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Analysis of the Impurity Atom Distribution Near the Diffusion Mask for a Planar p-n Junction

Abstract: Presented here are the results from a mathematical investigation of the impurity atom distribution within a planar *p-n* junction. Two fundamentally different diffusion processes are considered: In the first, a constant impurity atom concentration is maintained at the semiconductor surface; in the second, a fixed quantity of impurity atoms is involved in the entire diffusion process. The results of this investigation show than a one-dimensional approximation inadequately characterizes the impurity atom distribution within a planar junction, and that in theory, the planar junction is not at a constant distance from its impurity atom source. Instead, the junction is closer to its source at the semiconductor surface than deep within the bulk material. Further, it is shown that when diffusion takes place from a source of constant concentration density, the junction impurity atom gradient is maximum at the semiconductor surface. In contrast, this junction impurity atom gradient is shown to exhibit a minimum at the semiconductor surface when the total number of impurity atoms is time invariant throughout the entire semiconductor material.

Introduction

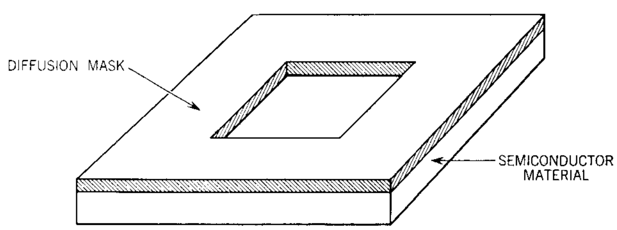
Modern *p-n* junction fabrication techniques are directed toward the formation of planar-type structures. The planar junction, unlike the mesa junction, requires several spatial variables to fully characterize its impurity atom distribution; this analytical problem has not been solved in a rigorous fashion. The purpose of this paper is to present two fundamentally different solutions for the impurity atom distribution in a planar junction: one which applies when a constant impurity atom concentration is maintained at the semiconductor surface, and another for the case in which a fixed quantity of impurity atoms is involved in the entire diffusion process. The first solution, assuming a constant surface concentration, involves mixed boundary conditions that restrict the analysis to two spatial variables. This particular solution is applicable to circular junctions and to regions of a rectangular junction that are far removed from the corners. The second solution of this problem, assuming a fixed number of impurity atoms, is presented in its full three-dimensional form for a rectangular junction configuration.

Analysis

Figure 1 illustrates the type of junction considered in this analytical investigation. Except for that portion of the surface from which diffusion takes place, the entire semiconductor surface is assumed to be covered with a diffusion mask. It should be emphasized that this analysis is based upon an idealization of surface masking techniques, and also upon an idealization of impurity atom diffusion within a semiconductor. The diffusion mask is presumed to be an impenetrable barrier for impurity atoms, thereby reducing to zero the impurity atom flux normal to the semiconductor surface. This idealization also implies that the semiconductor-diffusion mask interface does not offer an easy path for impurity atom diffusion in a direction tangent to the surface. In addition, an isotropic diffusion process is assumed within the semiconductor material, thus permitting the use of a single impurity atom diffusion constant.

The first analysis (for constant surface concentration) is conducted for a two-dimensional model (Fig. 2) representing a cross section of Fig. 1 in a region far removed from the diffusion mask corner. Because of its mathe-

Figure 1 Illustration of the diffusion mask for a planar junction of rectangular geometry.



matical simplicity, the configuration of Fig. 2 has semi-infinite geometry; this has little influence upon the applicability of the model. In a practical planar junction the depth of diffusion is small when compared with the dimensions of a diffusion mask orifice, and the region of interest is therefore adequately characterized by a model of semi-infinite geometry. A similar approach is used to establish the characteristics of a planar junction when a constant number of impurity atoms is maintained throughout the entire diffusion process. In this particular analysis, the semiconductor surface is located upon the plane $y = 0$ (Fig. 3), while diffusion takes place from the region $(0 < x < \infty; y = 0; 0 < z < \infty)$. The remaining surface in this analytical model is assumed to be covered by an ideal diffusion mask.

For simplicity, the two diffusion processes considered in this analysis are called, respectively, the constant-C_0 process and the instantaneous-source process. The constant-C_0 process represents the diffusion of impurity atoms into a semiconductor material when a constant impurity atom concentration (C_0) is maintained upon its surface. In a one-dimensional analysis, the constant-C_0 process yields the familiar complementary error-function type of impurity atom distribution. An instantaneous-source process represents the diffusion of impurity atoms into a semiconductor material when the total number of impurity atoms remains time-invariant. Prior to diffusion, a given number of impurity atoms is deposited upon each unit area of the semiconductor surface, thereby approximating the idealized mathematical concept of an instantaneous plane source. In one dimension, diffusion from an instantaneous plane source yields the familiar Gaussian type of impurity atom density distribution.

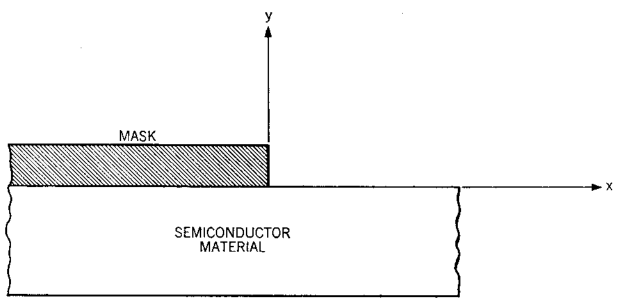
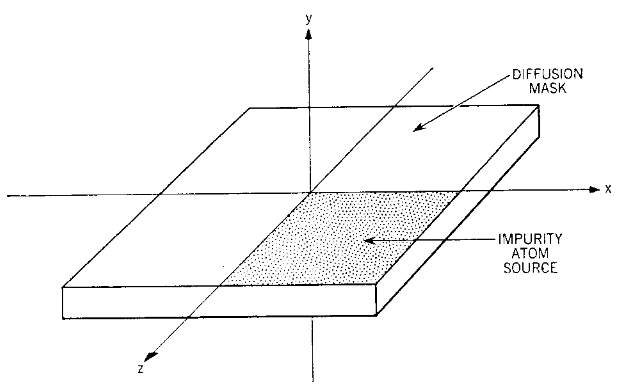


Figure 2 Two-dimensional analytical model for a planar junction.

Figure 3 Three-dimensional analytical model for a planar junction of rectangular geometry.

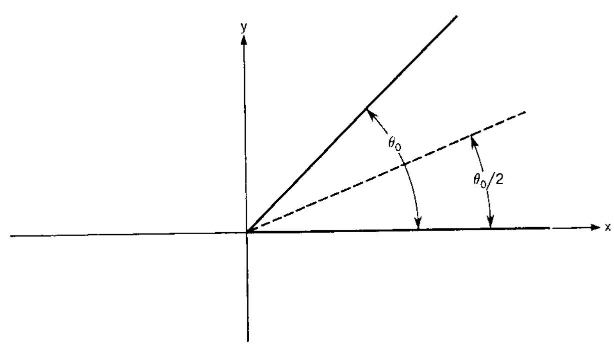


- *Two-dimensional analysis: constant C_0*

For analytical purposes the configuration of Fig. 2 has been replaced by an infinite wedge of semiconductor material[1] of angle θ_0, as shown in Fig. 4. The region surrounding this infinite wedge $(\theta_0 < \theta < 2\pi)$ is assumed to contain a constant impurity atom density (C_0) throughout the entire diffusion process. Furthermore, the wedge itself $(0 < \theta < \theta_0)$ is assumed to be characterized by an initial condition (at $t = 0$) that renders it free of impurity atoms of this particular species. After solving this infinite wedge problem, the wedge angle θ_0 is set equal to 2π, thereby solving the particular problem under consideration.

The reason for using this analytical technique is to permit us to solve the difficult mixed boundary value problem of a planar junction with a minimum of mathematical complication. The geometrical symmetry of a wedge provides a boundary at $\theta = \theta_0/2$ that effectively insulates each half of the wedge. The impurity atom flux originating at the wedge face $\theta = \theta_0$ will be equal in magnitude, but opposite in direction, to the flux originating at the wedge face $\theta = 0$; this means that the net flux crossing the boundary $\theta = \theta_0/2$ is always zero. After increasing the wedge angle θ_0 to 2π we obtain an impurity atom source upon the line $(0 < x < \infty; y = 0)$, and the resulting diffusion is into both the upper half-plane and the lower half-plane. An insulating barrier again appears at $\theta = \theta_0/2$, thus representing the masked semiconductor surface $(-\infty < x < 0; y = 0)$ in Fig. 2.

Figure 4 An infinite wedge of semiconductor material used in the analysis of a two-dimensional planar junction.



In Cartesian coordinates, the two-dimensional diffusion of impurity atoms in a homogeneous medium is characterized by the relation

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} = \frac{1}{D}\frac{\partial C}{\partial t}, \tag{1}$$

while in polar coordinates this expression has the form

$$\frac{\partial^2 C}{\partial x^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2} = \frac{1}{D}\frac{\partial C}{\partial t}. \tag{2}$$

Because the semiconductor wedge (Fig. 4) has the initial condition $C(r, \theta) = 0$, a Laplace transform of Eq. (2) yields the subsidiary equation

$$\frac{\partial^2 \bar{C}}{\partial x^2} + \frac{1}{r}\frac{\partial \bar{C}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{C}}{\partial \theta^2} - \frac{p}{D}\bar{C} = 0, \tag{3}$$

which is satisfied by the relation

$$\bar{C} = C_0\left\{\frac{1}{p} - \frac{4}{\theta_0}\sum_{n=0}^{\infty}\sin\left[(2n+1)\frac{\theta\pi}{\theta_0}\right] \cdot \int_0^\infty u^{-1}(Du^2+p)^{-1}J_h(ur)\,du\right\}, \tag{4}$$

where $J_h(ur)$ is a Bessel function of order h, and

$$h = (2n+1)\pi/\theta_0. \tag{5}$$

After taking the inverse Laplace transform of Eq. (4) we have

$$C(r, \theta, t) = C_0\left\{1 - \frac{4}{\theta_0}\sum_{n=0}^{\infty}\sin\left[(2n+1)\frac{\theta\pi}{\theta_0}\right] \cdot \int_0^\infty u^{-1}\exp(-Dtu^2)J_h(ur)\,du\right\}. \tag{6}$$

The integral in Eq. (6) can be expressed in terms of hypergeometric functions,[2]

$$\int_0^\infty u^{-1}\exp(-Dtu^2)J_h(ur)\,du = \frac{(r/2\ Dt)^h\Gamma(h/2)}{2\Gamma(h+1)}\,{}_1F_1\{h/2; (h+1); -r^2/4Dt\}, \tag{7}$$

and therefore when $\theta_0 = 2\pi$, Eq. (6) has the form

$$C(r, \theta, t) = C_0\left\{1 - \frac{2}{\pi}\sum_{n=0}^{\infty}\sin[(n+\tfrac{1}{2})\theta] \cdot \frac{[r/2\sqrt{Dt}]^{(n+\frac{1}{2})}\Gamma(n+\tfrac{1}{2})/2}{2\Gamma(n+\tfrac{3}{2})} \cdot {}_1F_1\left[\frac{n+\frac{1}{2}}{2}; (n+\tfrac{3}{2}); -r^2/4Dt\right]\right\}. \tag{8}$$

The symbol ${}_1F_1[h/2; (h+1); -r^2/4Dt]$ in Eq. (7) is the hypergeometric series given by

$${}_1F_1(\alpha, \beta, z) = \sum_{n=0}^{\infty}\frac{\alpha_n z^n}{n!\ \beta_n}, \tag{9}$$

where α_n and β_n represent expressions of the form

$$k_n = k(k+1)(k+2)(k+3)\cdots(k+n+1), \qquad k_0 = 1. \tag{10}$$

An important quantity derivable from Eq. (8) is its gradient ($\mathfrak{A}_0$) in a direction normal to the surfaces of constant impurity atom concentration density; this quantity partially determines the avalanche breakdown voltage of the resulting p-n junction.[3] The gradient $\mathfrak{A}_0$ is given by[4]

$$\mathfrak{A}_0^2 = \frac{\partial C^2}{\partial r} + \left(\frac{1}{r}\frac{\partial C}{\partial \theta}\right)^2, \tag{11}$$

and therefore, after substituting the derivatives of Eq. (8) into Eq. (11), we obtain

$$\mathfrak{A}_0^2 = \left(\frac{2C_0}{\pi}\right)^2\sum_{\substack{m=0\\n=0}}^{\infty}\{I_{m,n}(r)\cos[(m+\tfrac{1}{2})\theta]\cos[(n+\tfrac{1}{2})\theta] + K_{m,n}(r)\sin[(m+\tfrac{1}{2})\theta]\sin[(n+\tfrac{1}{2})\theta]\}, \tag{12}$$

where

(a) $I_{m,n}(r) = I_m(r)I_n(r)$

(b) $K_{m,n}(r) = \left\{\frac{\partial}{\partial r}\left[\frac{r}{(m+\frac{1}{2})}I_m(r)\right]\right\} \cdot \left\{\frac{\partial}{\partial r}\left[\frac{r}{(n+\frac{1}{2})}I_n(r)\right]\right\}$

(c) $I_m(r) = \dfrac{(m+\frac{1}{2})[r/2\sqrt{Dt}]^{(m+\frac{1}{2})}\Gamma(m+\frac{1}{2})/2}{2r\Gamma(m+\frac{3}{2})} \cdot {}_1F_1\left\{\dfrac{m+\frac{1}{2}}{2}; (m+\tfrac{3}{2}); -r^2/4Dt\right\}$

(d) $I_n(r) = \dfrac{(n+\frac{1}{2})[r/\sqrt{Dt}]^{(m+\frac{1}{2})}\Gamma(n+\frac{1}{2})/2}{2r\Gamma(n+\frac{3}{2})} \cdot {}_1F_1\left\{\dfrac{n+\frac{1}{2}}{2}; (n+\tfrac{3}{2}); -r^2/4Dt\right\}. \qquad (13)$

- *Three-dimensional analysis: instantaneous plane source*

The solution of this second problem is based upon an assumption that prior to diffusion a given number of impurity atoms is deposited upon the semiconductor surface. This situation is mathematically approximated by an instantaneous plane source located upon the surface ($0 < x < \infty$; $y = 0$; $0 < z < \infty$) in Fig. 3. From the three-dimensional impurity atom diffusion equation,

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} = \frac{1}{D}\frac{\partial C}{\partial t}, \tag{14}$$

a solution is obtained for two instantaneous point sources located symmetrically about $y = 0$,

$$G(x, y, z, t; x', y', z') = \frac{(\pi Dt)^{-\frac{3}{2}}}{8}\left\{\exp\left[-\frac{(y-y')^2}{4Dt}\right] + \exp\left[-\frac{(y+y')^2}{4Dt}\right]\right\} \cdot \exp - \frac{(x-x')^2 + (z-z')^2}{4Dt}. \tag{15}$$

Equation (15) is the Green's function solution for this boundary value problem.[5] The symmetrical distribution of this Green's function about the plane $y = 0$ results in a normal flux of zero upon the plane $y = 0$ when $0 < t < \infty$.

The initial conditions ($t = 0$) of this problem require an impurity atom density of zero throughout the entire semiconductor material, except at the surface $y = 0$; this condition is satisfied if $y' = 0$ in Eq. (15). In addition, at $t = 0$ any surface element $dx\, dz$ upon the plane $(0 < x < \infty; y = 0; 0 < z < \infty)$ must contain a constant number of impurity atoms. From Eq. (15), the impurity atom distribution at a surface element $dx\, dz$ is given by

$$C_0 G(x, y, z, t; x', z')\, dx'dz'. \tag{16}$$

If we now assume that identical impurity atom sources reside upon every element of the surface $(0 < x < \infty; y = 0; 0 < z < \infty)$, we obtain

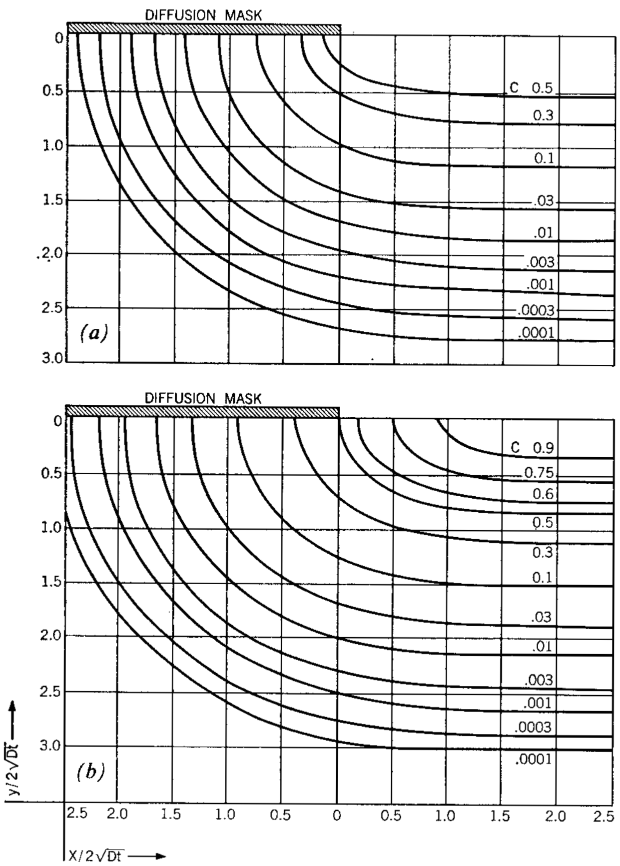
$$C(x, y, z, t) = \left\{\frac{C_0[\pi Dt]^{-\frac{3}{2}}}{4}\right\} \exp\left[-\frac{y^2}{4Dt}\right] \cdot \int_0^\infty \int_0^\infty \exp\left[-\frac{(x-x')^2 + (z-z')^2}{4Dt}\right] dx'dz'$$
$$= \frac{C_0}{4\sqrt{\pi Dt}} \exp\left[-\frac{y^2}{4Dt}\right]\left\{1 + \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right\} \cdot \left\{1 + \operatorname{erf}\left(\frac{z}{2\sqrt{Dt}}\right)\right\}, \tag{17}$$

which is the three-dimensional impurity atom distribution resulting from an instantaneous-source type of diffusion process.

As before, we can establish the density gradient ($\mathfrak{G}_0$) of impurity atoms in a direction normal to the surfaces of constant impurity atom density; this, again, is directly related to the avalanche breakdown voltage of a resulting *p-n* junction.[3] From Eq. (17) we have

$$\mathfrak{G}_0^2 = \left(\frac{\partial C}{\partial x}\right)^2 + \left(\frac{\partial C}{\partial y}\right)^2 + \left(\frac{\partial C}{\partial z}\right)^2 = \frac{C^2(x, y, z, t)}{\pi Dt} \cdot \left\{\frac{y^2}{4Dt} + \frac{\exp(-x^2/2Dt)}{[1 + \operatorname{erf}(x/2\sqrt{Dt})]^2} + \frac{\exp(-z^2/2Dt)}{[1 + \operatorname{erf}(x/2\sqrt{Dt})]^2}\right\}. \tag{18}$$

Figure 5 Calculated contours of constant impurity atom density at the mask edge in a two-dimensional planar structure: (a) Constant-C_0 diffusion process: $C = C(r, \theta, t)/C_0$; (b) Instantaneous-source diffusion process, $C = C(x, y, \infty, t)\sqrt{\pi Dt}/C_0$.



Discussion

Figure 5 illustrates the calculated contours of constant impurity atom concentration density for both a constant C_0 diffusion process, Eq. (8), and an instantaneous-source diffusion process, Eq. (17). Assuming a constant doping level within the semiconductor material prior to diffusion, this figure illustrates the families of *p-n* junctions resulting from various levels of bulk doping. It should be noted that Fig. 5 presents a comparison between the two-dimensional impurity atom distribution for these two types of diffusion processes. Although Eq. (17) describes a three-dimensional impurity atom distribution, this expression has been reduced to two spatial variables by setting $z = \infty$. Figure 5b therefore shows the impurity atom distribution resulting from an instantaneous-source type of diffusion process, as seen on an infinite plane located at $(-\infty < x < \infty; -\infty < y < 0; z = \infty)$.

Figure 5 illustrates an important difference between the conclusions derived from a one-dimensional analysis and those from a more rigorous mathematical solution of this diffusion problem. A one-dimensional analysis is based upon an assumption that identical impurity atom diffusion characteristics exist under the diffusion mask edge and in a direction perpendicular to the semiconductor surface. This assumption implies that the transition point of a resulting *p-n* junction will be everywhere equidistant from the exposed semiconductor surface (Fig. 1). Rigorous mathematical solutions of these diffusion problems show, as in Fig. 5, the inadequacy of a one-dimensional approximation; both types of diffusion processes (constant C_0 and instantaneous plane source) exhibit a greater penetration of impurity atoms in a direction perpendicular to the semiconductor surface. In Fig. 5, for example, the junction termination at the diffusion mask ($y = 0$) is closer to the exposed semiconductor surface than the junction location deep within the bulk material. This characteristic is particularly evident in shallow structures.

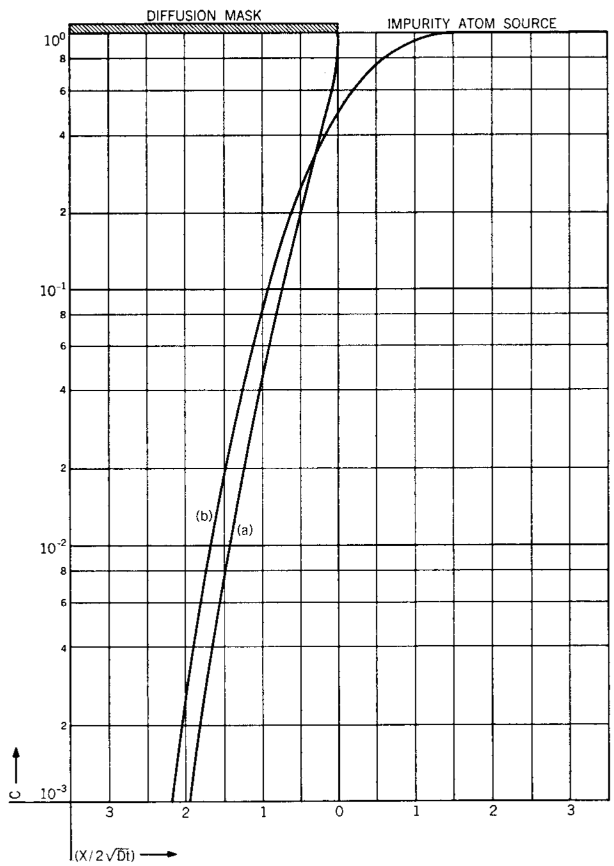
Figure 5a implies the possibility that a constant C_0 diffusion process yields an increased junction impurity atom density gradient ($\mathcal{A}_0$) beneath the diffusion mask, thereby decreasing its avalanche breakdown voltage. Furthermore, Fig. 5b indicates that an instantaneous-source diffusion process yields an impurity atom density gradient that extends well across the surface containing the impurity atom source. It is difficult to estimate from this type of illustration the resulting gradient at the semiconductor surface.

To investigate these questions relating to the density gradient in a planar *p-n* junction, Eqs. (8) and (17) were first used to calculate the impurity atom distributions at the semiconductor surface (Fig. 6). For a constant C_0 diffusion process, a large impurity atom gradient is found near the diffusion mask edge. At a distance of less than one diffusion length ($2\sqrt{Dt}$) from the mask edge, this gradient decreases in magnitude, and will be shown to become approximately equal to the gradient established by a one-dimensional solution of this diffusion problem. In contrast, Fig. 6 illustrates substantially different surface characteristics for an instantaneous-source diffusion process. The impurity atom gradient ($\mathcal{A}_0$) is everywhere less than the magnitude obtained from a one-dimensional solution; a decreased junction depth results from the modified impurity atom concentration density upon the exposed semiconductor surface.

In addition, the derivatives of Eq. (12) and Eq. (18) have been investigated in an attempt to locate a region of maximum impurity atom density gradient, $\mathcal{A}_0$; this consists of finding the extremum, subject to restraint by means of a Lagrange multiplier.[6] The results of this investigation clearly indicate the semiconductor surface as a region where $\mathcal{A}_0$ assumes either a maximum or a minimum value. For a constant-C_0 diffusion process, the derivative of $\mathcal{A}_0$ is zero at the semiconductor surface and, from Fig. 6, this surface is therefore a region of maximum $\mathcal{A}_0$. Similarly, in two dimensions, the instantaneous-source diffusion process yields a zero derivative for $\mathcal{A}_0$ at the semiconductor surface. In three dimensions this diffusion process yields a zero derivative for $\mathcal{A}_0$ along a line bisecting the diffusion mask corner at the semiconductor surface ($y = 0, x = z$).

Figure 7 presents a comparison between the calculated impurity atom density gradient ($\mathcal{A}_0$) at the semiconductor surface, as illustrated in Fig. 6, and that in a direction perpendicular to the semiconductor surface (Gaussian and erfc). It should be noted that this calculation is for a two-dimensional approximation of the diffusion process. In Fig. 7, the magnitude of $\mathcal{A}_0$ is plotted against the impurity atom density, thereby permitting an evaluation of $\mathcal{A}_0$ at two points along a junction transition surface (transition from *n*-type to *p*-type material): at the semiconductor surface, and at a location that is far removed from the diffusion mask.

Figure 6 Calculated impurity atom distribution at the surface of a two-dimensional planar structure: (a) Constant-C_0 diffusion process, $C = C(r, \pi, t)/C_0$; (b) Instantaneous-source diffusion process, $C = C(x, 0, \infty, t)\sqrt{\pi Dt}/C_0$.



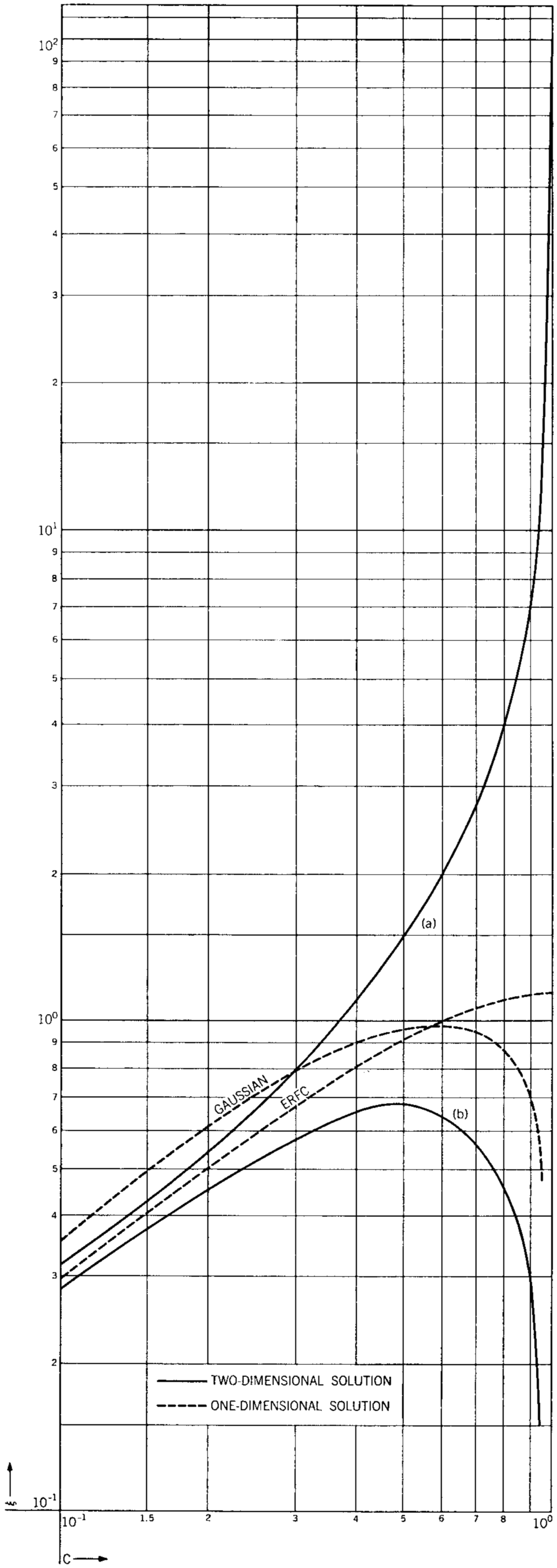
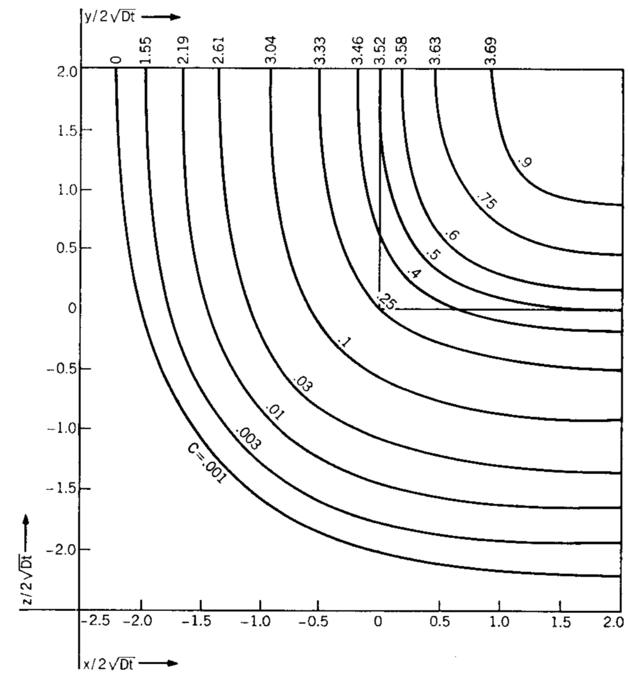


Figure 7 Calculated impurity atom density against the density gradient normal to the constant density surfaces in a two-dimensional planar structure. (a) Constant-C_0 diffusion process, $C = (r, \theta, t)/C_0$; $\xi = 2\alpha_0\sqrt{Dt}/C_0$; (b) Instantaneous-source process, $C = C(x, y, \infty, t)\sqrt{\pi}\ Dt/C_0$; $\xi = 4\alpha_0(Dt)/C_0$.

From Fig. 7 we conclude that a constant-C_0 diffusion process yields a substantially larger impurity atom density gradient (α_0) at the semiconductor surface than at regions far removed from the diffusion mask (erfc). It should be noted that this increased gradient has little influence upon a practical *p-n* junction. An increased gradient can occur in the junction transition region only when the bulk material doping, C_b, is less than an order of magnitude below the surface concentration, C_0. A situation of this type is seldom encountered in practical semiconductor devices.

An instantaneous-source diffusion process yields a substantially different impurity atom density gradient than does a constant-C_0 diffusion process. From Fig. 7, it is seen that an instantaneous-source diffusion process results in a minimum impurity atom gradient, α_0, at the semiconductor surface, in a region far removed from the corners of a diffusion mask. This gradient, in fact, is less than the

Figure 8 Calculated contours of constant impurity atom density at the surface of a planar junction fabricated by an instantaneous-source diffusion process. This illustration can also be interpreted as a map of the surface $C = 10^{-3}$ where $y/2\sqrt{Dt}$ is the penetration depth: $C = C(x, y, z, t)\sqrt{\pi}\ Dt/C_0$.

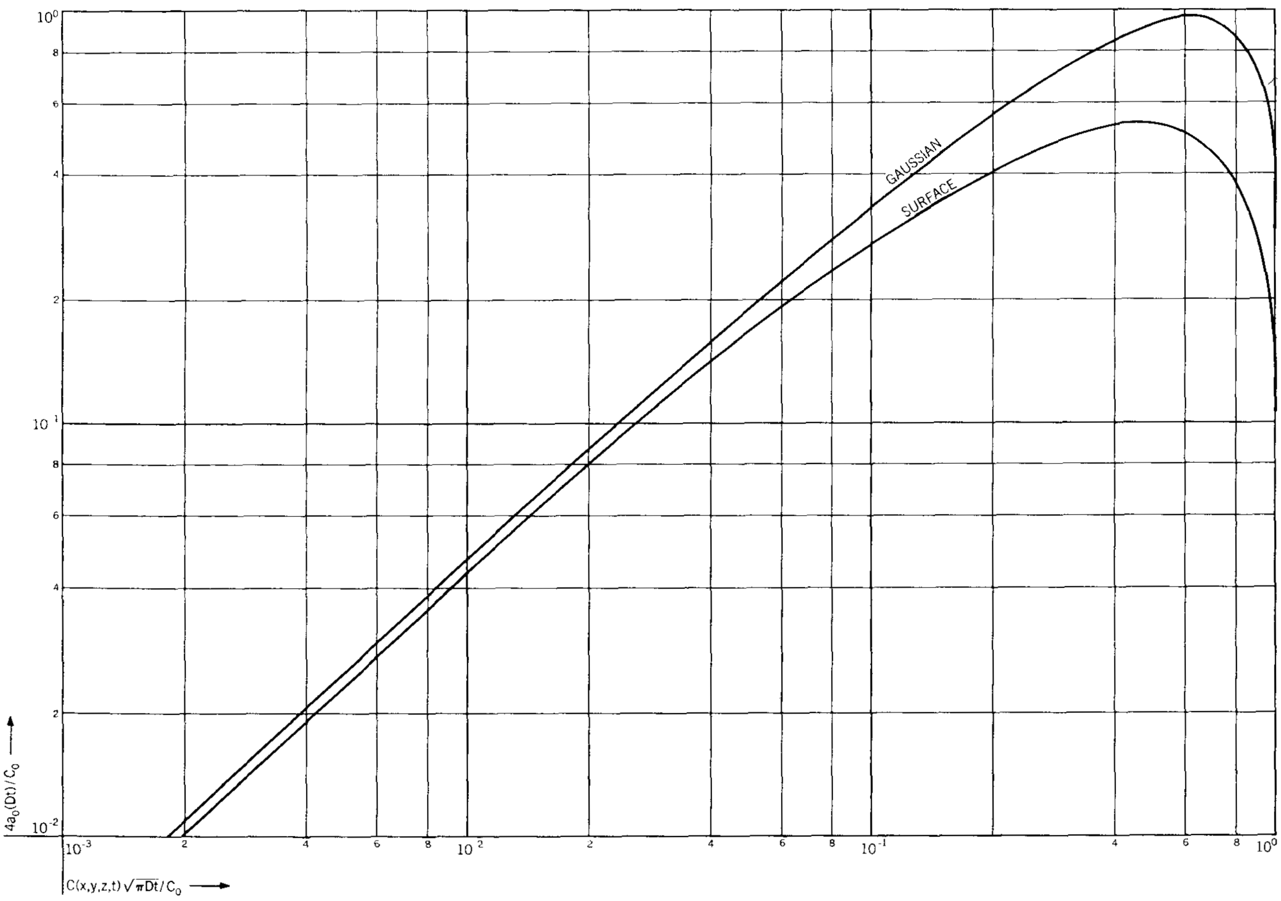


gradient one would calculate from a one-dimensional analysis of this same problem. It should be noted that a two-dimensional form of Eq. (17) is not applicable in the vicinity of a diffusion mask corner. In this region the impurity atom concentration density, and its gradient, must be established from three-dimensional forms of Eqs. (17) and (18), respectively.

Figure 8 illustrates contours of constant impurity atom density in the immediate vicinity of a diffusion mask corner. This illustration has two interpretations. The contours in Fig. 8 could represent a family of lines upon which surfaces of constant impurity atom density terminate at the semiconductor surface. If, therefore, a constant bulk doping level exists within semiconductor material, this Figure could illustrate the families of *p-n* junctions resulting from various levels of bulk doping. A second interpretation of Fig. 8 is obtained if each contour is associated with a specific normalized depth from the semiconductor surface ($y/2\sqrt{Dt}$). In this fashion, a topographical map is obtained for the constant density surface $C = 10^{-3}$ resulting from an instantaneous-source diffusion process.

The impurity atom density gradient $\mathcal{A}_0$ normal to the contours of Fig. 8 exhibits either a maximum or minimum along the line ($x = z$; $y = 0$). Figure 9 shows that $\mathcal{A}_0$ has, in fact, a minimum value at this location. Two regions have been plotted in Fig. 9 for purposes of comparison. The curve marked "surface" provides normalized values of $\mathcal{A}_0$ vs normalized concentration density of impurity atoms along a line bisecting the diffusion mask corner ($x = z$; $y = 0$). The curve marked "Gaussian" illustrates the normalized values of $\mathcal{A}_0$ along this same line, but in a direction normal to the semiconductor surface ($x = z = \infty$; $-\infty < y < 0$). Figure 9 shows that an instantaneous diffusion process yields a minimum value of $\mathcal{A}_0$ at the semiconductor surface, although this minimum will have a negligible influence upon most practical semiconductor devices.

Figure 9 Calculated impurity atom density vs density gradient normal to the surface of a *p-n* junction—instantaneous-source diffusion process. This illustration describes the impurity atom gradient along a line bisecting the diffusion mask corner ($x = z$; $y = 0$). The Gaussian distribution is obtained at infinite distance from this corner along a line directed into the material ($x = z = \infty$; $-\infty \leq y \leq 0$).



Conclusions

The foregoing analysis has shown that the location of the p-n junction in a planar device cannot be accurately determined from a one-dimensional solution of the diffusion equation. It is established that the transition surface of a planar junction (transition from n-type to p-type material) is not equidistant from its impurity atom source in the vicinity of a diffusion mask edge. This situation is encountered when diffusion takes place from a source of constant impurity atom density, and also when a constant number of impurity atoms is involved in the entire diffusion process.

The results of this analysis imply that the theoretical avalanche breakdown voltage of a planar p-n junction is the same as the theoretical breakdown voltage of an equivalent one-dimensional structure. A constant-C_0 diffusion process introduces an increased impurity atom gradient at the semiconductor surface (immediately under the diffusion mask), although when the bulk semiconductor doping, C_b, is substantially below the surface concentration C_1, this increased gradient will not appear in the junction transition region. In addition, it is shown that diffusion from an instantaneous plane source yields junctions with a maximum impurity atom gradient deep within the semiconductor material, and far removed from the diffusion mask. This maximum gradient is equal to the gradient determined from a one-dimensional solution of this same problem.

Acknowledgments

The authors wish to thank Dr. J. E. Thomas, Jr. for his continued interest and his support of this analytical effort. They also thank Mr. P. C. Murley for many valuable suggestions during the course of this work.

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Received January 4, 1965.