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Amplification of Sound by Hot Electrons

It is well known that sound waves can be amplified in a solid by electrons or holes having a drift velocity, in the propagation direction, greater than the velocity of sound.¹⁻⁴ At least at low temperatures, an electric field strong enough to induce a drift velocity of this magnitude, in a semiconductor, can result in a "hot electron" state.^{4,5} The purpose of the present communication is to show that the properties of hot electrons can themselves give rise to a distinct contribution to the amplification, one which does not change sign according as the drift velocity is greater or less than the velocity of sound, both for semiconductors typified by cadmium sulphide² and those typified by *n*-germanium.⁴

For simplicity, effects of modulation (variation due to the wave motion) of the fraction of the charge carriers in bound states will be excluded.⁶ Let $\phi(t, x)$ be the energy of a charge carrier (in one particular valley, for the many-valley case) due to the wave, where x measures distance in the propagation direction, proportional to the wave amplitude. Let the wave amplitude be proportional to $\exp(i\omega t - ikx)$, in the usual convention for harmonic motion. We may define a "dynamical density of states", $G(\omega, -k)$, by the relation

$$n' = -G \phi \quad (1)$$

where n is the density of carriers (in the valley) and the prime denotes a small deviation proportional to the wave amplitude. For the germanium case, ϕ is equal to the valley deformation potential, Ξ , times the strain. Assuming for simplicity a combination of orientations such that $\Sigma n' = 0$ (where Σ indicates summation over valleys), we have

$$2\rho s^2 \alpha_c / k = - \Sigma \Xi^2 \text{Im } G, \quad (2)$$

where α_c is the carrier contribution to the attenuation constant α , the mass density of the lattice is ρ , and $s = \omega/k$ is the propagation velocity, provided that $\alpha \ll k$. For a piezoelectric semiconductor, such as cadmium sulphide,

$$2\rho s^2 \alpha / k = D_0^2 \text{Im } (1/\epsilon) \quad (3)$$

where D_0 is the induced electric displacement per unit strain, ϵ the dielectric constant. (Again it is assumed that $\alpha \ll k$, and attenuation not due to the imaginary part

of ϵ is omitted in (3).) Since the contribution of the charge carriers to $\epsilon(\omega, k)$ is

$$\epsilon_c = (4\pi e^2 / k^2) \Sigma G, \quad (4)$$

again the sign of α_c agrees with the sign of the imaginary part of $-G$.

Let u be the drift velocity and N the flux density, in the propagation direction, of the carriers (in a particular valley). Then

$$N' = n'u + n\theta F' - \partial(\zeta n) / \partial x, \quad (5)$$

where θ is the differential mobility per unit charge, ζ the diffusion coefficient, for the propagation direction and F is the force on a carrier in that direction, so that

$$F' = -\partial\phi / \partial x, \quad \theta \equiv du / dF. \quad (6)$$

We may write, for the last term of (5),

$$\frac{\partial(\zeta n)}{\partial x} = \zeta \frac{\partial n}{\partial x} + n\Omega \frac{\partial F}{\partial x}; \quad \Omega \equiv \frac{d\zeta}{dF}. \quad (7)$$

The conservation equation is

$$\partial n / \partial t + \partial N / \partial x = R \quad (8)$$

where R , the rate of generation of carriers, is proportional to the wave amplitude. We shall write

$$-R = \nu(n' + g\phi + hF'), \quad (9)$$

where ν represents the recombination and intervalley scattering rates. The term in g comes from the strain dependence of the electron states, that in h from the field dependence of the distribution over states. In that we are excluding modulation of the occupation of bound states we are concerned with "exhaustion" conditions in which (9) refers to intervalley scattering only. Then for a "single-valley" semiconductor we will have $R = 0$ in (8) and accordingly should set the ν terms of (11) equal to zero. For the many-valley case, in general for each of the valleys R will be given not by (9) but by a similar sum over the n' , ϕ and F' of all valleys. For appropriately symmetrical orientations,⁴ however, (9) will be valid. The coefficients θ , ζ , Ω , ν , g , h are of course functions of F (of the applied electric field).

From Eqs. (5)–(9), on substituting $\partial n/\partial x = -ikn'$, etc., and $\partial n/\partial t = isk n'$, we get

$$-\frac{k}{n} \text{Im } G = \frac{a(s-u) - bc}{(s-u)^2 + c^2/k^2} \quad (10)$$

where

$$\begin{aligned} a &\equiv k^2 \theta + \nu g/n, \\ b &\equiv k^2 \Omega + \nu h/n, \\ c &\equiv k^2 \zeta + \nu. \end{aligned} \quad (11)$$

For thermal equilibrium $\Omega = h = 0$, so $b = 0$ and (10) reduces to the conventional form, and furthermore $a = \beta c$ for Boltzmann statistics, so evidently we recover the known results for this case.^{2,4}

The contribution to α_c with which we are concerned is that given by the second term in the numerator of (10). It gives amplification when b is positive; i.e., when Ω and/or h are positive. The longitudinal diffusivity, ζ , is difficult to calculate in terms of known quantities,^{7,8} but it appears probable that in general it is an increasing function of $|F|$ —i.e., ΩF is positive. Evaluation of one simple theoretical model for hot electrons confirms this. Recently the longitudinal diffusivity was measured⁹ for holes in germanium with electric fields up to $\sim 10^3$ V/cm, and it was found to increase monotonically with field strength (roughly proportional to $|F|$ in the hot-electron range). Where intervalley scattering is by phonon emission and absorption, it is clear that h is positive. For intervalley scattering by ionized impurities in germanium, from theoretical considerations¹⁰ it appears that the contribution to h will be negative but will become small. With a variable fraction of impurities ionized⁴ (a situation which has been excluded from this discussion), the question of the sign of h is more complicated.

If the terms in c are dominant in the numerator and the denominator of (10), if the terms in Ω and ζ dominate over the terms in ν , for b and c in (11), and if $\Omega/\zeta \sim 1/F$, then by (2) we have

$$-\alpha_c \sim (\Xi^2/\rho s^2)(nk^2/F).$$

For n -germanium, if $k \sim 10^5 \text{ cm}^{-1}$, $F/e \sim 10^2 \text{ V/cm}$, this gives $-\alpha_c \sim n \times 10^{-14} \text{ cm}^2$. Inasmuch as the condition $u > s$ is not a necessary one for amplification, the latter may be brought about at a lower level of power dissipation than previous theory would indicate. It is evident that in a many-valley semiconductor, by suitable choice of

orientations, amplification could be obtained with an applied electric field transverse to the propagation direction, with the hot-electron effects represented by the Ω and h terms coming from the rotation of the valley effective fields and depending on the valley anisotropies. It should be noted that the coefficients in the “phenomenological” equations (5)–(9) will vary with ω and k when the latter are appropriately large, and furthermore since these coefficients will then no longer be real the imaginary part of G will no longer be given by (10). Thus the ultimate high frequency limit, within the quasiclassical range, is not to be obtained simply from (10) with these coefficients given their values for small frequencies and wavevectors.

Note added in proof

Equation (2) can be derived by showing, by use of an argument given by E. I. Blount [*Phys. Rev.* **114**, 418 (1959), Section II], that the net rate of dissipation per unit volume is $-\Sigma(\phi \partial n/\partial t)$. The latter is negative when the positions of the maxima of n are displaced, from the minima of ϕ , in the direction of wave propagation. This forward displacement ensues from a positive value of b , which corresponds to a tendency for the carriers to flow to the minima of the total instantaneous force F in each valley.

The possibility of amplification in n -germanium (i.e., negative values of (10)) for small values of u/s may be shown by estimates, similar to that in the last paragraph above, of bc/a , and is indicated for “warm electron” conditions [T. N. Morgan, *Phys. Rev.*, in press] by similar estimates.

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References and footnotes

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8. Chapter 8 of *Fluctuation Phenomena in Solids*, R. E. Burgess, Ed., Academic Press, New York, 1965.
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