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The Effective Carrier Ionization Rate in a p-n Junction at Avalanche Breakdown*

A comparison between experiment and theory has indicated an inaccuracy in avalanche breakdown calculations for low voltage silicon *p-n* junctions. The source of this error has been located. It is shown that the published values of effective carrier ionization rate, at avalanche breakdown, do not apply for low values of avalanche breakdown voltage. This error results from an inadequate mathematical approximation used during the empirical determination of effective carrier ionization rates for both silicon and germanium *p-n* junctions.

Avalanche breakdown in *p-n* junctions has been treated extensively in the literature.[1-6] An important parameter involved in any avalanche breakdown process is the carrier ionization rate per centimeter, α_i, and its dependence on the space-charge layer electric field E. It has been experimentally shown[6] that a simple relation exists between the the carrier ionization rate and the electric field

$$\alpha_i = a \exp(-b/E), \tag{1}$$

where a and b are constants that differ for holes and electrons.

After assuming that α_i is the same for holes and electrons,[2,7] the condition for avalanche breakdown can be expressed by the integral

$$1 = \int_0^w \alpha_i \, dx, \tag{2}$$

where w is the junction space-charge layer width. Although holes and electrons individually exhibit substantially different ionization rates, they contribute to an avalanche breakdown process in approximately equal numbers.[4] This condition implies that a single effective carrier ionization rate can be used in Eq. (2) when conducting an avalanche breakdown calculation.[7] Extensive use has been made of this simplification, and it has proven adequate in the analysis of numerous types of *p-n* junctions.[8]

In a recent application of this approximate method, differences were observed between the calculated and measured avalanche breakdown voltage for low-voltage diffused (silicon) *p-n* junctions. For these devices, the calculated avalanche breakdown voltage was 9.2 volts, and their measured avalanche breakdown voltage was approximately 7.5 volts. This difference between experiment and theory is of a greater percentage than previously observed on high voltage devices; an investigation was therefore conducted to determine the source of this computational error.

In the initial publication of this approximate method,[7] Eq. (2) was combined with both Eq. (1) and an equation for the electric field distribution in an abrupt *p-n* junction:

$$\int_0^w \alpha_i \, dx = a \int_0^w \exp\left\{-b\Big/\left[E_m\left(1-\frac{x}{w}\right)\right]\right\} dx, \tag{3}$$

where E_m represents the maximum space-charge layer electric field. Thereafter the exponent in Eq. (3) was simplified by the use of a two term polynomial expansion

$$\int_0^w \alpha_i \, dx = a \int_0^w \exp\left[-\frac{b}{E_m}\left(1+\frac{x}{w}\right)\right] dx. \tag{4}$$

After integrating Eq. (4), the resulting equation was used to empirically establish an effective carrier ionization rate in both silicon and germanium abrupt *p-n* junctions. This empirical evaluation was accomplished by fitting the calculated avalanche breakdown voltage to the breakdown voltages measured upon various laboratory devices. Thereby, the carrier ionization parameters a and b in Eq. (4) were established:

For silicon, $a = 9 \times 10^5$, $b = 1.8 \times 10^6$;

and for germanium, $a = 1.2 \times 10^7$, $b = 1.4 \times 10^6$.

From Eq. (4) and these empirically determined ionization constants, Fig. 1 illustrates the previously calculated avalanche breakdown voltage for *abrupt p-n* junctions in both silicon and germanium[7] (curves A). Similarly, Fig. 2 shows the previously calculated avalanche breakdown voltage for *linearly-graded p-n* junctions in both silicon and germanium[7] (curves A).

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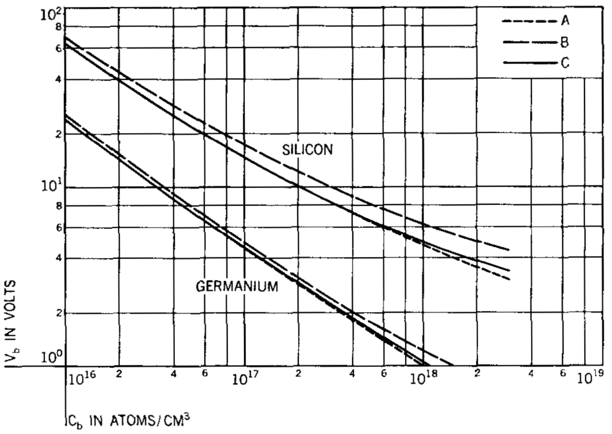

Figure 1 Avalanche breakdown voltage (V_b) for abrupt asymmetrical *p-n* junction. Curves A from Eq. (4); curves B from Eq. (6) uncorrected; curves C from Eq. (6), constants *a* and *b* corrected.

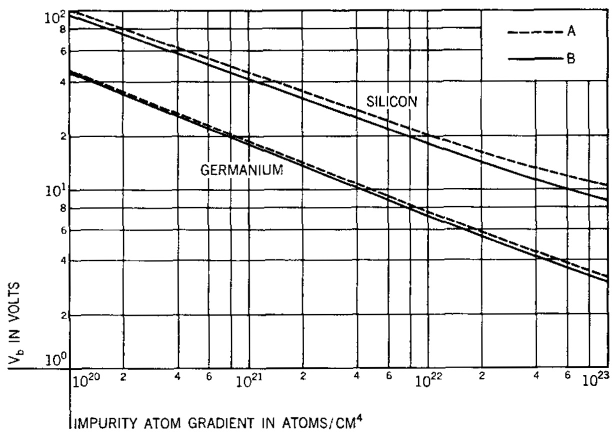

Figure 2 Calculated avalanche breakdown voltage (V_b) for linearly-graded *p-n* junctions. Curves A, constants *a* and *b* uncorrected; curves B, constants *a* and *b* corrected.

The present investigation has established that the polynomial approximation in Eq. (4) is inadequate when applied to low voltage abrupt (silicon) *p-n* junctions. If, instead, this approximation is carried out one more term, Eq. (4) becomes substantially more complicated:

$$\int_0^w \alpha_i\, dx = a \int_0^w \exp\left[-\frac{b}{E_m}\left(1 + \frac{x}{w} + \frac{x^2}{w^2}\right)\right] dx$$
$$= \frac{aw}{2} \exp\left(-\frac{3b}{4E_m}\right) \frac{\mathrm{Erfc}\,\left(\frac{1}{2}\sqrt{b/E_m}\right)}{\sqrt{b/E_m}}, \qquad (5)$$

and the expression for avalanche breakdown in an abrupt asymmetrical structure has the form

$$V_b = \frac{9b^2/8(q/\kappa\epsilon_0)C_b}{\left(\log_e\left\{\frac{\pi a^2 V_b^{3/2}}{b(2qC_b/\kappa\epsilon_0)^{1/2}}\left[\mathrm{Erfc}\frac{b^{1/2}}{2(2q\, V_b C_b/\kappa\epsilon_0)^{1/4}}\right]^2\right\}\right)^2}. \qquad (6)$$

Because the previously published carrier ionization constants (*a* and *b*) were derived from an inadequate equation for avalanche breakdown, Eq. (4), it must be assumed that the magnitudes of these constants are also incorrect. Figure 1 shows the assumption is true by presenting a comparison between the calculated avalanche breakdown voltage using these arbitrary constants (*a* and *b*) in Eq. (4) (curves A), and in Eq. (6) (curves B). For an abrupt silicon *p-n* junction, a significant error arises from the use of a simple two term polynomial expansion of the exponent in Eq. (3). For an abrupt germanium *p-n* junction, this error is substantially smaller, and probably of little consequence in the design of these semiconductor devices.

It should be emphasized that the initial evaluation of this effective carrier ionization rate was accomplished by obtaining agreement between the calculated (curves A of Fig. 1) and measured avalanche breakdown voltage for both silicon and germanium devices. For this reason, corrected values for the carrier ionization constants (*a* and *b*) in Eq. (1) were selected so that Eq. (6) is in substantial agreement with curves A of Fig. 1. The values satisfying this requirement (curves C in Fig. 1) are:

For silicon, $a = 1.47 \times 10^6$, $b = 1.875 \times 10^6$;

and for germanium, $a = 1.65 \times 10^7$, $b = 1.422 \times 10^6$.

Figure 2 also shows the calculated avalanche breakdown voltages for a linearly-graded *p-n* junction, using the corrected "effective" carrier ionization rates for silicon and germanium (curves B). These calculations were performed by combining Eqs. (1) and (2), and thereafter introducing an expression for the electric field distribution in this type of semiconductor device.

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