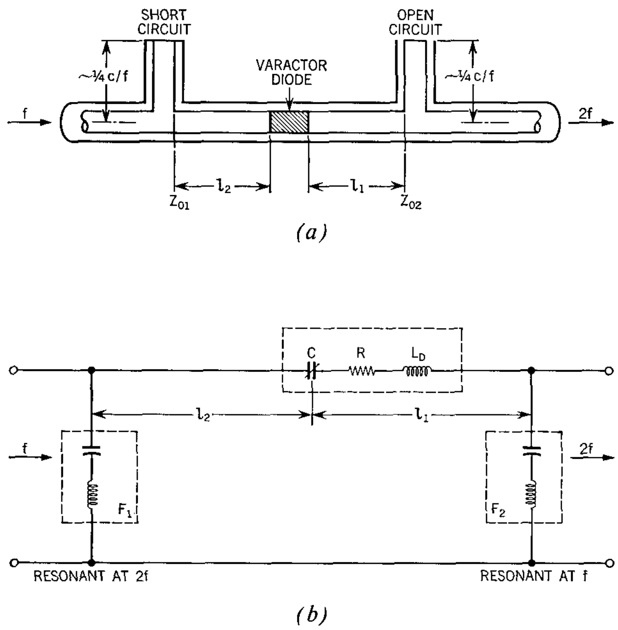
The design technique is based on the analysis of the energy relations in nonlinear reactances by J. M. Manley and H. E. Rowe.[6] Furthermore, the technique utilizes the results of analytical work by J. A. Morrison[7] on the maximization of the real power at the fundamental frequency in a nonlinear capacitance diode that can be converted to the second harmonic. In the next three sections these analyses are reviewed (to explain the design approach) and the design parameters are derived. Then the design parameters are given as functions of the diode properties.

The analyses by Manley and Rowe[6] and by Morrison[7] are valid for lossless nonlinear capacitance diodes. A varactor diode, however, is not a lossless device. Losses in the varactor diode's series resistance must be considered as perturbations. Furthermore, Morrison makes simplifying assumptions on the harmonic content of the charge waveform and of the output power. In addition, he idealizes the voltage-charge relationship in the diode's forward conduction region.

Because of the perturbation calculations and because of the simplifying assumptions, it was important to test experimentally the validity of the design technique on different types of varactor diodes and under different operating conditions.

To verify the design technique, we evaluated numerically the design parameters for two experimental models of an L-band frequency doubler. The design parameters were computed for operation of two types of varactor diodes in, or close to, the region of highest efficiency. Then experimental models were built and tested.

Figure 1 Series-type microwave frequency doubler. (a) physical configuration, (b) lumped-element equivalent circuit.



Power conversion capability

Because of the nonlinear capacitance-charge relationship of a varactor-diode, the charge contains harmonic terms in addition to the fundamental term. In the microwave frequency doubler, shown schematically in Fig. 1a, waves at the fundamental frequency and at the second harmonic can pass the appropriate filters. Waves at harmonics higher than the second are reactively terminated by the filter impedances. We will assume that the charge waveform contains no higher than the second harmonic. (This assumption is supported by the close agreement between experiment and analysis based on this assumption.) Then, the charge on the diode capacitance, when developed in a Fourier series containing only the fundamental frequency and the second harmonic is given in its normalized form:

$$q = \frac{Q + Q_\phi}{Q_{\max} + Q_\phi} = a + b \sin \omega t + c \cos \omega t + d \sin 2\omega t + e \cos 2\omega t, \tag{1}$$

where $Q_{\max} + Q_\phi$ is the charge at the breakdown voltage, defined by[4]

$$Q_{\max} + Q_\phi = \frac{1}{1-\gamma} C_{\min}(V_B + \phi), \tag{2}$$

$C_{\min}$ is the diode capacitance at the breakdown voltage,
V_B is the breakdown voltage,
ϕ is the contact potential,
γ is the exponent that characterizes the nonlinearity in the diode's voltage-charge relationship,
a, b, c, d, and e are the coefficients of the Fourier series representing the charge on the diode capacitance.

The charge on the diode capacitance yields the current as its time derivative. The voltage across the diode capacitance, expressed as a function of the charge, follows from the nonlinear voltage-charge relationship. In the frequency doubler the product of this current and voltage gives the real and reactive powers in the diode at the fundamental frequency and the second harmonic. Expressions for these real and reactive powers in general form follow directly from Manley and Rowe's analysis.[6] (This relation is shown in Appendix I to this paper.) When we make the assumption that the nonlinear capacitance diode is lossless, and further, that no waves at harmonics higher than the second harmonic are generated, then, for conservation of energy, the real power at the second harmonic must be equal to the real fundamental power. (Obviously, real power cannot exist in a linear and lossless capacitance. The real power in the varactor diode is possible only because of the diode's nonlinear voltage-charge relationship.)

The reactive powers in the nonlinear capacitance diode at the fundamental frequency and at the second harmonic are proportional to the average energy stored in the electric field of the diode's capacitance at these two frequencies.

The expressions defining the real and reactive powers at the n^{th} harmonic in a nonlinear capacitance diode (Equation A-4 in Appendix I) cannot be solved in closed form. Morrison[7] has evaluated these expressions by solving simultaneous transcendental equations. In his analysis he defines the real and reactive powers in a varactor diode in the n^{th} harmonic in normalized form:

$$p_n + jt_n = 2\pi^2 \frac{P_n + jT_n}{\omega(Q_{max} + Q_\phi)(V_B + \phi)}. \tag{3}$$

(The relation between the expressions of real and reactive powers, in general form, and in normalized form, is given in Appendix I.)

In Morrison's analysis the normalized real and reactive powers are expressed as functions of the charge on the nonlinear capacitance diode. Morrison, in solving the equations for the normalized real and reactive powers, has shown that a charge waveform can be derived for which the power transfer of the real power from the fundamental frequency to the second harmonic is maximized. He has computed this charge waveform; it varies periodically at the rate of the fundamental frequency. Each period of the charge waveform is composed of three intervals. In one interval the charge waveform takes on a prescribed maximum value; this maximum is made equal to the charge at the breakdown voltage. In the next interval, the charge waveform follows a curve; the form of the curve depends on the voltage-charge relationship of the nonlinear capacitance diode, and on the prescribed maximum and minimum in the adjacent intervals. In the third interval the charge waveform takes on a prescribed minimum.

Morrison has computed the charge waveform that yields maximum power transfer from the fundamental frequency to the second harmonic for operation in the reverse bias region and for operation extending partly in the forward conduction region. For the same operating regions, he computed the maximum real fundamental power (in normalized form) that can be converted to the second harmonic. (In his computation, he assumes an idealized voltage-charge relationship for the forward conduction region.)

Since the maximum real fundamental power in normalized form ($p_{1\,max}$) has been evaluated, the maximum real fundamental power in the nonlinear capacitance diode of the frequency doubler ($P_{1\,max}$) can now be derived. It follows from Eqs. 2 and 3 that

$$P_{1\,max} = \frac{1}{2\pi^2} \frac{p_{1\,max}}{1-\gamma} \omega C_{min}(V_B + \phi)^2. \tag{4}$$

A major step in accomplishing our objective of designing a high-efficiency frequency doubler is to devise a technique of converting the maximum real fundamental power flowing into the varactor diode to power at the second harmonic.

Manley and Rowe's, and Morrison's analyses assume a lossless nonlinear capacitance diode. However, a varactor diode is not a completely lossless device. In the equivalent circuit representing a varactor diode, a resistance is in series with the nonlinear junction capacitance. Losses that occur in the series resistance must be regarded as perturbations. When losses are taken into account, the real power at the fundamental frequency and at the second harmonic can no longer be equal. The real power at the fundamental frequency flowing into the diode then becomes equal to the sum of the real power converted to the second harmonic, and the power lost in the diode's series resistance at the fundamental frequency. Furthermore, the real power at the second harmonic flowing out of the diode becomes the converted power, reduced by the power loss in the series resistance at the second harmonic.

The power losses P_{1l} and P_{2l} in the diode's series resistance R_s are proportional to the square of the magnitude of the current at the fundamental and at the second harmonic:

$$\begin{aligned} P_{1l} &= \tfrac{1}{2}\,|I_1|^2\,R_s \\ P_{2l} &= \tfrac{1}{2}\,|I_2|^2\,R_s. \end{aligned} \tag{5}$$

The magnitudes of the currents as derived from Eqs. 1 and 2 are:

$$\begin{aligned} |I_1| &= \frac{1}{1-\gamma}(b^2 + c^2)^{1/2}\omega C_{min}(V_B + \phi) \\ |I_2| &= \frac{1}{1-\gamma}(d^2 + e^2)^{1/2}2\omega C_{min}(V_B + \phi). \end{aligned} \tag{6}$$

Equivalent impedances

In the frequency doubler the varactor diode at the fundamental frequency corresponds to a terminating impedance, and at the second harmonic, it corresponds to a generator. Consequently, we can define equivalent diode impedances at the fundamental frequency and at the second harmonic (The impedance representation was used previously for nonlinear resistance diodes.[8])

The equivalent impedances can be derived from the varactor diode's equivalent circuit, shown schematically in Fig. 1b. In the equivalent circuit the nonlinear junction capacitance is in series with the resistance, R_s, and the parasitic inductance, L_D. The series resistance and the parasitic inductance are constant parameters (their values are furnished by the manufacturer of the diode). The equivalent impedances of the diode junction, however, depend largely on the operation conditions, and are different at the fundamental frequency and at the second harmonic.

The equivalent impedances of the diode junction can be derived from the energy relations in the nonlinear capacitance diode junction. The equivalent impedance of the diode junction at each frequency can be defined as twice the ratio of the vector power in the diode junction (that is, the sum of the real and the reactive power) to the square of the magnitude of the diode current.[9] This definition, derived for linear circuit elements, is sufficiently general to be equally valid for nonlinear devices as long as the impedances at the fundamental and at the harmonic frequencies are treated separately.

The equivalent impedance of the diode junction at the fundamental frequency for the charge waveform that yields maximum fundamental real power follows from this definition; it is

$$Z_1 = 2\,\frac{P_{1\,\max} + jT_1}{|I_1|^2}\,, \qquad (7a)$$

and the equivalent impedance of the diode junction at the second harmonic is

$$Z_2 = 2\,\frac{P_{1\,\max} + jT_2}{|I_2|^2}. \qquad (7b)$$

$P_{1\,\max}$, $|I_1|$ and $|I_2|$ have already been defined in Eqs. 4 and 6. The reactive powers T_1 and T_2 can be derived from Eqs. 2 and 3:

$$T_1 = -\frac{1}{2\pi^2}\,\frac{t_1}{1-\gamma}\,\omega C_{\min}(V_B + \phi)^2$$

$$T_2 = -\frac{1}{2\pi^2}\,\frac{t_2}{1-\gamma}\,\omega C_{\min}(V_B + \phi)^2. \qquad (8)$$

The negative sign in Eq. 8 is not contained in Morrison's paper. It follows, however, directly from the phase relation in Morrison's paper between the charge component at the fundamental and the second harmonic in the charge waveform that yields maximum power transfer from the fundamental frequency to the second harmonic. (This is shown in Appendix II of this paper.)

The equivalent impedances of the varactor diode represented by the equivalent circuit in Fig. 1b (that includes the series resistance and the parasitic inductance) can now be given. The equivalent impedance of the varactor diode at the fundamental frequency is

$$Z_\omega = 2\,\frac{P_{1\,\max}}{|I_1|^2} + R_s + j\left(2\,\frac{T_1}{|I_1|^2} + \omega L_D\right), \qquad (9a)$$

and at the second harmonic is

$$Z_{2\omega} = 2\,\frac{P_{1\,\max}}{|I_2|^2} + R_s + j\left(2\,\frac{T_2}{|I_2|^2} + 2\omega L_D\right). \qquad (9b)$$

Power transfer

Power conversion capability and equivalent impedances of a varactor diode in a frequency doubler have been derived in the preceding sections. Now we must consider the transfer of the power from the incident wave to the diode, where it becomes converted to power at the second harmonic, and the transfer of the converted power to the load.

At the fundamental frequency the varactor diode corresponds to a complex terminating impedance. To obtain maximum power transfer from the incident wave to the diode, the input transmission line must be matched to the resistive component of the diode's equivalent impedance in Eq. 9a; and the input circuit must be in resonance. To match the resistive components in the input circuit, the characteristic impedance of the input transmission line, Z_{01}, must be made equal to the resistive component of the diode's equivalent impedance at the fundamental frequency. It follows from Eq. 9a that

$$Z_{01} = 2\,\frac{P_{1\,\max}}{|I_1|^2} + R_s. \qquad (10)$$

For resonance of the input circuit, an external reactance which is the conjugate of the diode's reactance at the fundamental frequency, must be added in series with the diode. The external reactance that must be added follows from Eq. 9a:

$$X_L = -2\,\frac{T_1}{|I_1|^2} - \omega L_D. \qquad (11)$$

Since the parasitic inductive reactance, ωL_D, in Eq. 11 is in most cases smaller than the diode junction's capacitive reactance, $2T_1/|I_1|^2$, an external inductive reactance must be added to the input circuit. In the frequency doubler in Fig. 1a this external inductive reactance is made part of the microwave structure. It is realized by the image impedance of filter 2, as transformed by the transmission line between filter 2 and the varactor diode. (Filter 2 represents a very small reactance at the fundamental frequency.)

For maximum power transfer of the converted power from the varactor diode to the load in the output circuit of the frequency doubler, the load impedance must be the complex conjugate of the equivalent diode impedance at the second harmonic. Hence, the characteristic impedance of the output transmission line Z_{02} must be made equal to the resistive component of the varactor diode's equivalent impedance at the second harmonic. It follows from Eq. 9b that

$$Z_{02} = 2\,\frac{P_{1\,\max}}{|I_2|^2} + R_s. \qquad (12)$$

The reactive load impedance must be the conjugate of the diode's reactance at the second harmonic. Since the reactance of the diode junction is capacitive, the load reactance must be inductive. The load reactance from Eq. 9b is

$$X_{2L} = -2 \frac{T_2}{|I_2|^2} - 2\omega L_D. \tag{13}$$

To realize this inductive reactance in the output circuit an external inductance must be added to the parasitic inductive reactance of the varactor diode. In the frequency doubler in Fig. 1a, the external inductive reactance is included in the microwave structure. It is realized by the image impedance of filter 1, as transformed by the transmission line between filter 1 and the varactor diode. (Filter 1 represents a very small reactance at the second harmonic.)

In this section we have related the design parameters of the microwave frequency doubler to the properties of the varactor diode. Furthermore, we have shown that the design parameters of the microwave frequency doubler can actually be computed from these relations. It is important to notice that the varactor diode parameters are not constant, but are dependent on the waveform and the maximum and minimum of the instantaneous charge on the diode junction capacitance. Therefore, the design technique described in this paper enables us to derive the design parameters solely for a definite charge. Only for this charge can the corresponding maximum real fundamental power of the diode be transferred completely to the second harmonic.

Design parameters

The design parameters of a high-efficiency microwave frequency doubler follow directly from the results in the previous sections. We wish (a) to derive design parameters of the frequency doubler that enable us to convert the maximum real fundamental power of the diode junction (defined previously) to the second harmonic; and (b) to compute these design parameters for the charge on the diode capacitance that yields highest conversion efficiency.

To achieve (a), we define the design parameters by the diode's maximum real power, and by the corresponding reactive powers and currents. To achieve (b), we evaluate the design parameters for the charge that yields highest conversion efficiency. (The charge serves as a reference only; it is not a design parameter.)

The numerical evaluation of the design parameters utilizes results of Morrison's[7] computation of the normalized parameters $p_{1\,\max}$, $(b^2 + c^2)$, $(d^2 + e^2)$, t_1 and t_2. He has computed these properties for operation of the nonlinear capacitance diode in the reverse bias region and for operation extending partly in the forward conduction region. The results are summarized in Table 1 for $\gamma = 0.5$ and $\gamma = 0.33$.

Specifically, b, c, d and e in Table 1 are the coefficients in the Fourier series representing the charge waveform that yields maximum power transfer from the fundamental frequency to the second harmonic. Further, $p_{1\,\max}$ is the corresponding normalized maximum real fundamental power and t_1 and t_2 are the corresponding normalized reactive powers at the fundamental frequency and at the second harmonic.

The values are given as functions of the prescribed minimum of the instantaneous charge on the diode junction capacitance. The prescribed maximum of the instantaneous charge is kept constant. Morrison[7] has expressed the minimum charge as a function of the maximum charge; where the maximum charge is the charge at the breakdown voltage, it is in normalized form $q_{\max} = 1$. The minimum charge in normalized form is $q_{\min} = -mq_{\max}$ where m is specified in Table 1. For operation of the nonlinear capacitance diode in the reverse bias region extending to the forward conduction region, $m = 0$. For operation extending partly in the forward conduction region $m > 0$.

First, in the design procedure, the diode type is determined to meet specific requirements on conversion efficiency and output power. The *conversion efficiency* in general form can be approximated by the ratio of the real power at the second harmonic available from the varactor diode to the input power at the fundamental frequency.[10]

$$\eta = \frac{P_{1\,\max} - P_{2l}}{P_{1\,\max} + P_{1l}}. \tag{14}$$

The conversion efficiency as a function of the diode properties, using Eqs. 4 to 6, is given by

$$\eta = \frac{1 - 4\dfrac{\pi^2}{1-\gamma}\dfrac{d^2+e^2}{p_{1\,\max}}\dfrac{\omega}{\omega_c}}{1 + \dfrac{\pi^2}{1-\gamma}\dfrac{b^2+c^2}{p_{1\,\max}}\dfrac{\omega}{\omega_c}}, \tag{15}$$

where $\omega_c = 1/R_s C_{\min}$ is the cut-off frequency of the diode at the breakdown voltage.

Table 1 Normalized parameters of nonlinear capacitance diodes.*

γ	m	$p_{1\,\max}$	$(b^2 + c^2)$	$(d^2 + e^2)$	t_1	t_2
0.5	0	0.2814	0.1482	0.0370	1.462	0.7310
	1/2	0.7773	0.3289	0.0865	1.966	1.060
	1	1.284	0.5947	0.1573	2.300	1.300
	2	2.198	1.451	0.3484	2.921	1.561
0.33	0	0.1623	0.1499	0.0366	1.514	0.7389
	1/2	0.6782	0.3257	0.0878	2.137	1.182
	1	1.246	0.5635	0.1652	2.428	1.529
	2	2.271	1.367	0.3682	3.023	1.882

* From Morrison's results[7]; used with author's permission.

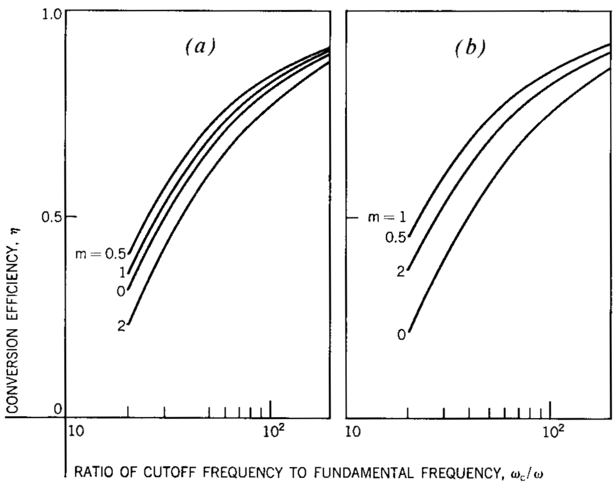


Figure 2 Conversion efficiency of varactor diode in frequency doubler as function of ω_c/ω. (a) $\gamma = 0.5$, (b) $\gamma = 0.33$.

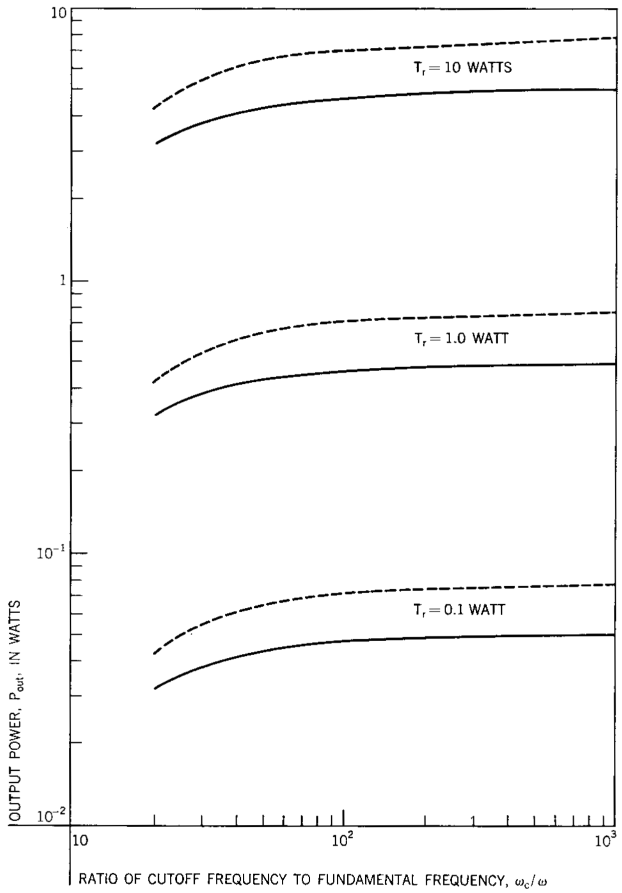


Figure 3 Output power of varactor diode in frequency doubler as function of ω_c/ω and of the nominal reactive power T_r for $m = 1/2$; solid lines: $\gamma = 0.33$, dashed lines: $\gamma = 0.5$.

We have evaluated numerically the conversion efficiency of varactor diodes as a function of ω_c/ω from Eq. 15 using the values in Table 1. The results are shown in Fig. 2 where the conversion efficiency is given for different prescribed minima of the charge on the diode junction capacitance; specifically for $m = 0$, 0.5, 1 and 2; and for $\gamma = 0.5$ and $\gamma = 0.33$. The conversion efficiency is higher when the varactor diode is operated partly in the forward conduction region ($m > 0$) than when confined to reverse bias ($m = 0$). As the operation is extended partly in the forward conduction region, the conversion efficiency increases and reaches a maximum. A further extension in the forward conduction region results in a decrease of the conversion efficiency.

A maximum of the conversion efficiency when operating the varactor diode partly in the forward conduction region (for given γ and ω_c/ω) can be anticipated. Obviously, the conversion efficiency in Eq. 14 relates the resistive components of the equivalent impedances of the diode junction at the fundamental frequency and the second harmonic to the resistance R_s that is in series with the diode junction. While the series resistance R_s is a constant parameter, the resistive components of the equivalent junction impedances in Eqs. 7a and 7b are functions of the operating region on the diode characteristic. Specifically, both properties in Eqs. 7a and 7b, that is, the maximum fundamental power that can be converted to the second harmonic as well as the current through the diode, are functions of the operating region.

When the operation of the varactor diode is extended from the reverse bias region partly in the forward conduction region, the nonlinearity of the voltage-charge relationship of the diode junction increases. Thus, the maximum fundamental power that can be converted to the second harmonic increases. The current through the diode junction at the fundamental frequency and at the second harmonic, that is the time derivative of the charge on the junction capacitance, increases as well. In fact, the current increases very fast as the operation is extended further in the forward conduction region. Thus, the functional relations of the resistive components of the equivalent junction impedances on the operation region (for given γ and ω_c/ω) are not monotonic but reach maximum values. These maxima occur in the region where $0.5 < m < 1$.

The highest conversion efficiency of a varactor diode as a function of the minimum, m, in the charge waveform corresponds to the maxima of the resistive components of the equivalent junction impedances.

The *output power* of the frequency doubler is equal to the maximum real fundamental power converted to the second harmonic, less the power lost in the diode's series resistance. The output power in general form is

$$P_{out} = P_{1\,max} - P_{2l}. \tag{16}$$

The output power as a function of the diode properties follows from Eqs. 4 to 6:

$$P_{out} = \frac{1}{2}\frac{\omega}{1-\gamma}\frac{C_{min}(V_B+\phi)^2}{\pi^2} \cdot \left[p_{1\,max} - 4\frac{\pi^2}{1-\gamma}(d^2+e^2)\frac{\omega}{\omega_c}\right]. \tag{17}$$

Figure 3 shows the output power as a function of ω_c/ω, for different values of the nominal reactive power T_r:

$$T_r = \frac{\omega C_{min}(V_B+\phi)^2}{\pi^2}.$$

The output power was computed for the charge that yields very high efficiency operation ($m = 0.5$), and for $\gamma = 0.5$ and $\gamma = 0.33$.

From Figs. 2 and 3 the designer can derive the properties ω_c, m, $C_{min}(V_B+\phi)^2$ and γ for his particular application, and can choose the diode type. The design parameters for the frequency doubler, operating with this diode, can now be evaluated.

The *input power* of the frequency doubler is equal to the sum of the maximum real fundamental power in the diode, which becomes converted to the second harmonic, and the power loss in the diode's series resistance at the fundamental. It is given in general form:

$$P_{in} = P_{max} + P_{1l}, \tag{18}$$

and as a function of the diode's properties:

$$P_{in} = \frac{1}{2}\frac{\omega}{1-\gamma}\frac{C_{min}(V_B+\phi)^2}{\pi^2} \cdot \left[p_{1\,max} + \frac{\pi^2}{1-\gamma}(b^2+c^2)\frac{\omega}{\omega_c}\right]. \tag{19}$$

The *characteristic impedances* of the input and output transmission lines of the microwave structure, for matched operation, have already been given in general form in Eqs. 10 and 12. The characteristic impedances as functions of the diode properties follow from Eqs. 4, 6, 10 and 12:

$$Z_{01} = \frac{1-\gamma}{\pi^2}\frac{p_{1\,max}}{b^2+c^2}\frac{1}{\omega C_{min}} + R_s \tag{20}$$

$$Z_{02} = \frac{1-\gamma}{4\pi^2}\frac{p_{1\,max}}{d^2+e^2}\frac{1}{\omega C_{min}} + R_s, \tag{21}$$

where Z_{01} is the characteristic impedance of the transmission line from the frequency doubler input to the varactor diode. Z_{02} is the characteristic impedance of the line from the varactor diode to the output. Z_{01} and Z_{02} are very nearly the same since $(b^2 + c^2)$ and $4(d^2 + e^2)$ are approximately the same.

The *spacing* l_1, between the varactor diode and filter 2 can be derived from the resonance condition of the input circuit. The spacing l_1, can be evaluated from the relationship in Eq. 22 which follows from Eq. 11:

$$Z_{02}\tan\left(\frac{\omega}{c}l_1\right) = -2\frac{T_1}{|I_1|^2} - \omega L_D. \tag{22}$$

Equation 22 is derived under the assumption that the image impedance of filter 2 is very small compared to Z_{02} at the fundamental frequency. Equation 22 as a function of the diode properties follows from Eqs. 6 and 8:

$$Z_{02}\tan\left(\frac{\omega}{c}l_1\right) = \frac{1-\gamma}{\pi^2}\frac{t_1}{b^2+c^2}\frac{1}{\omega C_{min}} - \omega L_D. \tag{23}$$

The *spacing* l_2 between the varactor diode and filter 1 follows from the resonance condition of the output circuit. This condition requires that the reactive load impedance be the conjugate of the diode's reactance at the second harmonic. The spacing l_2 can be derived from the relation in Eq. 24 which, in its general form, follows from Eq. 13:

$$Z_{01}\tan\left(\frac{2\omega}{c}l_2\right) = -2\frac{T_2}{|I_2|^2} - 2\omega L_D, \tag{24}$$

assuming that the image impedance of filter 1 at the second harmonic is very small compared to Z_{01}. Eq. 24 in terms of the diode properties, using Eqs. 6 and 8, is given by

$$Z_{01}\tan\left(\frac{2\omega}{c}l_2\right) = \frac{1-\gamma}{4\pi^2}\frac{t_2}{d^2+e^2}\frac{1}{\omega C_{min}} - 2\omega L_D. \tag{25}$$

The input and output impedances of the frequency doubler in Fig. 1a are purely resistive since the inductive reactances in the input and output circuit were included in the microwave structure. The input impedance of the frequency doubler is equal to the characteristic impedance of the input line, Z_{01}, in Eq. 20, and the output impedance is equal to the characteristic impedance of the output line, Z_{02}, in Eq. 21. Quarter-wave transformers at the input and output of the frequency doubler can transform the doubler impedances to the external generator and load impedances.

The *total dissipated power* in the varactor diode, P_l, is the sum of the power losses in the diode's series resistance at the fundamental frequency and at the second harmonic. The power losses are given in general form in Eq. 5. The total dissipated power in the varactor diode as a function of the diode properties follows from Eqs. 5 and 6:

$$P_l = \frac{1}{2}\frac{\omega}{(1-\gamma)^2} C_{\min}(V_B + \phi)^2 \frac{\omega}{\omega_c} \cdot [b^2 + c^2 + 4(d^2 + e^2)]. \qquad (26)$$

In case the varactor diode generates harmonics higher than the second harmonic, then power losses introduced by higher harmonic currents must be considered as well. However, in our experiments, powers in higher harmonics were at least 20 dB below the power in the second harmonic. Therefore, power losses at harmonics higher than the second harmonic were not considered.

We have now defined the design parameters of the frequency doubler. These design parameters can be computed for the prescribed minimum in the charge waveform that yields highest conversion efficiency. The prescribed minimum in the charge waveform on the junction capacitance, however, is not a design parameter; it serves as reference to evaluate corresponding design parameters. The close agreement between measured and computed conversion efficiencies (shown in the next Section) indicates that, by setting up the design parameters evaluated for the prescribed charge, we can actually obtain this charge on the diode junction capacitance.

The normalized diode parameters in Table 1 are given for discrete values of m only. The design parameters, therefore, can be computed for these values of the minimum in the normalized charge only. For values of m, not contained in the tables, the design parameters can be obtained by graphical interpolation.

The dc bias voltage to the varactor diode was not computed. In our experiments this bias voltage was adjusted until the reflected power at the fundamental frequency obtained a minimum and the power output at the second harmonic, a maximum.

Verification of design technique

To verify the design technique, we computed the design parameters of two series-type frequency doublers for operation with two types of varactor diodes. The characteristics of these diodes are shown in Table 2.

To obtain general information on the design technique, we computed the design parameters as functions of the minimum of the charge on the junction capacitance. The conversion efficiencies of the diodes were computed from Eq. 15. The characteristic impedances of the input and output transmission lines of the frequency doubler were evaluated, for matched operation, from Eqs. 20 and 21, and the input and output powers were calculated from Eqs. 17 and 19. The normalized parameters of the varactor diodes were taken from Table 1, and graphical interpolations were made for values that were not tabulated. In Figs. 4 and 5 the design parameters of the frequency doubler, operating with the diode type MA 4047 D1, are shown as functions of the minimum of the normalized charge. (To simplify the models, we made the characteristic impedance of the output line the same as the characteristic impedance of the input line $Z_{01} \approx Z_{02} = Z_0$). In Fig. 4 the fundamental frequency is 0.79 Gc/sec and $\omega_c/\omega = 85$; in Fig. 5 the fundamental frequency is 1.05 Gc/sec and $\omega_c/\omega = 64$. In Fig. 6 the same functional relations are given for the diode type L 4244 B when operated at 1.05 Gc/sec and $\omega_c/\omega = 53$.

The highest conversion efficiency of the diode MA 4047 D1 at a fundamental frequency of 0.79 Gc/sec (Fig. 4) is obtained for operation extending partly in the forward conduction region when the minimum of the normalized charge is approximately $m = 0.7$. The highest efficiency for this same diode operating at a fundamental frequency of 1.05 Gc/sec (Fig. 5) is obtained when $m = 0.8$. The input power for $f = 0.79$ Gc/sec and $m = 0.7$ was taken from Fig. 4 and the input power for $f = 1.05$ Gc/sec and $m = 0.8$ was taken from Fig. 5.

The characteristic impedance of the transmission line of the microwave structure, for matched operation, is 22 Ω for $f = 0.79$ Gc/sec and $m = 0.7$. The characteristic impedance is 14 Ω for $f = 1.05$ Gc/sec and $m = 0.8$. We then built our two laboratory models of a series-type frequency doubler. The first had a transmission line with a characteristic impedance of 22 Ω. The second model had a characteristic impedance of 14 Ω. Next, we computed the spacing l_1, between the varactor diode and filter 2 from Eq. 23, and the distance l_2, between the varactor diode and the filter 1 from Eq. 25. Furthermore, we evaluated quarter-wavelength impedance transformers for the input and output terminals of the frequency doubler. We computed all these design parameters for $f = 0.79$ Gc/sec and $m = 0.7$; and for $f = 1.05$ Gc/sec and $m = 0.8$.

We then used an identical design procedure for a frequency doubler operating with the varactor diode of the type L 4244 B. We computed the conversion efficiency, the characteristic impedance of the transmission line, and

Table 2 Characteristics of two silicon varactor diodes (from manufacturers' data).

Type	MA 4047 D1	L 4244 B
Manufacturer	Microwave Associates	Philco
$C_{\min}$	1.48 pF	0.75 pF
$V_B + \phi$	−90.6 V	−60.6 V
γ	0.325	0.4
R_S	1.6 Ω	3.8 Ω
L_D	~2 nH	~2 nH
ω_c	4.21×10^{11} rad/sec	3.50×10^{11} rad/sec
Θ_t	20°C/W	42°C/W

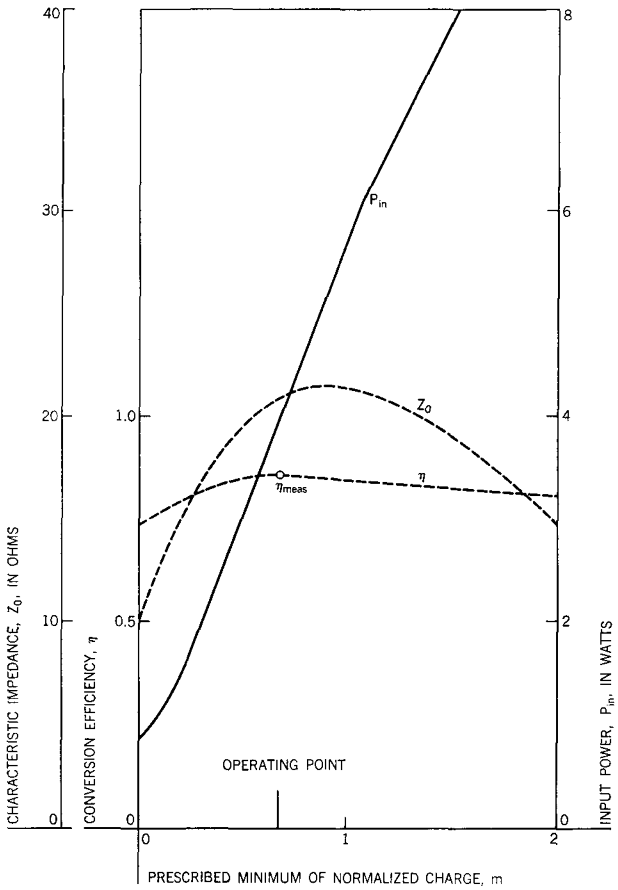


Figure 4 Conversion efficiency, η, characteristic impedance of transmission line, Z_0, and input power, P_{in}, of frequency doubler operating with varactor diode MA 4047 D1 as a function of the prescribed minimum, m, of the normalized charge ($f = 0.79$ Gc/sec).

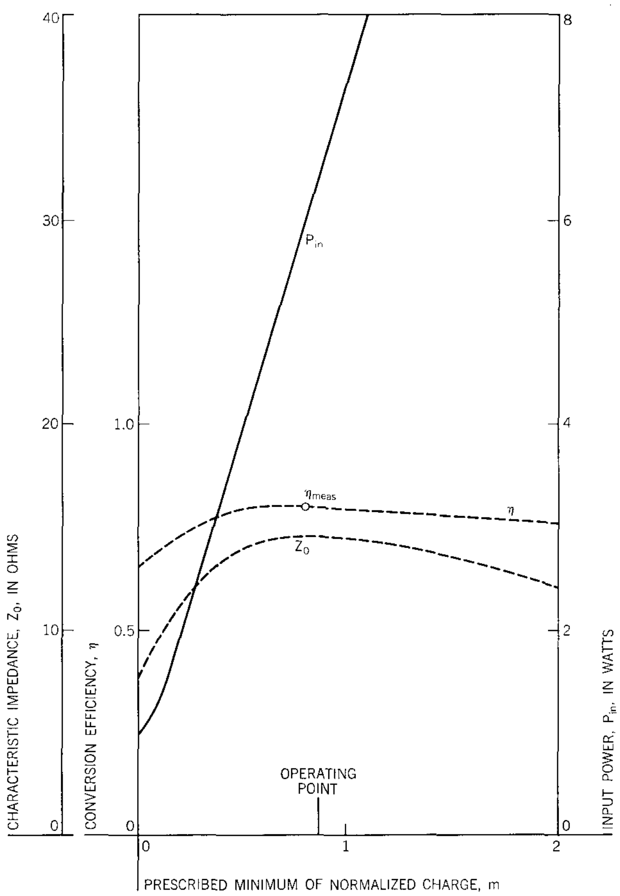


Figure 5 Conversion efficiency, η, characteristic impedance of transmission line, Z_0, and input power, P_{in}, of frequency doubler operating with varactor diode MA 4047 D1 as a function of the prescribed minimum, m, of the normalized charge ($f = 1.05$ Gc/sec).

the input power (given in Fig. 6). In order to operate this type diode at 1.05 Gc/sec in one of the experimental frequency doublers, the varactor diode had to be operated below its highest efficiency region, i.e., at $m = 0.15$.

The bias voltage of the varactor diode in the frequency doublers was supplied by an external d-c source; no power loss in the biasing circuit was observed.

We measured the conversion efficiencies of the frequency doublers, operated with the two types of varactor diodes. The measured values included the losses in the microwave circuit elements. We were, however, interested primarily in verifying the design technique; we wanted to compare the experimental results with the conversion efficiencies computed from Eq. 15, where only the losses in the diode's series resistance were considered. To obtain these values, we calibrated the losses in the microwave circuit elements at the fundamental frequency and at the harmonic, and then deducted them from the measured values. (The circuit losses occurred primarily in the line stretchers of the experimental setup. The losses ranged from 1 to 1.7 dB, depending on the operating frequency. No attempt was made to reduce the losses in the microwave circuitry.)

The experimental results of the conversion efficiencies of the varactor diodes in the frequency doubler (without circuit losses) are shown in Figs. 4, 5, and 6 and are summarized in Table 3. The experimental results are in close agreement with the values that were computed from Eq. 15. Hence, this design technique enabled us to obtain experimentally the predicted conversion efficiencies.

Next, we wanted to determine whether the resistive diode impedances were matched to the characteristic impedance of the transmission line of the frequency doubler. We measured the resistive component of the equivalent impedance of the varactor diode at the fundamental frequency, R_1. Then we computed the resistive diode impedance at the fundamental frequency from Eq. 20, which defines the characteristic impedance of the input transmission line Z_{01}. Equation 20, however, is equally valid

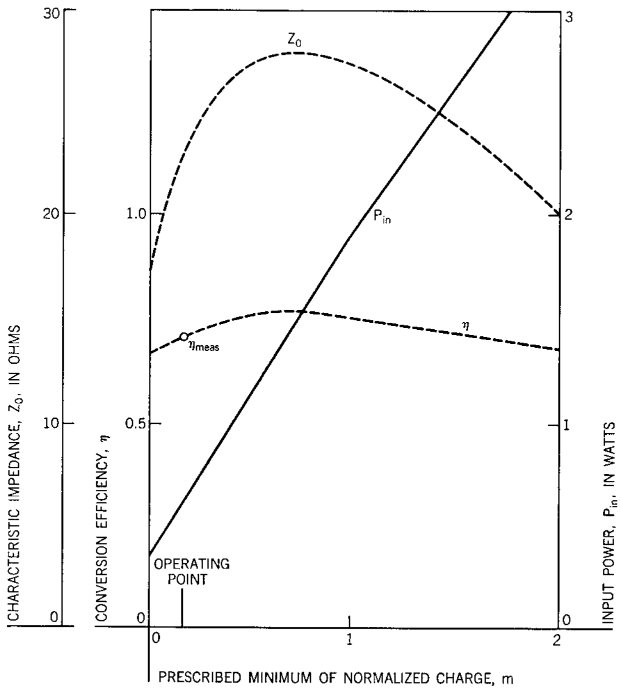


Figure 6 Conversion efficiency, η, characteristic impedance of transmission line, Z_0, and input power, P_{in}, of frequency doubler operating with varactor diode L 4244 B as a function of the prescribed minimum, *m*, of the normalized charge ($f = 1.05$ Gc/sec).

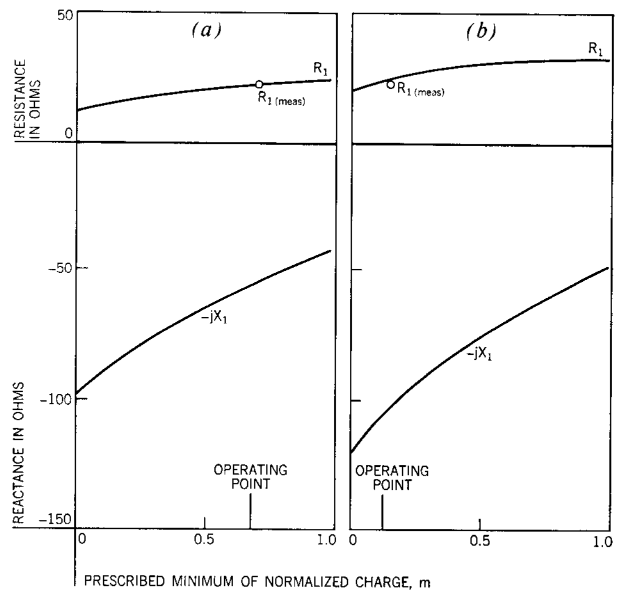


Figure 7 Impedance of frequency doubler at the fundamental frequency as a function of the prescribed minimum, *m*, of the normalized charge. (a) diode MA 4047 D1, $f = 0.79$ Gc/sec, (b) diode L 4244 B, $f = 1.05$ Gc/sec.

Table 3 Summary of experimental results.

Diode type	MA 4047 D1		L 4244 B
Fundamental frequency	0.79 Gc/sec	1.05 Gc/sec	1.05 Gc/sec
Conversion efficiency of varactor diode in frequency doubler	0.84	0.79	0.7
Input power to varactor diode	4.2 W	6.4 W	0.6 W
Output power from varactor diode	3.5 W	5 W	0.42 W
Power dissipated in varactor diode	0.7 W	1.4 W	0.18 W

for defining R_1, since Z_{01} in Eq. 10 was derived by setting Z_{01} equal to R_1.

To measure the resistive component of the varactor diode's equivalent impedance, R_1, under optimum operating conditions at the fundamental frequency, a slotted line was placed in the input line to the frequency doubler. An r-f probe was used and the ratio of incident to reflected wave at the fundamental frequency was measured with a spectrum analyzer. Further, a reference was established by replacing the varactor diode in the frequency doubler by a short circuit.

In Figs. 7a and 7b the computed and measured values of the resistive diode impedance at the fundamental frequency are compared. R_1 was computed as a function of the minimum of the normalized charge. R_1 of the diode MA 4047 D1 was measured at $f = 0.79$ Gc/sec and $m = 0.7$; R_1 of the diode L 4244 B was measured at $f = 1.05$ Gc/sec and $m = 0.15$. There is very close agreement between the computed and measured values of R_1. This means that, for both diodes, the resistive diode impedance R_1 was well matched to the characteristic impedance of the transmission line. Similar agreement between measured and computed values of the resistive diode impedance, at the second harmonic, was obtained. These results prove again that we had succeeded in operating the varactor diodes in the region for which the design parameters had been evaluated.

Finally, we evaluated from Eqs. 6, 7a and 7b the capacitive diode reactance at the fundamental frequency X_1, as a function of the minimum of the normalized charge. In Fig. 7a, X_1 of the diode MA 4047 D1 is shown, and in Fig. 7b, X_1 of the diode L 4244 B, is given. It is interesting to notice the fast decrease of the diode's capacitive reactance as the operating region is extended further in the forward conduction region.

Conclusions

A design technique for high-efficiency microwave frequency doublers that requires only a small experimental effort has been developed. The design parameters are derived analytically and can be evaluated numerically. The technique actually enables the designer (a) to obtain matched operation; (b) to convert the maximum real fundamental power of a varactor diode to the second harmonic; and (c) to operate the varactor diode in the region of its highest efficiency.

The design parameters are all expressed as functions of a prescribed charge on the diode junction capacitance. The prescribed charge, however, is not a design parameter but serves as a reference to evaluate corresponding design parameters.

The design technique is based on the power conversion capability of a nonlinear capacitance diode that is assumed to be lossless. Losses in the diode's series resistance are included in the design equations as perturbations. Furthermore, simplifying assumptions are made on the harmonic content of the charge waveform and of the power output of the nonlinear capacitance diode. In addition, the voltage-charge relationship in the forward conduction region is idealized.

We have verified experimentally the design technique and have obtained close agreement between measured and computed conversion efficiencies. We can assume, therefore, that we have attained the prescribed charge on the diode junction capacitance for which the design parameters were evaluated. Furthermore, we can assume that errors introduced by the perturbation calculation and by the simplifying assumptions are small.

However, the design technique was verified for high-efficiency operation only, where $\omega_c/\omega \geq 53$. The design technique might not be equally valid when the operation frequencies of the frequency doubler are quite close to the diode's cut-off frequency. At the present time, the limitations of the design technique have not been established.

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Appendix I

J. M. Manley and H. E. Rowe[6] have derived the power relations for the frequencies $mf_1 + nf_0$ (where m and n can take on all integral values, positive, negative, and zero) in a nonlinear capacitor. In Manley and Rowe's analysis the voltage and current in the nonlinear capacitor were expressed as functions of the charge. Then the charge flowing into the nonlinear capacitor was written as a double Fourier series; and the current was obtained by taking the derivative of the charge with respect to time. Next, the voltage was represented by a double Fourier series. Thereupon the sum of the real power and the reactive power at each particular frequency was defined.

The real and reactive powers in the nonlinear capacitance diode operated in a harmonic multiplier follow directly from Manley and Rowe's equations when we set the multiplying factor m of the frequency f_1 equal to zero. The sum of the real and reactive powers at the n^{th} harmonic then is

$$P_n + jT_n = 2V_nI_n^*; \tag{A1}$$

the real power in the n^{th} harmonic is

$$P_n = V_nI^* + V_n^*I_n, \tag{A2}$$

and the reactive power in the n^{th} harmonic is

$$T_n = j(V_n^*I_n - V_nI_n^*). \tag{A3}$$

V_n in Eqs. A1, 2 and 3 is the Fourier coefficient at the n^{th} harmonic in the Fourier series (in complex form) representing the voltage $(V + \phi)$ across the varactor diode capacitance:

$$V_n = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} (V + \phi)e^{-in\omega t}\, dt.$$

I_n in Eqs. A1, 2 and 3 is the Fourier coefficient at the n^{th} harmonic in the Fourier series representing the current, I, flowing into the nonlinear diode capacitance:

$$I_n = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} Ie^{-in\omega t}\, dt.$$

V_n^* and I_n^* are the corresponding complex conjugate Fourier coefficients.

When using the defining equations of the Fourier coefficients, the sum of the real and reactive powers in Eq. A1 becomes

$$P_n + jT_n = \frac{1}{2}\left(\frac{\omega}{\pi}\right)^2\left[\int_0^{2\pi/\omega} (V + \phi)e^{-in\omega t}\, dt\right] \cdot \left[\int_0^{2\pi/\omega} Ie^{in\omega t}\, dt\right]. \tag{A4}$$

J. A. Morrison[7] defines the real and reactive powers in the nonlinear capacitance diode in normalized form by introducing the normalized voltage

$$v = \frac{V + \phi}{V_B + \phi}$$

and the normalized current, that is, the time derivative of the normalized charge,

$$i = \frac{I}{\omega(Q_{\max} + Q_\phi)}.$$

(The normalization factors, $V_B + \phi$, and $Q_{\max} + Q_\phi$, were explained in Eq. 2.) Then the real power in Eq. A2 in normalized form (as expressed by Morrison), using the defining equations of the Fourier coefficients, becomes

$$\begin{aligned} p_n &= \frac{2\pi^2 P_n}{\omega(Q_{\max} + Q_\phi)(V_B + \phi)} \\ &= \left[\int_0^{2\pi} v \cos(n\omega t)\, d(\omega t)\right]\left[\int_0^{2\pi} i \cos(n\omega t)\, d(\omega t)\right] \\ &+ \left[\int_0^{2\pi} v \sin(n\omega t)\, d(\omega t)\right]\left[\int_0^{2\pi} i \sin(n\omega t)\, d(\omega t)\right], \end{aligned} \tag{A5}$$

and the reactive power in Eq. A3 in normalized form becomes

$$\begin{aligned} t_n &= \frac{2\pi^2 T_n}{\omega(Q_{\max} + Q_\phi)(V_B + \phi)} \\ &= \left[\int_0^{2\pi} v \sin(n\omega t)\, d(\omega t)\right]\left[\int_0^{2\pi} i \cos(n\omega t)\, d(\omega t)\right] \\ &- \left[\int_0^{2\pi} v \cos(n\omega t)\, d(\omega t)\right]\left[\int_0^{2\pi} i \sin(n\omega t)\, d(\omega t)\right]. \end{aligned} \tag{A6}$$

In Eqs. A5 and A6 the normalized voltage and current can both be expressed as functions of the charge on the nonlinear capacitor.

In the frequency doubler in this paper operating with the nonlinear capacitance diode, the input signal is at the fundamental frequency ($n = 1$) and the output signal is at the second harmonic ($n = 2$). We assume that harmonics higher than the second harmonic are not generated.

Appendix II

The definition of the reactive powers in a nonlinear capacitance diode at the fundamental frequency and at the second harmonic given in Eq. 8, follows from Morrison's analysis. However, the negative sign in Eq. 8 is not contained in Morrison's paper.

We will prove in the following that the negative sign follows directly from the phase relation between the charge component at the fundamental frequency and at the second harmonic in the charge waveform that yields maximum power transfer from the fundamental frequency to the second harmonic. Specifically, we will show that for operation of the nonlinear capacitance diode in the reverse bias region, the reactive fundamental power is negative. To do so, we will use Manley and Rowe's definition of the reactive powers in a nonlinear capacitance diode,[6] (Equation A3 in Appendix I).

The reactive power at the fundamental frequency is:

$$T_1 = j(V_1^* I_1 - V_1^* I_1) \tag{A7}$$

The Fourier coefficients V_1 and V_1^* of the voltage component at the fundamental frequency follow from the series expansion of the varactor diode's nonlinear voltage-charge relation in the reverse bias region:

$$V_1 = (V_B + \phi)\left[\frac{1}{1-\gamma}\frac{q_1}{q_0} + \frac{\gamma}{(1-\gamma)^2}\frac{q_1^* q_2}{q_0^2}\right] q_0^{1/(1-\gamma)}$$

and

$$V_1^* = (V_B + \phi)\left[\frac{1}{1-\gamma}\frac{q_1^*}{q_0} + \frac{\gamma}{(1-\gamma)^2}\frac{q_1 q_2^*}{q_0^2}\right] q_0^{1/(1-\gamma)}, \tag{A8}$$

where q_0, q_1, q_1^*, q_2 and q_2^* are the coefficients in the Fourier series (in complex form) that represents the normalized charge on the diode capacitance. The Fourier coefficients I_1 and I_1^* of the current component at the fundamental frequency follow from the time derivative of the charge:

$$\begin{aligned} I_1 &= j\frac{1}{1-\gamma}\omega C_{\min}(V_B + \phi) q_1, \\ I_1^* &= -j\frac{1}{1-\gamma}\omega C_{\min}(V_B + \phi) q_1^*. \end{aligned} \tag{A9}$$

Then the reactive power T_1 in Eq. A7 follows:

$$T_1 = -\frac{2}{(1-\gamma)^2}\omega C_{\min}(V_B + \phi)^2 q_0^{1/(1-\gamma)} \frac{|q_1|^2}{q_0} + \frac{\gamma}{1-\gamma}\frac{|q_1|^2\,|q_2|}{q_0^2}\cos(2\Theta_1 - \Theta_2). \tag{A10}$$

Morrison[7] has evaluated the Fourier coefficients of the charge waveform that yields maximum power transfer from the fundamental frequency to the second harmonic. They are given in Eq. 104 in Morrison's paper for operation extending from the breakdown voltage to forward conduction. The Fourier coefficients in complex form are:

$$\begin{aligned} q_0 &= \tfrac{1}{2} \\ q_1 &= \frac{1}{3\sqrt{3}} e^{j(\pi/6)} \qquad q_1^* = \frac{1}{3\sqrt{3}} e^{-j(\pi/6)} \\ q_2 &= \frac{1}{6\sqrt{3}} e^{j(5\pi/6)} \qquad q_2^* = \frac{1}{6\sqrt{3}} e^{-j(5\pi/6)}. \end{aligned} \tag{A11}$$

For the phase relations of the complex Fourier coefficients in Eqs. A11, Eq. A10 becomes

$$T_1 = -\frac{2}{(1-\gamma)^2}\omega C_{\min}(V_B + \phi)^2 q_0^{1/(1-\gamma)} \frac{|q_1|^2}{q_0}. \tag{A12}$$

The reactive power T_1 in Eq. A12 actually is capacitive. Furthermore, Eq. A12 is identical with Eq. 8 since the normalized reactive fundamental power t_1 as defined by Morrison is equal to:

$$t_1 = 4\pi^2 \frac{1}{1-\gamma} q_0^{1/(1-\gamma)} \frac{|q_1|^2}{q_0}.$$

We have shown that the reactive fundamental power T_1 in Eq. 8 for operation of the nonlinear capacitance diode in the reverse bias region is capacitive. We can assume that the reactive fundamental power remains capacitive when the operation is extended in the forward conduction region. Further, we can assume that the reactive power at the second harmonic is capacitive as well.

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