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Space-Charge Model for Surface Potential Shifts in Silicon Passivated with Thin Insulating Layers†

Abstract: Semipermanent changes in the semiconductor surface potential occur in insulator-covered semiconductors when external fields are applied for long times, particularly at elevated temperatures. An attempt to explain these changes in terms of the charging and discharging of interface states leads to conclusions that disagree with many of the experimental facts. Specifically, the semipermanent effects of interface-state charges can always be overcome by the application of a field smaller than that which is used to induce the effect, and of the same sign, while the experiments described in the accompanying papers generally show that a field much larger than the inducing field, and of the opposite sign, is required to return the insulator covered surface to its initial status. The accumulation of space charge in the insulating layer can give rise to very large fields at the semiconductor surface that persist after the removal of an external inducing field. The size and sign of such space-charge fields agree with the experimental observations. Measurements made after treatment at temperatures above 125°C show that the surface of silicon passivated with silicon dioxide can become strongly n-type as a result of such a positive space-charge layer formed at the interface. A model is presented based on the concept that this space charge arises from oxygen vacancies in the silicon dioxide. It is suggested that the improvement resulting from the use of phosphorus pentoxide on the outside surface is due to the elimination of vacancies by the oxidizing action of the phosphorus pentoxide.

Introduction

Environmental effects on semiconductor surface potential have generally been thought to involve surface states in some fashion.[1] One possibility is the slow change in the population of such states under the influence of the environment; another is the creation or destruction of surface states through chemical interaction of the semiconductor and its ambient. Neither of these explanations is adequate to explain all the phenomena described in the accompanying papers.§ Instead, the data suggest the presence of large space charges in the insulating layers. While such charges might be thought of as being associated with the filling or emptying of trap levels in the insulator (or possibly with the creation and destruction of such levels) the lack of any knowledge of the energy level structure of amorphous insulators suggests a simpler approach based on space-charge-limited conduction in the insulator.

The most familiar insulating layer used over silicon is silicon dioxide. The large *n*-shifts at the silicon surface associated with this insulator, and the rarity or absence of *p*-shifts, suggest that positive space charge can easily exist in the SiO_2 but that negative space charges never occur there. The accompanying papers show that chemically reducing environments tend to aggravate the formation of positive space charge and that the presence of a P_2O_5-SiO_2 glass layer on the outside surface tends to stabilize against formation of positive space charge. These results suggest that the positive space charge is due to oxygen ion vacancies in the SiO_2 structure and that P_2O_5 serves as a source of oxygen to prevent this effect.

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§ See pages 376 to 429, this issue.

The presence of the P_2O_5-SiO_2 glass reduces the high-temperature electrical conductivity, which suggests that the oxygen ion vacancies are mobile in SiO_2.

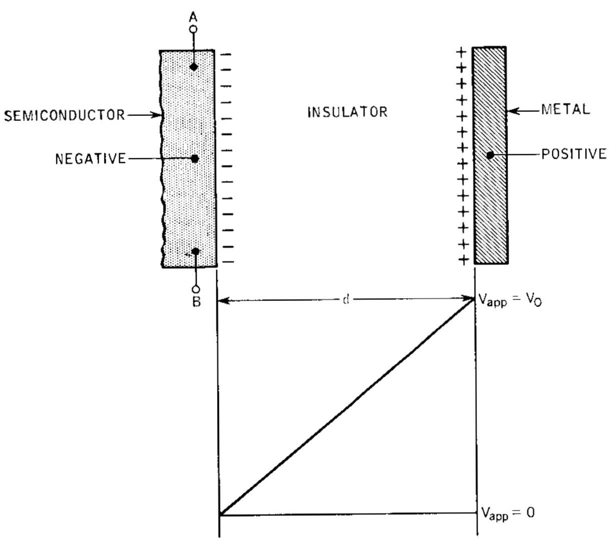
The surface-state case

We find that the changes in silicon surface potential that are observed are not due to surface state effects as ordinarily defined. To make this clear we discuss the results that would be expected when charges enter and leave surface states.

In Fig. 1 we see the simple case of the field effect caused by an electrode external to a body of semiconductor. A negative charge is shown, bound at the surface of the semiconductor. This charge can be entirely in the form of an increment in the number of free carriers, it can be charge trapped in surface states, or it can be partly free and partly trapped. If the mobility of carriers at the surface is known, the fraction of incremental charge in surface states can be ascertained from a conductance measurement between Terminals A and B.

In Fig. 1, Q_t designates the total surface charge per unit area on the semiconductor, equal and opposite to the charge on the electrode. The charge Q_n is due to the increment in current carriers, Q_n being used for convenience whether the carriers involved are electrons, as assumed in Fig. 1, or holes. The charge in surface states

Figure 1 **Above: Semiconductor - insulator - metal system with no charges in the bulk of the insulator.**
Below: Potential vs distance in the insulator.
$Q_t = -\epsilon V_0/d = Q_n + Q_s$, in which Q_n is charge due to free electrons and Q_s, charge due to surface states. $Q_s < Q_t$, $Q_n \leq Q_t$.



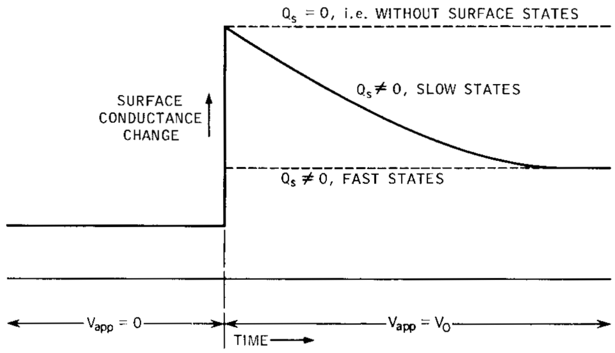


Figure 2 **Surface conductance change with time following the application of an external field normal to the surface.**

Q_s is always of the same sign as Q_n and thus is less than Q_t. The permittivity of the insulator is ϵ.

In many cases[2], the conductance is a function of time after the application of a potential to the metal electrode. This is customarily explained as a delayed filling and emptying of the surface states or, in other words, a decrease with time of the fraction of the total charge attributable to free carriers. If the charge in surface states establishes itself instantaneously, or nearly so, the surface states are called *fast states*, and if an appreciable time is required, they are called *slow states*. Time constants of many minutes for slow states are fairly common in measurements on germanium surfaces.[3] Figure 2 shows the surface conductance response to the application of external field for the three cases: no surface states, slow states only, and fast states only.* There is little doubt that the surface state explanation for the germanium work cited is correct.

The accompanying papers report various observations of semipermanent changes in surface potential as a function of high temperature treatment, with and without applied external field. Direct and indirect measurements of Q_n are used as indicators of surface potential. The most natural assumption to make concerning the source of these changes is the slow filling or emptying of surface states. Since the time for filling and emptying of all traps is a function of temperature, the temperature dependence of these effects is not unreasonable.

The superficially similar phenomena reported on germanium surfaces are shown in Fig. 3. The charge in surface states accumulates in the time interval when $V_{app} = V_0$. When V_{app} returns to zero, Q_s does not immediately change, with the result that a field of opposite sign to that

* For simplicity, Figs. 2 and 3 are drawn for the case in which the surface is already n-type, and the first-order effect of an external positive voltage (small negative increment in Q_n) is to increase the conductance of the semiconductor.

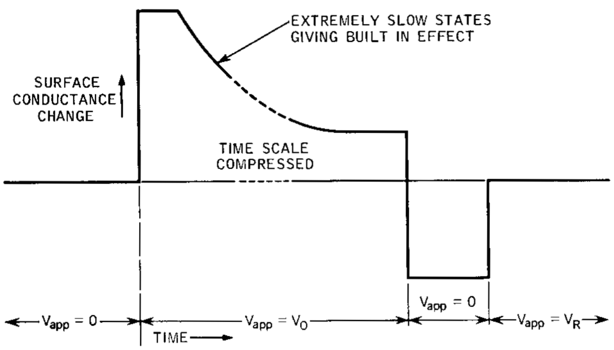


Figure 3 **Surface conductance changes associated with slow charge accumulation in surface states.**
Let $V_R/V_0 = M$; here $0 < M < 1$, since $Q_s < Q_0$. But we see $M < 0$, $|M| \gg 1$, usually.

caused by V_0 now extends into the semiconductor and Q_n changes sign, i.e.,

$$Q_n + Q_s = \epsilon V_0/d \quad \text{for} \quad V_{app} = V_0, \quad \text{and}$$

$$Q_n = -Q_s \quad \text{for} \quad V_{app} = 0.$$

The reversal of Q_n is indicated by a decrease of surface conductance *below* its initial value, assuming that the immediate effect of increasing V_{app} had been to increase the conductance. A convenient measure of the charge stored in surface states is the applied field necessary to overcome its effect, which can be described in terms of V_R, the voltage on the external electrode required to return the surface conductance to normal. If all the charge Q_n has decayed to zero during the period of application of the external field, Q_s will equal Q_t and have the maximum possible value. Setting $V_{app} = 0$ will then result in a decrease in surface conductance equal in magnitude (to the first order) to the original increase which resulted from raising V_{app} up to V_0. In this case V_R will be equal in sign and magnitude to V_0.

If we define a factor $M = V_R/V_0$, it will thus always obey $0 < M < 1$ if the semipermanent surface potential changes are caused by charges in surface states. In the experiments described in the accompanying papers, V_R is frequently larger than V_0, and instead of finding M positive, M is usually negative. Because V_R is observed to be larger than V_0, M will be called the *magnification factor.*

The insulator space-charge case

Alternately we can neglect interface states ($Q_s = 0$) and assume that the insulator contains current carriers of a single sign mobilized by an increase in temperature. We assume further that these carriers can be neutralized at one electrode but cannot be supplied at the other electrode or generated in the bulk; as a result we can have a situation in which M can be negative and much greater than 1.0 in magnitude.

In the case of SiO_2, the carriers are presumed to be oxygen ions that move in the direction of the positive electrode if suitable oxygen ion vacancies exist as receptors. The vacancy, which in itself constitutes a net positive charge, thus moves toward the negative electrode by exchanging position with the negative oxygen ion. The final result is the accumulation of a positive space charge next to the negative electrode. The fact that in SiO_2 *positive* space charges are observed, but not negative, suggests that oxygen vacancies are possible, but that an excess of oxygen ions, beyond that needed for stoichiometry, does not occur.

For glass insulators, either sign of charge appears to occur at the silicon-insulator interface. Whether oxygen is responsible for either or both signs of space charge cannot be determined from the present data.

In Fig. 4,* the curves illustrate the sequence of events for the case of negative carriers, or the equivalent positive-vacancy situation, when V_{app} is positive. In a situation such as this, in which displacement currents are comparable with conduction currents, significant amounts of space charge will always accumulate at any interface which constitutes an impedance to current flow. We assume here, as noted above, that the semiconductor-to-insulator interface presents an impedance that dominates the flow of current and that, as a result, the current eventually drops to a low value. The first effect, however, is to bow the initially linear potential gradient upward as positive charge accumulates in the insulator. The positive charge gravitates to the semiconductor surface where it accumulates as a space-charge layer. Since current must be continuous throughout the insulator, it can stop only if conditions for no current exist everywhere. Thus, we get a cessation of current when the space-charge region is depleted of carriers and the rest of the insulator has no field,† as shown in the top curve of Fig. 4.

It is clear from Fig. 4 that the field at the semiconductor can be much larger than the original applied field. Clearly Q_n can be much greater than the charge induced initially (Q_0), but it is usually not possible to measure Q_n at the temperatures required to activate conduction in the

* Figures 4, 5, and 6 are drawn with positive electric potential upward, to clarify the electrical relations in the insulator, contrary to the usual practice in energy band diagrams. Also, the familiar changes in shape of the energy bands at the surface of the silicon are ignored, since the small associated changes of potential can be negligible compared to V_{app} and compared to the potentials associated with the space charge. Figure 8 returns to the more customary conventions, since the surface potential changes can no longer be ignored.

† There will, of course, be a transition region in which carriers diffuse to the left and drift to the right for a net of zero current. The width of this region will depend on the mass and density of the carriers. Since neither of these numbers are known at present, we will assume an abrupt transition at the edge of the space charge.

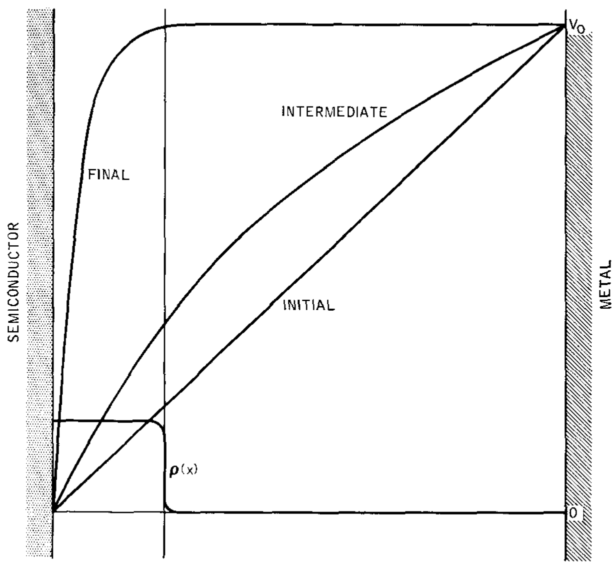


Figure 4 **Space charge and potential curves resulting from negative current carriers which cannot be supplied at the electrodes.**

$Q_s = 0,\ Q_n = -\epsilon dV/dx|_{x=0},\qquad |Q_n| \gg Q_0.$

insulator. However, the space charge can remain fixed when the temperature is lowered and V_{app} returned to zero, thus giving a semipermanent change in Q_n. The potential V_R to return Q_n to normal can be evaluated and thus M can be experimentally determined.

When the space charge layer is located next to the semiconductor surface, most of its field will terminate on the semiconductor. This field will induce mobile charges in the semiconductor and immobile surface charges at the semiconductor-to-insulator interface. The immobile charges are not included in the following calculations, but in any case it can be seen that they do not affect the magnitude of V_R. For the case shown by Fig. 4, a large negative value of Q_n will result and a large *negative* value of V_R will be needed to drive Q_n back to normal. Thus M will be negative and much larger than 1.0 in magnitude, as observed in the experiments described in the accompanying papers.

If the entire current which flows at elevated temperature in the insulator serves only to establish the space charge near the semiconductor interface, the time integral of the current should equal the total stored charge. Provided that the stored charge does not change after the system is cooled to room temperature, it can be measured in terms of V_R. Kerr et al.[4] report such measurements; they find approximate equality between the total charge which flows in the first half-hour or so of temperature-bias treatment and the stored charge, as revealed by the V_R data. There is, however, a steady component of current in the insulator which does not contribute to the stored charge.

A steady current would result in an *I-R* drop through the insulator which would reduce the effective value of the applied voltage on the electrode. In the calculations below, the steady component of current is presumed to be zero.

While the space-charge concept seems to give the best explanation for the gross changes in surface carrier density which occur under temperature bias treatment, several phenomena in MOS devices[5] and field effect transistors[6] can only be explained by the presence of fast Si-insulator interface states.

Space-charge calculations

The experimental observations have been made in the following sequence.

(1) The sample is heated to a specified temperature.
(2) A voltage is applied for a given time.
(3) The sample is cooled to room temperature.
(4) Measurements are made of device characteristics.

In all of the following, the primed notation refers to the conditions present after the above treatment. We assume that a space charge formed at high temperatures is frozen in place and remains constant at room temperature. The space-charge layer is taken to be next to the semiconductor and to have thickness d_1. The overall insulator thickness is d.

We can solve Poisson's equation for two cases: (1) potential between metal and semiconductor equal to the potential difference across the insulator space-charge layer, i.e., no field in the uncharged insulator, and (2) no potential difference between metal and semiconductor, but with the same space charge as (1). We will let Q_0 be the charge on the semiconductor which would terminate a uniform field V_0/d and Q_n be the charge which terminates the space-charge field while the external voltage is present. Q_n' will represent the charge required on the semiconductor to terminate the space-charge field after the removal of V_{app}. Q_n' may be thought of as a "built-in" charge of free carriers after the external potential has been removed.

Figure 5 shows the potential curves in the insulator. $V(x)$ is the potential after the space-charge has accumulated but with the external voltage still present, while $V'(x)$ is the potential after the external voltage has been returned to zero. In the case shown in Fig. 5, the carriers are assumed negative and the space charge positive.

The details of the solution are given in the Appendix; the result can be expressed without reference to the actual value of ρ, the space-charge density, as follows:

$$Q_n' = Q_0[2(d/d_1) - 1].$$

The value V_R will be opposite in sign to V_0 and, by the principle of superposition, will have to satisfy

$Q_n' = \epsilon V_R/d.$

Since

$M = V_R/V_0$ and $Q_0 = -\epsilon V_0/d,$

$M = -Q_n'/Q_0 = -[2(d/d_1) - 1].$

Figure 6 shows the interesting case for the opposite polarity applied to the metal and the same sign of carrier but with the metal presumed to be the blocking electrode instead of the semiconductor. Here, although the sign of V_0 is reversed, the sign of the space charge is the same as in the previous example. Thus, the sign of Q_n' is unchanged, although the magnitude of Q_n' is greatly reduced because most of the field from the space charge terminates on the metal. It can be seen that Q_n' turns out to be independent of both ρ and d_1. In fact, $Q_n' = Q_0$, as though there were no space charge and V_{app} had been positive rather than negative. Thus the potential required to overcome the effects of the space charge will be of the same sign and

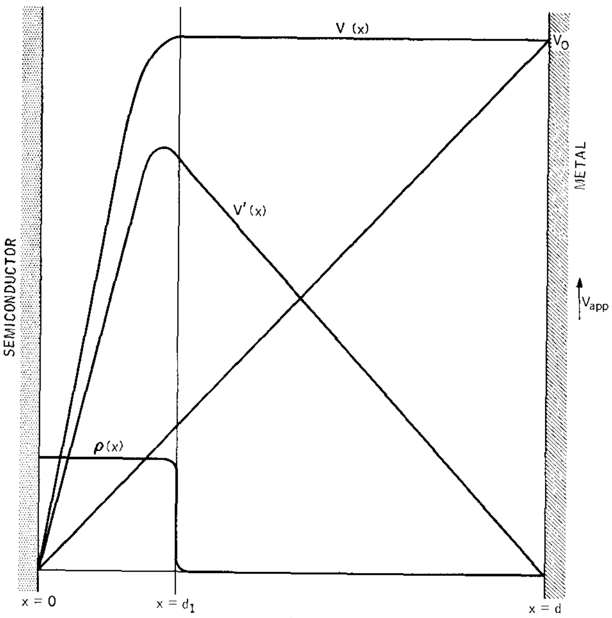
Figure 5 **Details of space charge situation with and without external potential.**

$$Q_0 = -\frac{\epsilon V_0}{d} \qquad Q_n' = -\epsilon \left.\frac{dV'}{dx}\right|_{x=0},$$

where Q_n' = "built-in" charge of free carriers with $V_{app} = 0$. Solve Poisson's equation and obtain

$$Q_n' = Q_0(2\,d/d_1 - 1), \qquad M = -Q_n'/Q_0,$$

$$M < 0, \qquad |M| \gg 1.$$



equal in magnitude to the potential that caused the space charge in the first place, i.e., $M = 1$.

This result would fortuitously resemble the case of full charge in surface states. If an example was found of this type, it could be differentiated from the surface-state case by removing the outer layers of the insulator. If the induced surface conductance disappeared, it would be safe to assume that the charge responsible for the conductance was at the outside rather than the inside of the insulator. This experiment would be performed with a removable electrode. Any fields caused by stray charge accumulation on the outside surface of the insulator while the outer layers were being removed would be neutralized when the electrode was restored to intimate contact with the insulator.

It is unlikely that our simple model of a fully depleted layer of positive space charge is realistic. A more likely situation is a graded layer having a maximum net charge density at the interface and grading to zero density 50 Å or so into the insulator. The carriers in this layer would still develop the full potential drop and would carry zero net current by the balance of a drift current toward the silicon and a diffusion current away.

Vacancy model for SiO_2

In preceding sections, it has been shown that the large shifts in semiconductor surface potential resulting from temperature-bias treatments must be due to the formation of a space-charge layer at the insulator-silicon interface. In particular, for silicon dioxide, it has been shown in a companion paper[4] that a positive space charge can be formed at temperatures as low as 125°C, whereas the formation of a negative space charge is not usually observed. A relatively large current flow in the SiO_2 accompanies the formation of the space-charge layer. It has also been shown that the presence of a SiO_2-P_2O_5 glass layer on the outside surface of the silicon dioxide results in a significant improvement in the stability of the silicon surface.[4] Extensive studies have shown in general that the presence of an oxidizing agent on the outside surface is helpful, whereas the presence of a reducing agent is detrimental. As indicated earlier, these observations lead us directly into the consideration of an oxygen vacancy model. The model is based on the following concepts:

1. *Oxygen ions are removed at the outside surface of the silicon dioxide as a result of the reaction with a metal such as aluminum or by the application of an electric field.* This process is similar to the anodic reaction occurring in ordinary electrolysis. The removal of the oxygen ions leaves oxygen vacancies in the silicon dioxide structure which have associated with them a positive charge.

2. *Oxygen ions can migrate through silicon dioxide at temperatures as low as 125°C, but only if oxygen vacancies*

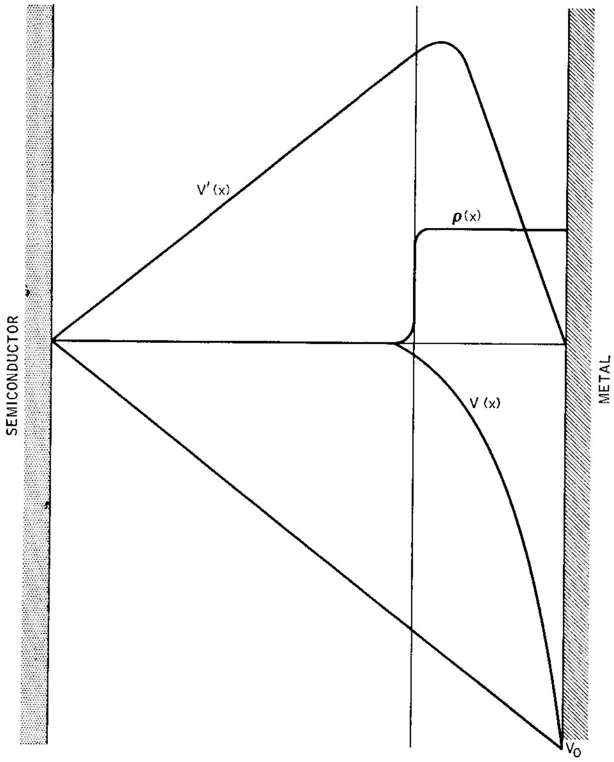


Figure 6 **Space-charge potential curves for charge layer at metal electrode.** *$dV'/dx = V_0/d$, ∴ $M = 1$, i.e., the field which caused the effect is of the proper sign and magnitude to neutralize the built-in effect.*

are present. This is equivalent to the concept that silicon dioxide becomes saturated with oxygen when exactly stoichiometric, and that when the SiO_2 is in this state, oxygen mobility is greatly inhibited.

3. *Phosphorus pentoxide serves as an oxygen source to inhibit the formation of oxygen vacancies in the SiO_2 structure.* This is based on the observation that phosphorus pentoxide tends to eliminate positive space charge effects.[4]

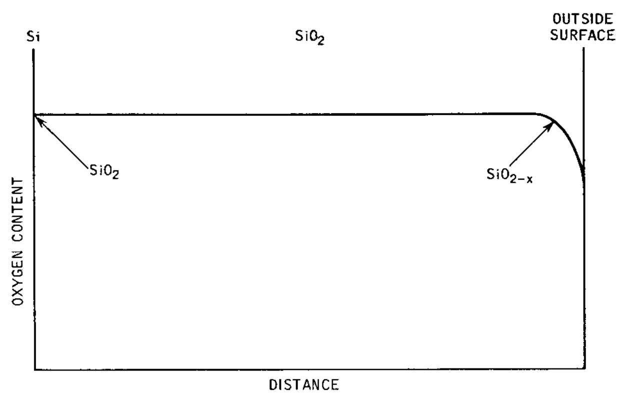
On the basis of these concepts, it is possible to explain the observations reported earlier. This can be shown by referring to Fig. 7, where it has been assumed that the silicon dioxide is deficient in oxygen at the outside surface.

If a negative voltage is applied to the outside surface, oxygen ions will be forced in the direction of the silicon; however, since the SiO_2 is saturated, a significant number cannot be transported and a small current flow is observed. Under these conditions, there will not be a significant space charge built up at the Si-SiO_2 interface. If a positive voltage is applied, oxygen ions will be attracted to the outside surface to fill the vacancies. This will introduce a new vacancy that is closer to the Si which can then be filled with another oxygen ion, etc., and the vacancy will eventually migrate to the Si-SiO_2 interface. This process is similar to the conduction due to holes in a semiconductor with oxygen ions playing the role of electrons in the valence band. In addition, a field applied in this direction can be expected to generate additional vacancies at the outside surface which, in turn, can be propagated to the interface. Since the oxygen vacancy will have the effect of a net positive charge, this migration will result in attracting more electrons in the silicon to the Si-SiO_2 interface and an n-type shift in the Si surface potential will result. In other words, using the notation of the previous section, Q_n' will be large and negative. This migration of positive charge to the Si-SiO_2 interface will produce the relatively large current that is observed to flow in the SiO_2. The current that is observed for the outside electrode positive is 100 times greater than for the outside electrode negative.[4,7] This large anisotropy in conductivity is explained by the model for the Si-SiO_2-metal system that we propose.

Phosphorus pentoxide in the silicon dioxide will act as an oxidizing agent and will thus fill the oxygen vacancies, with the net result that the silicon dioxide reduction will be inhibited and vacancies will not be transported to the interface. As expected with P_2O_5 present, the observed current for a positive electrode voltage is reduced to become approximately equal to that for a negative voltage.

In the absence of P_2O_5, the experiments show that a much larger voltage V_R is needed to overcome the effect of the space charge than the voltage V_0 which was used to induce it.[4] Also, V_R is opposite in sign to V_0. While it is not possible to make a quantitative comparison with theory from the data available, there is obviously qualitative agreement with the notion that V_R and V_0 should be related by the large ratio $-2d/d_1$, where d is the oxide thickness and d_1 is the space-charge layer thickness.

Figure 7 **Effect of electric field on oxygen content of SiO_2.**



Burkhardt[8] has estimated d_1 to be 25 Å by low-frequency capacitance measurements on MOS structures. This estimate was based on the following approximation: $d_1 \simeq d(C_{\text{high freq}}/C_{\text{low freq}})$, where $C_{\text{low freq}}$ is measured at 150°C. The high-frequency capacitance is independent of temperature. His measurements indicated a ratio ($C_{\text{low freq}}/C_{\text{high freq}}$) of 200. This result clearly demonstrates that the region of high electric field strength is indeed a narrow region in the insulator at the Si-SiO_2 interface.

The concept of oxygen as a mobile species in SiO_2, is consistent with the accepted theory[9–11] of the oxidation mechanism of silicon, in which it is assumed that oxygen is the mobile species and that negative oxygen ions are involved. It is not known if there is a connection between the vacancy model we present and the high temperature oxidation process.

Reactive metals

Lehman[12] has shown that aluminum on SiO_2 produces n-shifts which depend on the temperature to which the system has been heated. He has further shown,[13] that the effect can be prevented by a bias as low as minus 2 volts applied to the aluminum with respect to the silicon. Kerr[14] has reported that below 500°C, the effect of aluminum is lessened if the aluminum is not electrically connected to the silicon but above 500° the n-shift is the same whether the aluminum and silicon are connected or not. These observations are all consistent with the oxygen vacancy hypothesis.

Since aluminum reduces SiO_2, it is reasonable to suppose that oxygen vacancies would be created in profusion by heating the two in contact. It can be shown that the electrochemical potential of aluminum with respect to silicon should be equivalent to a positive electrode voltage.

Referring to Fig. 8, we see a detailed potential diagram for a silicon - silicon dioxide - aluminum sandwich. The positive of electric potential is downward, contrary to the other Figures, in order to show the details of the band structure with the usual sign convention for energy levels.

The built-in chemical potential difference will create a field in the insulator when the Fermi levels of the aluminum and the silicon are lined up by external connection. This field can result in the flow of ionic current, as in a normal battery, until polarization of the electrolyte—SiO_2, in this case—causes the current to stop. The polarization will redistribute the potential, exactly as in the previously discussed cases, to result in a semipermanent n-type shift in the surface.

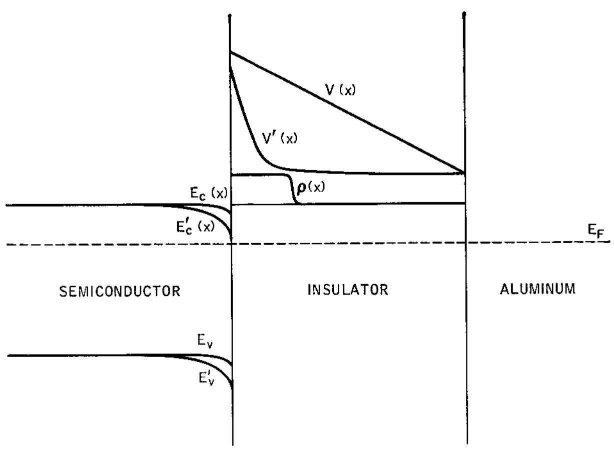
A small initial shifting down of the conduction band edge, shown as $E_c(x)$, is caused by the initial conditions of the built-in field. However, when the ionic current has flowed sufficiently to establish the positive space-charge polarization, the bands will be bent down much further with a resultant shift toward n-type of considerable magnitude. The deflected conduction band is shown in Fig. 8 as $E_c'(x)$. If the aluminum is not connected to the silicon, a small current will flow initially toward the silicon. However, the potential of the aluminum will have to become more negative to supply the current, with the result that the Fermi level of the aluminum will rise relative to that of the silicon, using the sign convention of Fig. 8. When this happens, the built-in field will disappear. A little thought will show that the maximum total charge that can be obtained in the space charge ρ, under these conditions, is the charge that was locked onto the aluminum by the original, undistorted, built-in field. Thus, the n-shifts available with a closed circuit between silicon and aluminum, and with an open circuit, should be related by a number of the order of M as calculated previously.

If any means are provided for transfer of charge from the silicon to the aluminum, the open circuit case can be made to resemble the closed. At higher temperature, it is reasonable to suppose that surface conduction over the insulator, or some other conduction mechanisms, will provide the current and maintain the alignment of the Fermi levels in the aluminum and the insulator. This would serve to explain Kerr's observation[14] that the open and closed circuit cases converge above 500°C.

Sufficient negative potential applied to the aluminum would overcome the electrochemical driving force urging the vacancies toward the silicon and thus prevent the n-shift, exactly as observed by Lehman.

An extensive electrochemical treatment of the system Si-SiO_2-metal has been developed by Seraphim et al.[15] and the oxygen vacancies that we discuss are referred to as V^{++} in their model.

Figure 8 **Potential curves for the system aluminum-silicon dioxide-silicon.** *Here the details of the band structure near the surface are shown, and the polarity convention is inverted with respect to the previous Figures, i.e., more positive, downward.*



Glass passivation

In his work on glass, Kerr[16] found results quite different from the SiO_2 situation. Some glassed surfaces shift more easily in the direction of more *p*-type and some are fairly symmetrical. In order to have a large value of M with both polarities of voltage applied, it seems necessary to assume that space charges of either sign can be locked in, very close to the surface of the silicon. M-values equal to -40 were observed in some cases. For such a situation to exist, the silicon must be presumed to be a blocking electrode that can neither neutralize carriers from the insulator nor supply them.

The glasses most prone to accumulate space charges of either sign, at low temperatures, appear to be those which contain significant quantities of sodium. This observation is consistent with the known high mobility of sodium ions in most glasses but beyond that, little can be said about the mechanism. In particular, it is not clear why the silicon surface should be an electrical blocking electrode for neutralization of the sodium ion. Other current carriers are probably active in glass, particularly the ultra-low sodium content glasses, but the present data offer few clues.

Appendix

Starting with Poisson's equation

$$\frac{d^2 V}{dx^2} = -\frac{\rho}{\epsilon}$$

we can solve for $V(x)$ and $V'(x)$ subject to the conditions shown in Fig. 5. It can be verified by substitution that, if

$$d_1 = (2 V_0 \epsilon/\rho)^{1/2},$$

then $$V = V_0 \left[1 - \frac{(x - d_1)^2}{d_1^2} \right] \quad \text{when} \quad 0 \le x \le d_1,$$

and $$V = V_0 \quad \text{when} \quad d_1 \le x \le d.$$

Also $$V' = V_0 \left[\left(\frac{2}{d_1} - \frac{1}{d} \right) x - \frac{x^2}{d_1^2} \right] \quad \text{when} \quad 0 \le x \le d_1,$$

and $$V' = V_0(1 - x/d) \quad \text{when} \quad d_1 \le x \le d.$$

If $$Q_n' = -\epsilon \left. \frac{d V'}{dx} \right|_{x=0},$$

then $$Q_n' = -\epsilon \frac{V_0}{d} \left(2 \frac{d}{d_1} - 1 \right) = Q_0 \left(2 \frac{d}{d_1} - 1 \right).$$

$V'(x)$ as shown in Fig. 6 will be the mirror image of the solution for Fig. 5.

Acknowledgments

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