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Filter Performance in Integrated Switching and Multiplexing*

Abstract: To achieve the advantages of a new technique (ISAM) for integrating the functions of time-division switching and frequency-division multiplexing, it is necessary to design filters that are somewhat different from those used in conventional switching and multiplexing systems. This paper analyzes the performance of ISAM filters. Since in the new technique signals are resonantly transferred between band-pass filters, the theory of resonant transfer for this general case is developed. The conditions for obtaining resonant transfer between ideal filters are determined and then the effects of using nonideal filters are investigated. An example is given showing the synthesis of a set of filters designed to meet ISAM requirements.

Introduction

This paper is primarily concerned with determining the design requirements for the filters used in a new communications technique. A previous paper[1] discussed the systems concepts of a technique in which the functions of frequency-division multiplexing and time-division switching are combined as a single process. The integrated process of switching and multiplexing (called ISAM) gives the same results as are obtained with a conventional system in which the two functions are performed separately. The novelty of the technique is in its practical use of the sampling-frequency harmonics generated in a time-division switch. Sidebands of these harmonics are selected by appropriate bandpass filters, thus allowing the signals to be multiplexed in a manner equivalent to that of the usual single-sideband, suppressed carrier system.

The earlier paper suggested that the integration of functions can lead to considerable saving of electronic hardware in some applications. However, requirements on filter performance are more stringent for ISAM than for the conventional techniques. Discussion in the present paper will be directed toward the definition of these requirements.

To develop this discussion, we will first briefly review the signal expressions for conventional time-division switching and frequency-division multiplexing. Comparing these expressions with those of ISAM, we will demonstrate the functional equivalence of the new and conventional techniques. The expressions will also point out the problem areas of ISAM.

We will note that a method is needed for overcoming the large attenuation of a signal that is switched from input filter to output filter. As with conventional time-division switching, the method used in ISAM is one called "resonant transfer." The technique of resonant transfer has been extensively analyzed for the usual lowpass to lowpass case (see, for example, Cattermole[2]). Since ISAM involves switching between bandpass filters or between lowpass and bandpass filters, we extend the analysis of resonant transfer to both these cases. Then, with an understanding of the ISAM resonant transfer process, we will proceed to show that the filters require not only a distinct amplitude and phase response, but also a well-defined ring-off behavior. In addition, we will investigate the influences of unequal filter input capacitances and finite out-of-band attenuation. Finally, we give an example of filter design using insertion-loss parameters.

Signal analysis

- *Time-division switching*

In a time-divided telephone switching interchange the sampling switches close at time intervals $T = 1/f_s$ where

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the sampling frequency f_s is larger than twice the cut-off frequency of the lowpass filters which band-limit the incoming message signals. The switches stay closed during a short time interval $\tau < T/n$, where n is the number of switchable channels. Proper spacing of sampling pulses provides that no two pulses overlap at the time-divided highway. By synchronously closing any input-output switch pair, signal switching from any input line to any output line can be achieved conveniently. The demultiplexed sampled data pulse trains are fed into individual lowpass filters for reconstruction of the original waveform. The quality of the reconstructed waveform depends on the cut-off slope of the filters and the ratio of sampling frequency to information bandwidth.

In the interval $0 \leq t \leq T_m$ the input message $m(t)$ can be represented by the complex Fourier series

$$m(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_m t}, \qquad n \neq 0 \tag{1}$$

where

$$\omega_m = 2\pi/T_m,$$

$$C_n = \frac{2}{T}\int_0^T m(t)e^{-in\omega_m t}\,dt.$$

It is assumed that $m(t)$ has no dc component, i.e., $C_0 = 0$. The periodic closure of the sampling switch at intervals T is expressed as a Fourier series expansion

$$S(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} C_k e^{ik\omega_s t}, \tag{2}$$

where

$$\omega_s = 2\pi/T,$$

$$C_k = \int_{-\tau/2}^{\tau/2} A_1 e^{-ik\omega_s t}\,dt$$

$$= A_1\tau\,\frac{\sin k\omega_s\tau/2}{k\omega_s\tau/2}.$$

The sampled message is then

$$S(t)\cdot m(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} C_k C_n e^{i(k\omega_s+n\omega_m)t}, \tag{3}$$

where $n\omega_m < \omega_s/2$ to prevent aliasing.

Finally, sending the sampled message through a lowpass filter of bandwidth $\omega_s/2$ leaves only the signal component for $k = 0$ at the output; or

$$[S(t)\cdot m(t)]_{k=0} = \frac{\tau}{T} A_1 \sum_{n=-\infty}^{\infty} C_n e^{in\omega_m t}$$

$$= \frac{\tau}{T} A_1 m(t), \tag{4}$$

which is, for $A_1 = 1$, exactly the input waveform multiplied by an attenuation factor τ/T. To overcome this sampling loss, the resonant transfer technique[2] is applied to time-division switching systems.

- *Frequency-division multiplexing*

In a single-sideband, suppressed carrier FDM system the band-limited input function $m(t)$ is multiplied by a high-frequency carrier $A_1 \cos \omega_c t$. Representing $m(t)$ as a complex Fourier series and using exponential notation for the carrier yields the modulator output:

$$A_1 \cos \omega_c t \cdot m(t) = \frac{A_1}{2}(e^{i\omega_c t} + e^{-i\omega_c t})$$

$$\cdot \sum_{n=-\infty}^{\infty} C_n e^{in\omega_m t}, \qquad n \neq 0. \tag{5}$$

Since the upper sideband component is assumed to be selected by filtering, the transmitted wave is given by

$$E_u(t) = \frac{A_1}{2}\sum_{n=1}^{\infty}[C_n e^{i(\omega_c+n\omega_m)t}$$

$$+ C_{-n}e^{-i(\omega_c+n\omega_m)t}]. \tag{6}$$

The product detector at the receiver multiplies $E_u(t)$ with the local carrier, $A_2 \cos(\omega_c t + \phi_c)$, where ϕ_c may represent any phase (ϕ_c = const) or frequency differences ($\phi_c = \phi_c(t)$) between the local carrier and the signal carrier. This leads to

$$\frac{A_1 A_2}{4}[e^{i(\omega_c t+\phi_c)} + e^{-i(\omega_c t+\phi_c)}]$$

$$\cdot\sum_{n=1}^{\infty}[C_n e^{i(\omega_c+n\omega_m)t} + C_{-n}e^{-i(\omega_c+n\omega_m)t}], \tag{7}$$

which, after lowpass filtering yields

$$E_L(t) = \frac{A_1 A_2}{4}\sum_{n=-\infty}^{\infty} C_n e^{i(n\omega_m t+\phi_c)}, \qquad n \neq 0. \tag{8}$$

If the carriers are synchronized in both phase and frequency, $\phi_c = 0$ and thus $E_L(t) = (A_1A_2/4)m(t)$. If the carriers are synchronized in frequency but not in phase, ϕ_c = const, the phase transfer function shows a zero intercept phase distortion which will not influence the envelope delay, $d\phi/d\omega$. This intercept phase distortion is not very important in audio transmission but has to be carefully considered in data transmission.

- *Integrated switching and multiplexing* (ISAM)

The fundamental idea behind ISAM is that the pulse modulated message of a time-division switch contains higher harmonics of the sampling frequency with their associated upper and lower signal sidebands. Placing appropriate bandpass filters along the frequency axis generates the usual frequency multiplex of a SSSC system. Recovery of the original message is effected by synchronously sampling the various filtered-out sideband signals.

This second sampling process is equivalent to multiplying the transmitted waveform with the original sampling frequency and its higher harmonics. In this case the transmitted wave, $E_u(t)$, is the upper sideband of the kth harmonic generated by the first sampling and selected by bandpass filtering. It has the form,

$$E_u(t) = \frac{C_k}{T} \sum_{n=-\infty}^{\infty} C_n e^{j(k\omega_s + n\omega_m)t}, \qquad n \neq 0. \tag{9}$$

The second sampling operation is expressed by

$$S_2(t) = \frac{1}{T} \sum_{l=-\infty}^{\infty} C_l e^{j(l\omega_s t + \phi_s)}, \tag{10}$$

which is the same as the expression for the first sampling operation except for the constant phase shift, ϕ_s. Multiplying Eq. (9) by Eq. (10) results in

$$\frac{C_k}{T^2} \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} C_n C_l e^{j[(l+k)\omega_s t + n\omega_m t + \phi_s]}, \qquad n \neq 0. \tag{11}$$

Subsequent filters can now select any desired harmonic $l + k$. Selection of $l = 0$ for instance, would place the signal back to its original place on the frequency axis. Any other choice $l + k \geq 0$ allows for shifting the message to other desired sideband slots. The particular case $l + k = 0$ calls for an output lowpass filter which recovers the original baseband message

$$E_L(t) = A_1 A_2 (\tau/T)^2 \sum_{n=-\infty}^{\infty} C_n e^{j[n\omega_m t + \phi_s]}, \qquad n \neq 0 \tag{12}$$

phase shifted by ϕ_s and, for $A_1 = A_2 = 1$, multiplied by $(\tau/T)^2$. Just as in the purely time-division switching case, the attenuation factor $(\tau/T)^2$ can be avoided by using resonant transfer between filters. The influence of the phase shift ϕ_s can also be overcome.

If we assume frequency synchronous operation between stations and equal delays for message and carrier, the sampling phase difference is $\phi_s = \omega_c t_d$, where ω_c is the applicable carrier frequency $k\omega_s$ and t_d is the time difference between sampling instants at the input and output of the line. For a fixed sampling time scheme at input and output, it will introduce a constant phase shift ϕ_s. Since the human ear is relatively insensitive to signal phase relationships, the constant phase shift is of little or no concern for the transmission of voice. For data transmission, however, it is important to require that ϕ_s be zero or a multiple of π radians. This could theoretically always be achieved by properly synchronizing the corresponding sampling times at sender and receiver, such that $t_d = n\pi/\omega_c$, $n = 1, 2, 3, \cdots$. Such a synchronizing scheme is possible but requires additional hardware. However, modems[3] which are used for data transmission in SSSC systems automatically eliminate the effect of any zero intercept phase distortion.

Analysis of ideal generalized resonant transfer transmission

The preceding review of signal relations has noted that the ISAM technique employs resonant transfer of signals between input and output filters. In various ISAM applications, the transmission could be between bandpass filters, between a low pass and a bandpass filter, or between lowpass filters. Since previous analyses of resonant transfer have been restricted to the lowpass to lowpass case, it is important to develop an understanding of the general mode of resonant transfer operation. We will do this in the following paragraphs, using ideal filters for the analysis.

In Fig. 1, a simplified resonant transfer circuit is shown. The capacitors C_1 and C_2 are the equivalent input capacitances that the input filter and output filter present to the switch at a frequency $1/2\tau$. The initial conditions are $V_1(0) = V_0$ and $V_2(0) = 0$, with $L = L_1 + L_2$ and $1/C = (1/C_1) + (1/C_2)$. Assuming the circuit is tuned so that $\tau = \pi\sqrt{LC}$, the following expressions are obtained for resonant current and voltages:

$$\begin{aligned} i &= \pi(V_0 C/\tau) \sin \pi(t/\tau), \\ V_2 &= V_0[C_1/(C_1 + C_2)][1 - \cos \pi(t/\tau)], \\ V_1 &= V_0 - V_2. \end{aligned} \tag{13}$$

For $C_1 = C_2$, which is the condition for optimum power transfer, $V_2(\tau) = V_0$ and $V_1(\tau) = 0$, or, in other words, the voltages on the capacitors C_1 and C_2 are interchanged after every switch closure. Another necessary condition for optimum power transfer is that the initial conditions are properly fulfilled. If $f(t)$ is the band-limited output of the open circuited input filter, one has to assure that $V_1(nT) = f(nT)$ and $V_2(nT) = 0$, where $n = 1, 2, 3 \ldots$, $T \leq 1/2\Delta f$, and Δf is the bandwidth of the input filter. By proper design of the adjacent filter networks at both sides of the switch, the initial conditions can indeed be met before every switch closure, as will be shown later.

The following analysis is based on the fact that the resonant transfer circuit serves to generate a good approximation to a weighted current "delta function" at every sampling instant. The circuit may be analyzed by considering that a weighted current impulse is applied to the input

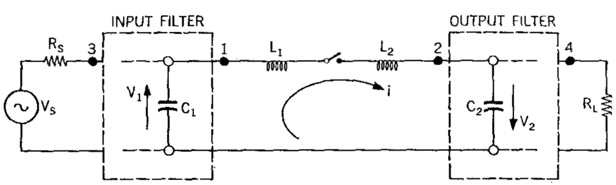

Figure 1 Basic resonant transfer circuit.

of the output filter and the negative of this impulse to the output of the input filter at the sampling instants. The operation will be traced from input to output. To carry out this analysis, certain impulse responses and the voltage transfer function of the input filter under open circuit conditions are considered.

- *Impulse responses and transfer function*

In Fig. 1, assume the two networks represented by the boxes are lossless; they are either lowpass or bandpass filters covering baseband or upper or lower sideband regions along the frequency axis. Assume that the load resistance, R_L, is equal to the source resistance, R_S. Further, for the moment assume that the input and output filter networks cover the same frequency range. Let these filters be ideal in the sense that

$$Z_{24} = Z_{13} = \begin{cases} R_L e^{-i\omega t_0} & \text{within the band} \\ 0 & \text{elsewhere,} \end{cases} \tag{14}$$

where

$Z_{24} \equiv E_4/I_2$,

$Z_{13} \equiv E_3/I_1$, and

t_0 is the phase shift constant. This leads to the following statements for the unit current impulse voltage responses at the output of the output filter, point 4, and at the input of the output filter, point 2.

$$g_{24}(t) = \frac{2R_L}{\pi t}\left[\sin\frac{\pi f_s(t-t_0)}{2} \cdot \cos\frac{\pi f_s(t-t_0)}{2}(2n-1)\right],$$

$$g_{22}(t) = g_{in}(t) = \frac{4R_L}{\pi t}\left[\sin\frac{\pi f_s t}{2} \cdot \cos\frac{\pi f_s t}{2}(2n-1)\right], \qquad t > 0 \tag{15}$$

where $n = 1$ for a baseband filter, $n = 2$ for a first lower sideband filter, $n = 3$ for a second lower sideband filter, etc.

These equations will be used subsequently in describing the overall circuit operation. The assumption of an ideal filter characteristic also leads to a statement regarding the open circuited voltage transfer function for the input filter in the forward direction; that is,

$$\frac{E_1}{E_3} = \begin{cases} e^{-i\omega t_0} & \text{within the band} \\ 0 & \text{elsewhere.} \end{cases} \tag{16}$$

This shows that ideally no drop occurs across R_S, a fact that will also be used subsequently.

- *Circuit operation*

As indicated, the input to the output filter may be considered to be a series of weighted current impulse functions occurring at the sampling instants. The weight of these impulses may be determined by integrating the expression for the resonant transfer current pulse, Eq. (13), from zero to τ; that is,

$$A = \frac{\pi V_0 C}{\tau}\int_0^{\tau}\sin\frac{\pi t}{\tau}\,dt = 2V_0C. \tag{17}$$

The conclusion to be drawn is that the weighting factor for the unit impulse would be $2V_0C$ or $2V_1(nT)C$. The weighted current impulse may be written

$$2V_1(nT)C\delta(t-nT)$$

where $\delta(t-nT)$ is the unit impulse function. To get the voltage at point 4 and point 2 due to such an impulse, simply convolve this current impulse with $g_{24}(t)$ and $g_{22}(t)$ respectively. The result would be

$$\begin{aligned} V_4(t) &= 2V_1(nT)Cg_{24}(t-nT) \\ V_2(t) &= 2V_1(nT)Cg_{22}(t-nT). \end{aligned} \tag{18}$$

The voltages at points 4 and 2 due to a succession of such impulses would be just the superposition of the impulse responses due to successively occurring impulses generated by the resonant transfer process at the sampling instants. There are two especially significant points to note. The first is that by considering the expression for $g_{22}(t-nT)$, it may be seen that the voltage at the point of insertion of the impulses into the output filter, point 2, will always be passing through zero at the sampling instant. This holds true for the case where the filter is coextensive with the baseband or any sideband. This guarantees fulfillment of the initial condition that $V_2(nT) = 0$ at these times. This particular point is discussed in more detail and in a slightly different way in the subsequent section entitled "ring-off behavior." The second point to note is that the amplitude of $g_{24}(t-nT)$ is one-half that of $g_{22}(t-nT)$. The effect of this is that the continuous voltage wave reconstructed by the output filter will have an amplitude one-half that of the voltage wave that is sampled by the resonant transfer gate.

The next question to examine is just what voltage wave is it that is so sampled. That is, what do the weights $V_1(nT)$ represent? To determine this, examine the behavior of the voltage at the output of the input filter, or at point 1. A convenient way to look at this is to consider that when an impulse of a given polarity is applied to the output filter, an impulse of the opposite polarity is applied to the output of the input filter. Since the input circuit, when viewed from the switch, looks just like the output circuit as viewed from this point, the voltage response to this current impulse will be the negative of that occurring at point 2. This voltage may then be superimposed on the voltage that would be coming out of the filter if it were open circuited to get the resultant waveform. Such a superimposition will show that at the sampling instant, $V_1(nT)$ drops from $f(nT)$ to zero and then in the next T seconds builds back up to $f[(n+1)T]$. The way in which it builds up will be dependent on what sideband region the filter covers. Thus, the other initial condition is guaranteed; that is, $V_1(nT) = f(nT)$. Since the open circuited voltage transfer function, Eq. (16), implies that $f(nT)$ is just a delayed version of the input voltage, it may be said that $V_1(nT)$ is a delayed version of the input voltage.

In summary, then, the input voltage, V_s, is simply transported in sampled form to the open circuited output of the input filter. It is undiminished in amplitude and delayed by an amount depending on the filter phase shift constant, t_0. These samples produce a voltage "ring-off" into the output filter, producing a continuous output signal with a voltage level of one-half of the input voltage and with a further phase delay dependent on t_0.

This is the result to be expected, since this makes the ideal generalized resonant transfer transmission circuit equivalent to the ideal lossless filter connected between a generator of internal resistance, R_S, and a load, R_L where $R_S = R_L$. In such an ideal filter, half of the input voltage will appear across R_S and half across R_L, producing a 3 dB loss from generator to load.

- *Band shifting*

Thus far in the analysis, both halves of the circuit have been assumed to cover the same frequency band, baseband or sideband. This has allowed a comparison to be drawn between the resonant transfer mode and the case of the ideal filter operating between source and load. The situation may be easily further generalized by allowing the input and output filters to define different sideband regions, or for one filter to define a baseband and the other a sideband region. The same kind of operation will occur. The only difference is that the impulse responses of each filter will differ. It is the fact that the two impulse responses do not have to be the same, that makes it possible to change bands. The only difference in operation between the cases where the bands are the same and where they are not is that when they are the same, the buildup of the voltage at point 1 and the decay of the voltage at point 2 are dependent on the same impulse response, while when the bands are different, the buildup at point 1 and the decay at point 2 are not dependent on the same impulse response. The methods outlined for determining these voltage variations when the bands are the same are equally applicable when the bands are not the same.

Filter considerations for ISAM

- *Amplitude response*

The required amplitude response for ISAM low pass filters is very similar to the response required for conventional SSSC low pass filters. For a system with 4 kc/sec channel spacing the spectrum should be sufficiently attenuated at 4 kc/sec, without introducing signal degradation at passband frequencies. For purposes of proper filter ring-off, as is shown later, this passband should be made as wide as possible without compromising the 4 kc/sec attenuation requirements. In this respect, a resonant transfer ISAM lowpass filter has a more severe constraint than a resonant transfer filter for purely time-division switching, which does not have the 4 kc/sec attenuation requirement.

The amplitude response for an ISAM bandpass filter is basically different from the usual SSSC channel filter. While the SSSC filter that follows the modulator need discriminate against only the second sideband, the ISAM bandpass filter must discriminate against a multitude of carrier harmonics and their sidebands. If the nominal filter bandwidth is, for instance, $f_c = 4$ kc/sec, f_c denoting the carrier frequency, the SSSC filter can cut off at a frequency slightly greater than $f_c + 4$ kc/sec because no energy will enter the conventional SSSC filter beyond this frequency. An ISAM filter, however, has to be sufficiently down at $f_c + 4$ kc/sec to prevent the energy spill-over from existing sideband harmonics. Similar considerations hold true for the bandpass filter preceding the demodulator. The significance of these considerations is then, that the attenuation of the required ISAM channel filters has to be sufficiently large at both sides of the band as well as at the side of the carrier.

- *Ring-off behavior*

It was previously mentioned that optimum power transfer is achieved in resonant transfer ISAM only if the input voltage to the channel filter is zero shortly before the next pulse transfer takes place. This condition, $V_2(nT) = 0$, will clearly be satisfied if the input pulse response of the filter has zeros at $t = nT$, where $T = 1/f_s$ is the switching time interval. Ideally, filters with a bandwidth $f_s/2$ and one sided termination will indeed have the required ring-off behavior if properly located along the frequency axis.

The response of the passive filter input impedance Z_{11} to a unit impulse is

$$V(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z_{11}(j\omega) e^{j\omega t} \, d\omega. \tag{19}$$

Considering that this input impedance consists of a real part and an imaginary part, or

$$z_{11}(j\omega) = \mathrm{Re}\,(Z_{11}) + j\,\mathrm{Im}\,(Z_{11}) \tag{20}$$

one obtains[4] for $t > 0$

$$V(t) = \frac{2}{\pi}\int_0^{\infty} \mathrm{Re}\,(Z_{11}) \cos \omega t \; d\omega. \tag{21}$$

For a lossless reactance network which is terminated by a 1-ohm load resistor, it is easy to show[4] that Re $(Z_{11}) = |Z_{12}(j\omega)|^2$ where Z_{12} is the transfer impedance of the considered network. For the ideal bandpass filter $|Z_{12}(j\omega)| = A$ for $\omega_1 < \omega < \omega_2$, and 0 otherwise; with these assumptions, the input pulse response of the bandpass filter is

$$\begin{aligned} V_{2B}(t) &= \frac{2}{\pi}\int_{\omega_1}^{\omega_2} A^2 \cos \omega t \; d\omega \\ &= 4A^2\Delta f \cos \omega_0 t \, \frac{\sin \Delta\omega t/2}{\Delta\omega t/2}, \end{aligned} \tag{22}$$

where

$\omega_0 = (\omega_1 + \omega_2)/2 \quad$ = center frequency of filter,
$\Delta\omega = 2\pi\Delta f = \omega_2 - \omega_1$ = bandwidth of filter.

This shows that the ring-off of an ideal bandpass filter has a sin x/x component which is modulated with the high frequency carrier cos $\omega_0 t$.

The pulse response of the corresponding lowpass filter $V_{2L}(t)$ is obtained from $V_{2B}(t)$ for the conditions $\omega_1 = 0$, $\omega_2 = \Delta\omega$, and takes the form

$$V_{2L}(t) = 4A^2\Delta f \frac{\sin \Delta\omega t}{\Delta\omega t}, \qquad t > 0\cdot \tag{23}$$

The important zero crossings of the bandpass and lowpass filters occur, respectively, at

$$\begin{aligned} t_B &= \frac{2n-1}{2}\frac{\pi}{\omega_0}, \\ t_L &= \frac{n\pi}{\Delta\omega}, \qquad n = 1, 2, 3, \cdots. \end{aligned} \tag{24}$$

The longest possible sampling time interval, T, consistent with the requirement that sampling occur at the ring-off zero is, for $n = 1$,

$$t_L = T = \frac{\pi}{\Delta\omega}\cdot \tag{25}$$

Any ISAM bandpass filter will have more than one zero crossing during this interval T. Compatibility is assured as long as the first zero crossing of the lowpass filter coincides with a zero crossing of the bandpass filter. This gives the compatible values of the bandpass center frequency:

$$\omega_0 = [(2n-1)/2]\Delta\omega. \tag{26}$$

This development shows that the necessary initial condition for resonant transfer, $V_2(nT) = 0$, is achieved for a lowpass filter if $\Delta f = 1/2T = f_s/2$ and for a $f_s/2$ wide bandpass filter if center frequency $f_0 = (2n-1)f_s/4$.

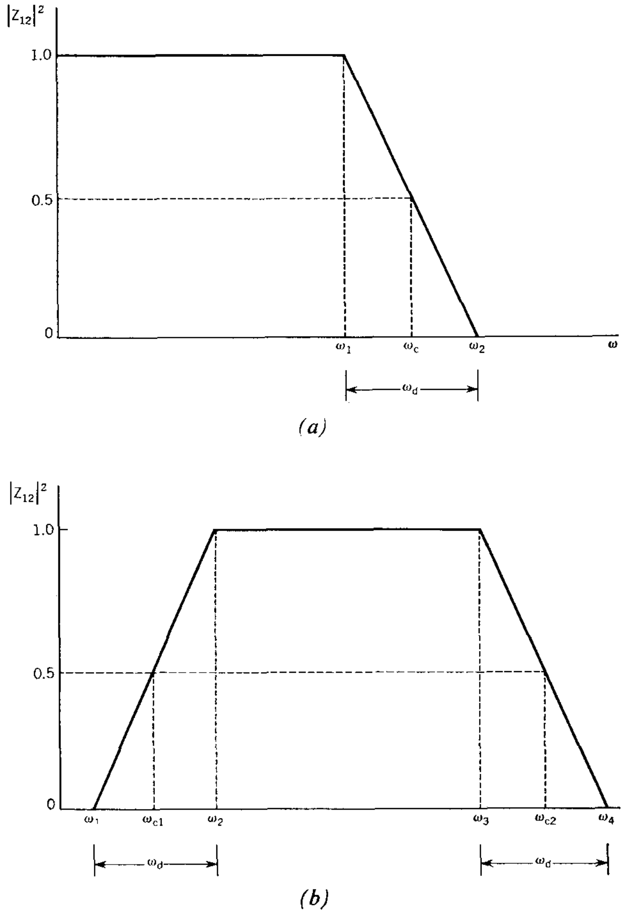

Figure 2 Amplitude response characteristics for nonideal filters. (a) Lowpass filter; (b) Bandpass filter.

All these considerations have assumed ideal rectangular filter characteristics. The influence of a nonideal cut-off characteristic will be studied next.

- *Effect of nonideal response characteristics*

No practical filter will have the assumed ideal rectangular amplitude response. The actual filter slope between passband and stopband can be approximated by a straight line (Fig. 2). This simplified model allows us to obtain some general results on the influence of filter cut-off slope on the time-domain filter ring-off.

The transfer impedance of the approximated nonideal lowpass filter is

$$|Z_{12}(j\omega)|^2 = \begin{cases} 1 & \text{for} \quad \omega < \omega_1 \\ \dfrac{\omega_2 - \omega}{\omega_d} & \text{for} \quad \omega_1 \le \omega \le \omega_2 \\ 0 & \text{for} \quad \omega > \omega_2. \end{cases} \tag{27}$$

Using the relations from the previous section, one obtains the response of the lossless reactance filter network terminated by an impedance of 1 ohm:

$$V_{2L}(t) = \frac{2}{\pi}\left[\int_0^{\omega_1} \cos\omega T\, d\omega + \frac{1}{\omega_d}\int_{\omega_1}^{\omega_2} (\omega_2 - \omega)\cos\omega T\, d\omega\right]. \tag{28}$$

Evaluation of this integral leads to

$$V_{2L}(t) = \frac{2}{\pi}\frac{1}{\omega_d t^2}(\cos\omega_1 t - \cos\omega_2 t) = \frac{2}{\pi}\omega_c \frac{\sin\omega_c t}{\omega_c t}\frac{\sin\omega_d t/2}{\omega_d t/2}, \tag{29}$$

where $\omega_c = (\omega_1 + \omega_2)/2$ and $\omega_d = \omega_2 - \omega_1$. The zero crossings of $V_{2L}(t)$ are mainly determined by the ω_c term, since in a typical ISAM system application $\omega_c > \Delta\omega/2$. Thus the nonideal lowpass filter with a 3 dB bandwidth of ω_c will have the same ring-off zeros as the ideal filter of bandwidth ω_c.

The ring-off behavior of a nonideal symmetric bandpass filter can be investigated similarly. The transfer impedance is here approximated by

$$|Z_{12}|^2 = \begin{cases} 0 & \text{for } \omega < \omega_1 \\ (\omega - \omega_1)/\omega_d & \text{for } \omega_1 \le \omega \le \omega_2 \\ 1 & \text{for } \omega_2 \le \omega \le \omega_3 \\ (\omega_4 - \omega)/\omega_d & \text{for } \omega_3 \le \omega \le \omega_4 \\ 0 & \text{for } \omega > \omega_4. \end{cases} \tag{30}$$

Using the previously developed relation for the response of a lossless reactance network terminated with 1 ohm yields

$$V_{2B}(t) = \frac{2}{\pi}\left[\frac{1}{\omega_d}\int_{\omega_1}^{\omega_2}(\omega - \omega_1)\cos\omega t\, d\omega + \int_{\omega_2}^{\omega_3}\cos\omega t\, d\omega + \frac{1}{\omega_d}\int_{\omega_3}^{\omega_4}(\omega_4 - \omega)\cos\omega t\, d\omega\right]. \tag{31}$$

Evaluation of these integrals leads to

$$V_{2B}(t) = \frac{2}{\pi}\frac{1}{\omega_d t^2}(\cos\omega_2 t - \cos\omega_1 t - \cos\omega_4 t + \cos\omega_3 t), = \frac{2}{\pi}\Delta\omega\cos\omega_0 t\frac{\sin\Delta\omega t/2}{\Delta\omega t/2}\frac{\sin\omega_d t/2}{\omega_d t/2}, \tag{32}$$

where

$$\omega_0 = (\omega_3 + \omega_2)/2, \qquad \omega_d = \omega_2 - \omega_1 = \omega_4 - \omega_3,$$

and

$$\Delta\omega = (\omega_3 + \omega_4 - \omega_1 - \omega_2)/2 = 3dB \quad \text{bandwidth}.$$

For $\omega_d = 0$ it can be shown that this expression is the same as the one previously found for the ideal bandpass filters. The nonideal bandpass filter will have all the zero crossings of the ideal bandpass filter. In addition, it will have zeros caused by the ω_d term.

The conflict between amplitude and time response in a nonideal $\omega_s/2$ wide ISAM lowpass filter arises because of the high attenuation requirement at $\omega_s/2$. The finite slope has to shift the 3 dB frequency to a lower value, $\omega_s/2 = \omega_d/2$, which will produce ring-off zeros at $T' = 1/(f_s - f_d)$ instead of $1/f_s$. Similar arguments hold for the nonideal ISAM bandpass filters. The detailed influence of this unavoidable, nonideal match between sampling time intervals and filter ring-off zero spacing has been partially analyzed and will be the subject of further investigations.

Effect of unequal capacitive input impedance

Resonant transfer requires capacitive filter input impedances at frequency $1/2\tau$. In the following, it is shown how the energy transfer depends on the relative size of the two resonance capacitors C_1 and C_2. Considering only the energy W_2 transferred from C_1 to C_2 during one switch closure of duration τ leads to

$$W_2 = \int_0^\tau i V_2\, dt. \tag{33}$$

Using the current and voltage expressions of Eq. (13), with $\tau = \pi\sqrt{LC}$, yields

$$W_2 = V_0^2 \frac{C}{C_2}\sqrt{\frac{C}{L}}\int_0^\tau \sin\frac{t}{\sqrt{LC}}\left(1 - \cos\frac{1}{\sqrt{LC}}\right) dt. \tag{34}$$

Solution of this integral leads to

$$W_2 = 2V_0^2\frac{(C_2C_1)^2}{(C_1 + C_2)^2}, \tag{35}$$

where

$$C = C_1C_2/(C_1 + C_2).$$

Substituting $C_2 = \alpha C_1$

$$W_2 = 2V_0^2 C_1\frac{\alpha}{(1+\alpha)^2}. \tag{36}$$

The maximum for W_2 will occur for $\alpha = 1$. Its value is $W_{2(\max)} = \frac{1}{2}V_0^2 C_1$, which is equal to the energy on C_1 shortly before switch closure for a voltage V_0 across C_1. Substitution of $W_{2(\max)}$ yields finally

$$\frac{W_2}{W_{2(\max)}} = \frac{4\alpha}{(1+\alpha)^2}. \tag{37}$$

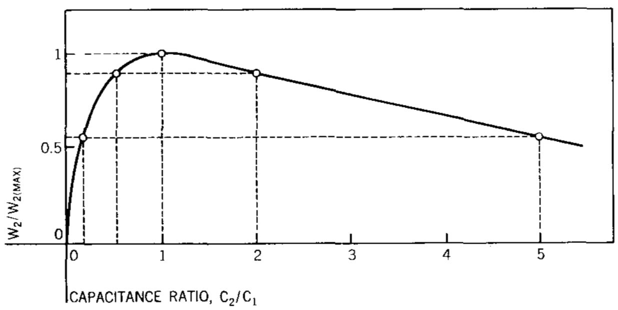

Figure 3 Effect of filter capacitance mismatch on resonant transfer of energy.

A plot of this function is shown in Fig. 3. It can be seen that the transferred energy is relatively insensitive to capacitor mismatch. For $\alpha = 2$ or $1/2$, corresponding to $C_2 = 2C_1$ or $C_2 = \frac{1}{2}C_1$, it is seen that W_2 is only about 10 percent less than $W_{2(\max)}$. For $\alpha = 5$ or $1/5$, the transferred energy will be 0.56 $W_{2(\max)}$.

• *Out-of-band selectivity*

Most practical filters have a guaranteed, small but nonzero transfer impedance outside the passband. The most economical Cauer parameter filters approximate this small constant transfer impedance in an optimum way (equal ripple). It is thus very important to find out how a filter of this type operates in a resonance transfer system.

Most critical is the filter output in response to pulse inputs. Since the power of the widely spaced input pulses must be equal to the power of the continuous filter output signal, the pulse amplitude is much larger than the output signal. Suppression of these large input pulses requires a relatively small transfer impedance outside the passband.

Assume an ideal bandpass filter with the following transfer impedance

$$Z_{12}(j\omega) = \begin{cases} A & \text{for} \quad -\omega_2 \le \omega \le -\omega_1 \\ & \text{and} \quad \omega_1 \le \omega \le \omega_2 \\ pA & \text{for} \quad |\omega| > \omega_2 \\ & \text{and} \quad |\omega| < \omega_1, \qquad p \ll 1. \end{cases} \tag{38}$$

Further assume that the filter receives input pulses of the form

$$f_1(t) = 2Bf_p \frac{\sin \omega_p t}{\omega_p t}. \tag{39}$$

These pulses serve as approximation to the half-wave sine function of the resonance circuit during switch closure. This particular approximation has been chosen since the corresponding Fourier transform has now the simple form of

$$F_1(j\omega) = \begin{cases} B & \text{for} \quad |\omega| < \omega_p \\ 0 & \text{for} \quad |\omega| > \omega_p. \end{cases} \tag{40}$$

The output signal of the filter has then the frequency domain representation

$$F_2(j\omega) = Z_{12}(j\omega) \cdot F_1(j\omega). \tag{41}$$

The corresponding output time function is finally

$$\begin{aligned} f_2(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_2(j\omega) e^{j\omega t}\, d\omega \\ &= \frac{AB}{2\pi} \left[\int_{-\omega_2}^{-\omega_1} e^{j\omega t}\, d\omega + \int_{\omega_1}^{\omega_2} e^{j\omega t}\, d\omega \right. \\ &\quad \left. + p \int_{-\omega_p}^{-\omega_2} e^{j\omega t}\, d\omega + p \int_{\omega_2}^{\omega_p} e^{j\omega t}\, d\omega \right]. \end{aligned} \tag{42}$$

Evaluation of these integrals leads to

$$\begin{aligned} f_2(t) &= 2AB \left[\Delta f \cos \omega_0 t \frac{\sin \Delta\omega t/2}{\Delta\omega t/2} (1 - p) \right. \\ &\quad \left. + p f_p \frac{\sin \omega_p t}{\omega_p t} \right], \end{aligned} \tag{43}$$

where again $\omega_0 = (\omega_1 + \omega_2)/2$, $\Delta\omega = \omega_2 - \omega_1$, and $\Delta\omega = 2\pi\Delta f$. For $p = 0$ one obtains a waveform that is identical in shape to the one obtained for the input ring-off behavior of the ideal filter, but has only half its amplitude. From the computed pulse response of the bandpass filter, the corresponding lowpass filter response is obtained for $\omega_0 = 0$ as

$$\begin{aligned} f_2(t) &= 2AB \left[\Delta f \frac{\sin \Delta\omega t/2}{\Delta\omega t/2} (1 - p) \right. \\ &\quad \left. + p f_p \frac{\sin \omega_p t}{\omega_p t} \right], \end{aligned} \tag{44}$$

where $\Delta\omega$ is the cut-off frequency. The $\Delta\omega$ term in both expressions represents the ring-off behavior of the ideal filter while the ω_p term characterizes an undesired residual of the input pulse.

The relative amplitude of this residual term can be expressed as $K = pf_p/\Delta f$. This formula allows us to make a quick estimate on the required out-of-band filter characteristic, p, if we know the resonance transfer frequency f_p, the filter bandwidth Δf and the relative amplitude of the residual pulses, K, at the output of the filter. This computed value of p will be smaller than required in a practical system, because of our $\sin x/x$ pulse approximation and the additional selectivity obtained in a practical filter where pA is only the maximum transfer impedance.

Filter synthesis

The chosen synthesis procedure is based on the prescribed squared magnitude of a transfer impedance $|Z_{12}|^2$, which

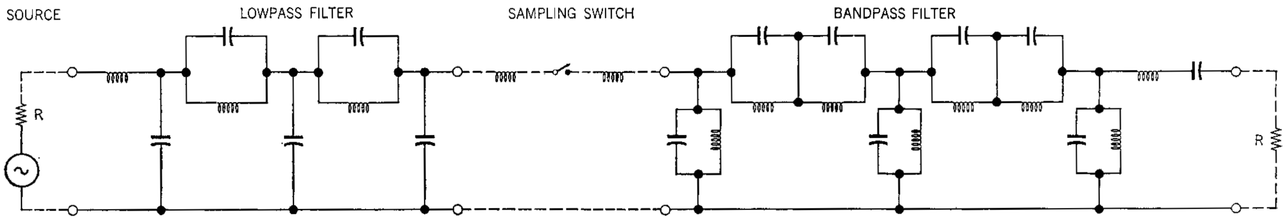

Figure 4 ISAM filter designed with 6th-order Cauer parameters.

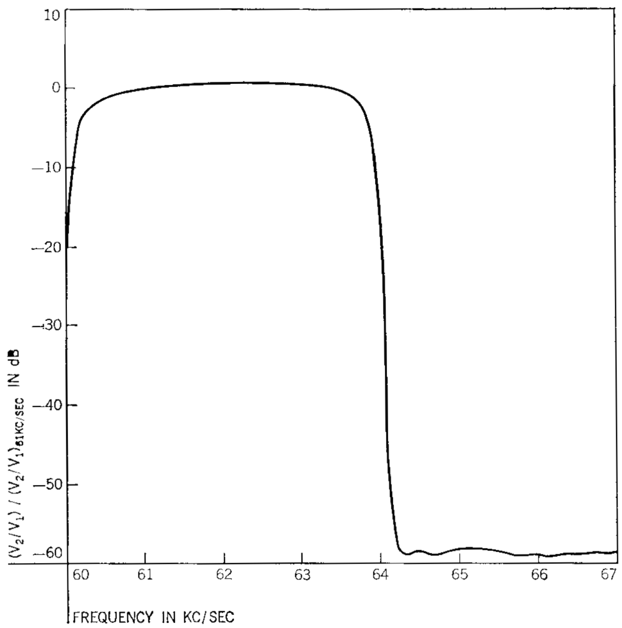

Figure 5 Amplitude response achieved with 8th-order Cauer parameters.

ideally is constant within the band and zero elsewhere. Instead of approximating $|Z_{12}|^2$, it is more convenient to approximate the slightly transformed function

$$F(\omega) = \sqrt{\frac{1}{|Z_{12}|^2} - 1} \tag{45}$$

Cauer[5] has first solved this approximation problem with functions that have equal ripple inside and outside the passband. A table of these functions[6] was used to get the Cauer-parameters and from these, $F(\omega)$.

For trade-off between in-band ripple amplitude against out-of-band ripple amplitude, an additional factor h was included to yield

$$|Z_{12}(s)|^2 = \frac{1}{1 + h^2 F^2(\omega)}, \qquad s = j\omega. \tag{46}$$

Generation of $Z_{12}(s)$ from $|Z_{12}(s)|^2$ will require the solution of an nth-order polynomial, n being the order of the original approximating function $F(\omega)$.

The resonant transfer filter is terminated at only one side. For such filters Guillemin[4] has shown that, given $Z_{12}(s) = m/(m_2 + n_2)$, one can obtain two impedance matrix elements of the filter: $z_{22} = m_2/n_2$ and $z_{12} = m/n_2$, where m and m_2 are even polynomials of s and n_2 is an odd polynomial of s. The final synthesis of the filter is achieved by developing the open circuit secondary driving point impedance z_{22} into a ladder-type configuration such that the zeros of the transfer impedance are being implemented.

An example of a 6th-order ISAM low pass filter is shown in Fig. 4. The matching bandpass filter was generated by a lowpass to bandpass transformation, as given by Saal and Ulbrich,[7] which produced inductors and capacitors of convenient size. The channel bandpass filters at the upper end of the frequency range between 60 kc/sec and 108 kc/sec presented some implementation problems since inductors with sufficiently high Q are not readily available. To compensate against the influence of imperfect inductors, predistortion methods were applied which produced almost perfect amplitude response of individual filters. Due to the influence of predistortion on the filter input impedance and hence the filter ring-off, the overall response of an up-down conversion process proved not as good as without predistortion.

Figure 5 shows a typical amplitude response for the up conversion process which was achieved with 8th-order Cauer parameter filters without predistortion. Better transfer characteristics are expected by use of high-Q crystal resonators.

Conclusions

It has been shown that for successful use of the ISAM technique, it is necessary to design filters that are more sophisticated than those required for conventional resonant transfer operation. Amplitude transfer and ring-off behavior of the filters were shown to be of vital importance in ISAM systems operation. Unequal capacitive input impedances were demonstrated to be of moderate influence over a wide range of capacitance ratios, while insufficient filter out-of-band selectivity was shown to produce output voltage spikes, whose amplitude can be easily estimated.

Amplitude characteristics achieved so far with *LC* filters are still affected by insufficient inductor *Q*. However, this problem is believed to be no more difficult than that usually encountered in conventional FDM channel-filter design.

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