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Two-Dimensional Laser Deflection Using Fourier Optics

Although several methods of rapid light deflection have been proposed by Bergmann and others,[1] the problem of two-dimensional deflection is still not completely solved. The purpose of this communication is to examine the feasibility of a method of diffracting the collimated monochromatic light of a laser by means of the diffraction spectrum of crossed objects having variable frequency, periodic amplitude or phase. By such means it is possible to move an intense light spot over a two-dimensional field. A possible application of this principal would be in read-only computer stores.

Description of the scheme

One realization of this two-dimensional light deflection scheme is shown in Fig. 1. A laser, *L*, is used to produce collimated monochromatic light that is incident on a pair of cells *x* and *y* containing a liquid such as water. These cells control the *x* and *y* axes, respectively.

Plane compression waves at ultrasonic frequencies are made to traverse these cells in directions that are mutually perpendicular and are perpendicular to the direction of light propagation. These cells then act as a pair of crossed diffraction gratings, and a lens is used to produce the Fraunhofer diffraction spectrum in the plane *P*.

The light distribution in the plane *P* depends on the frequencies of the ultrasonic waves propagated through cells *x* and *y* and on the velocity of the sound waves in the liquid, but the general appearance will be as in Fig. 2.

Points of light along the *x* and *y* axes are due to the diffraction orders of the individual gratings produced in the two cells. Off-axis light points are due to the moiré effect, i.e., the *interaction* of the grating effects. The intensity distribution of any light points, assuming the lens to be perfect, is given by $J_i^2(A_x)J_j^2(A_y)$, where i denotes the diffraction order in the x direction, j the diffraction order in the y direction, and A_x and A_y are related to the amplitudes of the compression waves in the liquid. The quantity J is the usual symbol for the Bessel functions of the first kind. The (x, y) positions of a moiré light point due to the interaction of, say, first-order diffraction spectra are directly proportional to the frequencies ν_x, ν_y of the ultrasonic waves. By controlling the amplitudes A_x and A_y it is possible to make the energy in, say, the first order $J_1^2(A_x)J_1^2(A_y)$, large compared with the energy in any other off-axis moiré order and by varying the ultrasonic frequencies ν_x and ν_y the four points (indicated by closed circles, •, in Fig. 2), can be moved so that each will sweep over a quadrant of the plane *P*.

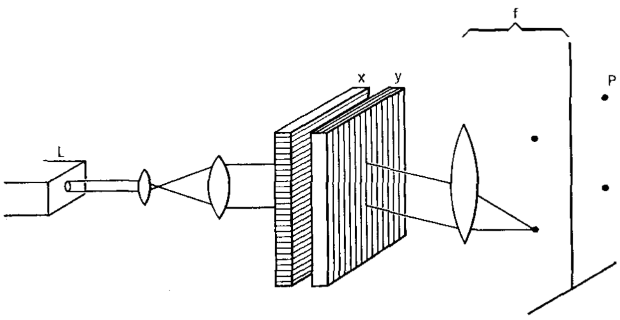
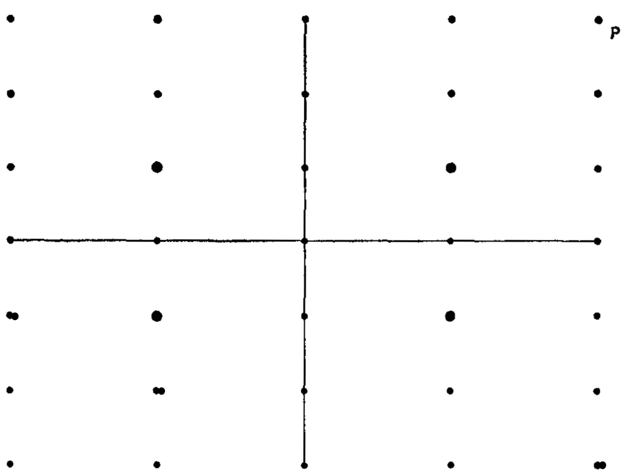


Figure 1 One configuration of the two-dimensional light deflection scheme.

Figure 2 General appearance of light distribution in the plane *P*.



It turns out that considerable light energy can be achieved for $J_1^2(A_x)J_1^2(A_y)$ while allowing an almost equal amount of energy to appear in the second orders $J_2^2(A_x)J_1^2(A_y)$, $J_1^2(A_x)J_2^2(A_y)$ and $J_2^2(A_x)J_2^2(A_y)$. In that case, one should restrict the frequency ranges (ν_x, ν_y) to be greater than one-half of the maximum frequency, (corresponding to the maximum angular deviation of the first diffraction order) and in this case each of the light points due to first- and second-order diffraction will sweep out uniquely the respective areas indicated in Fig. 3.

Thus, if the plane P were a storage plane containing clear and opaque spots as bits, it would be possible to address and read out simultaneously a word of 16 bits by utilizing the first and second moiré orders in the four quadrants of the P plane. It will be shown below that access time of the order of 10 μs to each of approximately 10^6 *words* of 16 bits each should be achievable. It should be noted that in this situation most of the light is distributed among 16 light points so that each spot contains approximately 5% of the energy in the laser beam, neglecting losses due to absorption in the optical system.

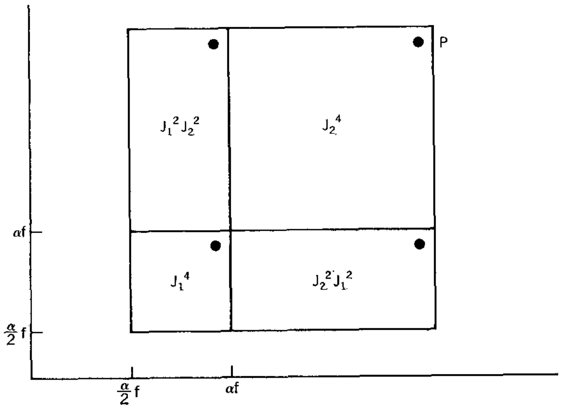


Figure 3 Areas swept out in plane P by light points due to first- and second-order diffraction.

Limitations and trade-offs

- *Deflection, resolution and access time*

The data presented here are based on the material contained in Born and Wolf.[2] These authors state that ultrasonic waves up to angular frequency of 3×10^9 sec^{-1} have been achieved. They give the velocity of sound in water as $v = 1.2 \times 10^5$ cm/sec so that the wavelength of the travelling-wave phase grating produced in the cells can be small as $\Lambda = 2.5 \times 10^{-4}$ cm or 2.5 microns. This means that the angular deflection, α, achievable with light of wavelength $\lambda = 0.5$ micron (green light) is approximately

$$\alpha = \frac{\lambda}{\Lambda} = \frac{0.5}{2.5} = 0.2 \text{ radians, or } 13°$$

for the first-order spectrum. For red light ($\lambda = 0.7\ \mu$) the angular deflection is limited to about 20°.

If the beam incident on the cells is $w = 1$ cm wide the number of points which can be resolved in either direction is

$$N = (\alpha f) \Big/ \left(\frac{\lambda}{w} f\right) = \frac{w}{\Lambda} = \frac{1}{2.5 \times 10^{-4}} = 4000,$$

where f is the focal length of the lens.

By requiring the angular deflection to be greater than $(1/2)\alpha$ (see Fig. 3) and allowing two linear resolution elements per bit, we still have left a total of 10^6 bits for each of the moiré orders.

Since the velocity of the ultrasonic wave in water is 1.2×10^5 cm/sec it will take approximately 8 μs for a complete change of frequency distribution across a 1-cm diameter beam. Consequently the random access time for *any* bit position is about the same as the access time for an adjacent bit position for a tape. By changing the beam diameter to 1 mm the time to change frequency (and hence, spot positioning) can be reduced to less than 1 μs. But in that case the linear resolution is reduced by a factor of 10, so that storage capacity is reduced by a factor of 100. Conversely, storage density could be increased at the expense of access time by using a wider beam diameter. For some applications, as in character generation for printing, it may only be necessary to have a 10×10 field of spots and in this case access time can be cut to 0.1 μs so that complete characters can be generated at the rate of 10^5 per second.

- *Energy limitations and trade-offs*

The energy of any light spot in the Fraunhofer plane is given by $BJ_i^2(A_x)J_j^2(A_y)$, where i and j denote the diffraction order, respectively, in the x and y direction and B is the intensity of the incident light. We shall take $A_x = A_y = A$ and relate the amplitude of the ultrasonic waves to the relevant optical parameters. For collimated light, normally incident on the ultrasonic waves, we have $A = (d\pi\epsilon_1)/(\lambda\sqrt{\epsilon_0})$, where ϵ_0 is the dielectric constant of the undisturbed liquid through which the ultrasonic waves travel. The maximum increment in the dielectric constant ϵ_1 is due to the ultrasonic compression waves, and d is the cell thickness, i.e., the path length of the light beam during its interaction with the ultrasonic wave.

Born and Wolf give a value of ϵ_1 in the order of 10^{-4} so that using a refractive index of $n = \sqrt{\epsilon_0} = 1.5$ and a cell diameter $d = 1.0$ cm, we obtain

$$A = \frac{\pi}{\lambda} \frac{\epsilon_1}{\sqrt{\epsilon_0}} d = \frac{3.14}{0.5 \times 10^{-4}} \cdot \frac{10^{-4}}{1.5} 1 = 4.2.$$

Table 1 Bessel functions for several values of A.

A	J_0	J_1	J_2	J_3
1.0	0.765	0.440	0.115	0.005
1.5	.512	.558	.232	.061
2.0	.224	.577	.353	.129
2.5	.002	.497	.400	.143
3.0	−.260	.339	.486	.309

Needless to say, A can be varied by means of variations in either ϵ_1 or d. The values of the Bessel functions of interest for various values of A are shown in Table 1.

From Table 1 we can see that by operating at $A = 1.5$ we obtain an intensity of $J_1^4(1.5) = 0.097$, or almost 10% of the incident energy in each of the four first-order moiré spots. By working in the neighborhood of $A = 2.5$ essentially all of the light points on the x and y axes are extinguished since $J_0 \approx 0$ and approximately 5% of the light can be directed to appear in each of 16 spots represented by the terms $J_i^2 J_j^2$ with $i = 1, 2$; $j = 1, 2$. The latter condition could be used for reading information in parallel out of a large memory, while the former condition could be used for dynamic display, in which the energy, rather than the information rate, must be maximized.

At high ultrasonic frequencies the attenuation of the ultrasonic energy in water can become the limiting factor in determining resolution. For example, at 1.5×10^8 cps the distance for a $1/e$ attenuation is only about 0.2 cm for a resolution of about 200 lines per field. By using a solid medium such as quartz, the attenuation is reduced by almost two orders of magnitude; however, in this case the beam must be widened by a factor of three to maintain the same resolution since the velocity of sound in quartz is that much larger (which affects Λ).

Some experimental results that point toward the feasibility of this deflection scheme have appeared recently in connection with one-dimensional light deflection.[3,4]

Acknowledgments

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Appendix: Derivation of light distribution in the Fraunhofer plane

Let x, y be coordinate values in the plane of the ultrasonic waves, and ξ, η be coordinates in the Fraunhofer plane. Let a monochromatic plane wave of unit amplitude be incident normally on the plane of the ultrasonic waves. The effect of the ultrasonic compression wave is that of modifying the phase $\phi(x, y)$ of the light distribution in such a manner that if $\phi(x, y) =$ constant then after passing through the ultrasonic disturbance we have $\phi(x, y) = \phi(x) + \phi(y) = A_x \sin \nu_x x + A_y \sin \nu_y y$. (We can neglect time variations since in the end we are concerned only with intensities.) In the Fraunhofer plane, we have the amplitude distribution $U(\xi, \eta)$ given by:

$$U(\xi, \eta) = \left(\frac{1}{2\pi}\right)^2 \iint e^{i\phi(x,y)} e^{i(\xi x + \eta y)}\, dx\, dy$$

$$= \int e^{iA_x \sin \nu_x x} e^{i\xi x}\, dx \int e^{iA_y \sin \nu_y y} e^{i\eta y}\, dy.$$

Since

$$e^{iA \sin \alpha} = \sum_{k=-\infty}^{\infty} J_k(A) e^{ik\alpha}$$

and since

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik\alpha x} e^{i\xi x}\, dx = \delta(\xi - k\alpha)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik\beta y} e^{i\eta y}\, dy = \delta(\eta - l\beta)$$

and

$$\delta(\xi - k\alpha)\delta(\eta - l\beta) = \delta(\xi - k\alpha, \eta - l\beta)$$

we obtain for $U(\xi, \eta)$ the expression:

$$\begin{aligned} U(\xi, \eta) = {} & J_0(A_x) J_0(A_y) \delta(\xi, \eta) \\ & + J_0(A_x) J_1(A_y) \cdot \delta(\xi, \eta \pm \nu_y) \\ & + J_1(A_x) J_0(A_y) \delta(\xi \pm \nu_x, \eta) \\ & + J_1(A_x) J_1(A_y) \cdot \delta(\xi \pm \nu_x, \eta \pm \nu_y) + \cdots \end{aligned}$$

Consequently, the light is concentrated at the points

$$(\xi = k\nu_x, \eta = l\nu_y) k = 0, \pm 1, \pm 2, \cdots \quad l = 0, \pm 1, \pm 2, \cdots$$

and the intensity at these points is given by

$$|U(\xi, \eta)|^2 = |U(k\nu_x, l\nu_y)|^2$$

$$= J_k^2(A_x) J_l^2(A_y). \qquad \begin{aligned} l &= 0, \pm 1, \pm 2 \cdots \\ k &= 0, \pm 1, \pm 2 \cdots \end{aligned}$$

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* These references were added in proof.

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