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Effect of the Self-Magnetic Field on Galvanomagnetic Effects in Bismuth

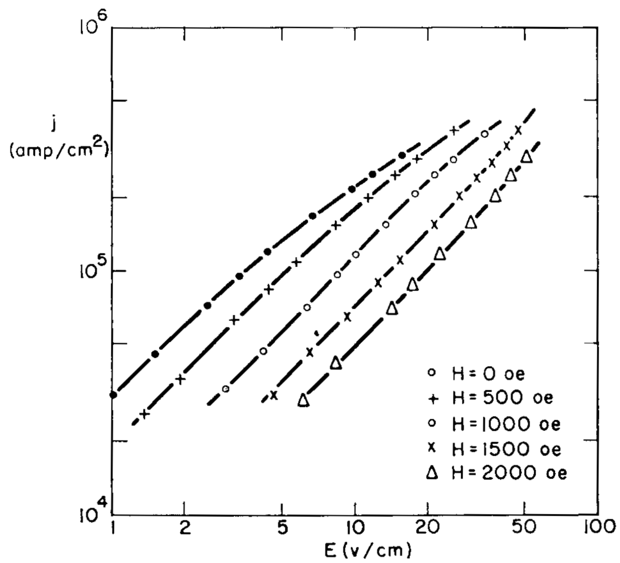
Abstract: The magnetoresistance and the Hall coefficient for pure bismuth at 77°K have been calculated to the second order in the self-magnetic field, i.e. current density. The calculations show that the observed dependence of the galvanomagnetic effects on the current density at high currents can be qualitatively explained by the self-magnetic field. However, to obtain quantitative agreement it is necessary to include the contribution of diffusion currents in the calculation. The theory for the self-magnetoresistance, including diffusion, has been carried out for the pre-pinch regime of currents. The resulting curve for the self-magnetoresistance agrees well with the observed one if the scattering times between ellipsoids are taken to be of order 10^{-9} sec (assuming that the diffusion effects are not dominated by the surface, i.e. that the surface-recombination velocity is sufficiently small).

I. Introduction

High current experiments on a single-crystal bismuth sample at 77°K under an applied magnetic field have been done recently by Hattori.[1] He found that the *I-V* characteristic deviates from Ohm's law, and the Hall coefficient decreases with increasing current at sufficiently high currents. The dimensions of the sample were 0.0394 cm width, 0.00428 cm thickness and 0.506 cm length between voltage probes. The orientation of the sample was chosen such that the binary, bisectrix and trigonal axes are parallel to the 0.0394 cm, 0.00428 cm and 0.506 cm dimensions respectively.

Figure 1 shows the *j-E* characteristics at 77° for various values of applied magnetic fields (**H** applied parallel to bisectrix axis). For low magnetic fields, the *j-E* curves are sublinear. These departures from Ohm's law can be attributed to the self-magnetoresistance, as was discussed by Hattori and Steele.[2] For high magnetic fields, however, the *j-E* curves are superlinear. In these cases, the self-magnetic field contributes to the conductivity in two ways. The self-magnetic field is antisymmetric with respect to the center of the cross section of the sample. Therefore, the applied and self-magnetic fields are additive in one half of the cross section, and subtractive in the other half.

Figure 1 **The j-E characteristics at 77°K for various values of the applied magnetic field.**



Calculations show a net decrease of the resistance relative to the low-current case, because the increase of resistance in the region of stronger fields is smaller than the decrease in the region of weaker fields. Also, at each point off center the vector addition of the local self-field to the external

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field gives a net field rotated with respect to the external field, i.e., the net field has an orientation deviating away from the bisectrix axis, in which direction the magnetoresistance coefficient is larger, toward the binary axis, in which direction the coefficient is smaller. The net result gives a decrease of the resistance as the current is increased. The Hall coefficient will be discussed in the same current regime.

II. Effect of the self-magnetic field on galvanomagnetic effects in bismuth

The calculations were made using the mobility tensor for bismuth proposed by Abeles and Meiboom.[3] The conductivity $\sigma(H, j)$ along the trigonal axis is expanded, out to second order, in the self-magnetic field. Since the self-magnetic field is proportional to the current density j, the result for $\sigma(H, j)$, averaged over the cross section, is written in the following form:

$$\sigma(H, j) = \sigma(H)[1 + a(H)j^2 + \cdots], \quad (1)$$

where $\sigma(H)$ is the low-current conductivity, and the second term in the square bracket gives the contribution of the self-magnetic field. Figures 2 and 3 show calculated values, neglecting diffusion, of $\sigma(H)$ and $a(H)$ respectively. The calculation gives qualitative agreement with the observation.

The Hall coefficient $R(H,j)$ is also calculated to the second order in the self-magnetic field and is written in the following form:

$$R(H, j) = R(H)(1 - b(H)j^2 + \cdots). \quad (2)$$

Table 1 shows the calculated and observed values of $R(H)$ and $b(H)$. The calculation again gives qualitative agreement with the observation.

In the course of these experiments, Hattori also found size effects on $\sigma(H)$ and $R(H)$. These size effects might not be related to the mean free path, but to the diffusion length. After all, the scattering times between ellipsoids of

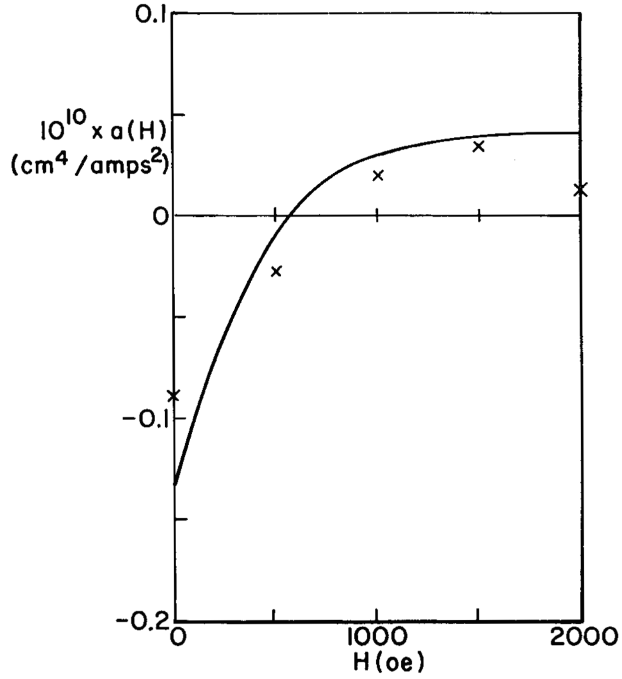

Figure 3 **a(H) vs H.**

Table 1 **Hall coefficient.** *$R(H)$ in particular units and $b(H)$ in units of 10^{-10} (cm^4/amp^2) are listed.*

H (gauss)	$R(H)$ [calc]*	$R(H)$ [obs]	$b(H)$ [calc]*	$b(H)$ [obs]
500	−7.40	−3.46	0.094	0.013
1000	−7.20	−2.71	0.046	0.010
1500	−7.19	−2.42	0.029	0.009
2000	−7.17	−2.25	0.042	0.010

* Diffusion effects not included in the calculation.

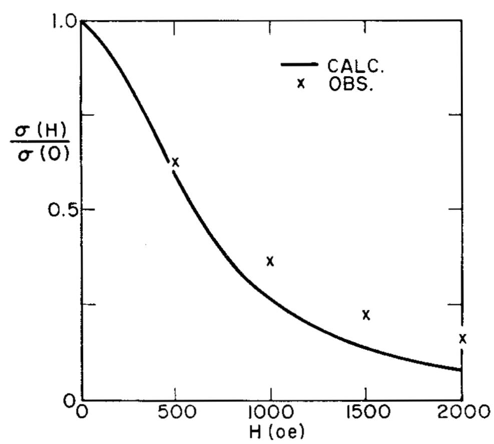

Figure 2 **The low current conductivity σ(H) vs H.**

electrons and holes are finite, and if the thickness is comparable with the diffusion length, the carrier density distribution can be distorted by the transverse particle flow. The distortion of the density distribution creates a diffusion force which opposes the Lorentz force. This effect partially compensates the magnetoresistance and the Hall coefficient. Taking into account this diffusion will give quantitative agreement with the observation.

III. Self-magnetoresistance effect

In order to investigate the effect of the diffusion we will consider the problem of the self-magnetoresistance. Hattori[4] has done the relevant experiments. The dimensions of his sample were 0.0386 cm width, 0.0052 cm thickness and 0.481 cm length between voltage probes.

The orientation of the sample was chosen such that the binary, bisectrix and trigonal axes were parallel to the 0.481 cm, 0.0386 cm and 0.0052 cm dimensions, respectively. In this orientation a significant contribution of the diffusion can be expected.

For this thin slab geometry, the carrier density will be essentially uniform along the bisectrix axis in the pre-pinch regime, and the diffusion forces in this direction can be ignored except near the two ends of the cross section. Furthermore, the magnetoresistance coefficient[3] $q_{11}(3)$ is about 1/6 of $q_{11}(2)$. The neglect of the diffusion forces does not give any serious error. Therefore, the effect of the inward-driven particle flow along the bisectrix axis, due to the magnetic field component H_3 (along the trigonal axis), approximately gives a contribution $q_{11}(3)\langle H_3^2\rangle$ to the self-magnetoresistance, where $\langle\ \rangle$ denotes an average over the cross section.

On the other hand, for the inward-driven particle flow parallel to the trigonal axis due to H_2 (component along the bisectrix axis), the diffusion forces give an important correction to $q_{11}(2)\langle H_2^2\rangle$. It is also assumed that the carrier densities and the electric field are uniform along the binary axis.

The calculations were based on the Abeles and Meiboom model[3] for Bi, and were made for the thin slab geometry. Complete results have been obtained for the pre-pinch regime in which the current is low enough to make the self-magnetic pressure smaller than the kinetic pressure of the electron-hole plasma. The conductivity σ is calculated to the second order in the self-magnetic field or j.

The steady-state equations describing the problem are: 1) The current flow equations for electrons in each ellipsoid and holes, respectively. The diffusion contribution is included in these equations. 2) The charge neutrality equation. 3) The Maxwell equations. 4) The conservation equations for electrons in each ellipsoid and for holes, respectively. The inter-ellipsoid transitions are included in these conservation equations. The transition probability between two electron ellipsoids is specified by the transition time $\tau_{e,e}$, while the transition probability between electron and hole ellipsoids is specified by $\tau_{e,h}$. 5) The boundary conditions, specified in terms of the surface recombination velocity S. Here we assume equal surface recombination velocities for electrons in each ellipsoid.

After straightforward calculations, the current density j along the binary axis is obtained as a function of the applied electric and the self-magnetic fields. Averaging j over the cross section gives the conductivity:

$$\begin{aligned}\sigma/\sigma_0 = {} & 1 - q_{11}(3)\langle H_3^2\rangle - q_{11}(2)\langle H_2^2\rangle \\ & \times\left[1 - \frac{1}{1+\gamma}\, g(b/L_D,\, b/S\tau_{e,h}) - \frac{\gamma}{1+\gamma}\, g(b/L_D',\, b/S\tau)\right] \\ & + q_{11}(2)\langle H_2^2\rangle\left[\frac{b^2/D_a\tau_0}{1+b/S\tau_{e,h}}\,\frac{1}{1+\gamma} \times g(b/L_D,\, b/S\tau_{e,h}) + \frac{b^2/D_e\tau_0}{1+b/S\tau}\,\frac{\gamma}{1+\gamma}\, g(b/L_D',\, b/S\tau)\right], \end{aligned} \qquad (3)$$

where

$$g(x,\alpha) = \frac{3(1+\alpha)}{x^2}\,\frac{x-\tanh x}{x+\alpha\tanh x}$$

σ_0 = low current conductivity

$$\gamma = (\mu_3+\nu_3)(\mu_1-\mu_2)^2/2\nu_3(2\nu_1+\mu_1+\mu_2)^2$$

$$1/\tau = 1/\tau_{e,h} + 3/\tau_{e,e}$$

$$\tau_0 = 4\pi\sigma_0 b^2/c^2$$

$$L_D = [D_a\tau_{e,h}]^{1/2}$$

$$L_D' = [D_e\tau]^{1/2}$$

$$D_a = (\mu_3 D_h + \nu_3 D_e)/(\mu_3+\nu_3).$$

μ_1, μ_2, μ_3 are the electron mobilities along the binary, bisectrix and trigonal axes of each ellipsoid respectively; ν_1, ν_2 and ν_3 are the hole mobilities along the binary, bisectrix and trigonal axes respectively; b is the half-thickness of the slab; D_e and D_h are the diffusion constant along the trigonal axis for electrons and holes respectively; and c is the velocity of light in vacuum.

The second term, and $q_{11}(2)\langle H_2^2\rangle$ in the third term on the right hand side of Eq. (3), are the pure self-magnetoresistance contributions. The $\tau_{e,h}$-dependent term in the third term is the diffusion contribution associated with the hole and net electron flows. The τ-dependent term is the diffusion contribution associated with the difference between the electron flows of different electron ellipsoids. If both $\tau_{e,h}\to\infty$ and $\tau_{e,e}\to\infty$, then $q_{11}(2)\langle H_2^2\rangle$ is perfectly compensated by these diffusion contributions. In this case the diffusion contribution to $q_{11}(3)\langle H_3^2\rangle$ should also be taken into account.

The last term represents the contribution of the surfaces. Owing to the inward-driven particle flow, the carrier densities on the surfaces become smaller than their thermal equilibrium values and, as a result, the carriers are generated on the surfaces. The $\tau_{e,h}$-dependent term is related to hole and net electron flows, and the τ-dependent term is related to the difference between electron flows of different electron ellipsoids as mentioned in the preceding paragraph. This generation on the surfaces increases the number of carriers, thus increasing the conductivity. If $S\to 0$ or $\tau_{e,h}\to 0$, there are no changes in the number of carriers in the plasma and the last term goes to zero.

Eq. (3) can be written in the form of Eq. (1). If Eq. (1) is fitted to the observed data,[4] in the region of onset of the departure from Ohm's law, it is found that $a(0) = -5.8 \times 10^{-12}$ $(\mathrm{amp/cm^2})^{-2}$, which is smaller than the Abeles-Meiboom value. In order to obtain values of $\tau_{e,h}$ and $\tau_{e,e}$ from $a(0) = -5.8 \times 10^{-12}$ $(\mathrm{amp/cm^2})^{-2}$, something must be known about the relative values of $\tau_{e,h}$, $\tau_{e,e}$ and b/S. Here we assume that $b/S\tau_{e,h}$ and $b/S\tau$ are much larger than unity. There is some justification for this assumption based on the recent experimental work of Zitter[5] on the magnetoresistance of Bi. The computation of $\tau_{e,h}$ and $\tau_{e,e}$ used the known mobilities[3] of carriers at 77°K, the diffusion constants D_e and D_h, and the value of $a(0)$ as determined from the experimental data. The diffusion constants are calculated by use of the Einstein relation, knowing the effective temperatures and mobilities of carriers. The effective temperature is defined by:

gradient of the partial pressure of carriers
$= kT_{\mathrm{eff}} \times$ gradient of the carrier density.

Using the ellipsoidal model for the electrons and holes, and taking the Fermi energies to be 0.022 eV for electrons and 0.014 eV for holes,[6] we estimate the effective temperatures 200°K for electrons and 140°K for holes at $T = 77$°K. Table 2 shows $\tau_{e,h}$ and $\tau_{e,e}$ computed for different values of the ratio $\tau_{e,e}/\tau_{e,h}$.

IV. Conclusion

The observed dependence of galvanomagnetic effects in Bi on the current density can be explained by taking into account the self-magnetic field. Conductivity and Hall coefficients were calculated to the second order in the self-magnetic field, i.e. current density. Calculations using the mobilities proposed by Abeles and Meiboom for Bi gave only qualitative agreement with the observation. To obtain quantitative agreement, it appears necessary to include the effects of diffusion in the calculation. The theory for the self-magnetoresistance, including diffusion, has been worked out for the pre-pinch regime of currents. To fit the calculated value of the self-magnetoresistance to the observed one, the transition times between ellipsoids must be of the order of 10^{-9} sec, if the diffusion effects are not dominated by the surface, i.e., if S is sufficiently small.

The size effects on the low current conductivity and the Hall coefficient can also, very likely, be explained quantitatively by taking into account the effects of diffusion. Such calculations are now in progress.

Table 2 **Inter-ellipsoid transition times $\tau_{e,h}$ and $\tau_{e,e}$ (seconds).**

$\tau_{e,e}/\tau_{e,h}$	$\tau_{e,h}$	$\tau_{e,e}$
1/10	9.3×10^{-9}	0.93×10^{-9}
1	1.6×10^{-9}	1.6×10^{-9}
10	0.9×10^{-9}	9×10^{-9}

Acknowledgments

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