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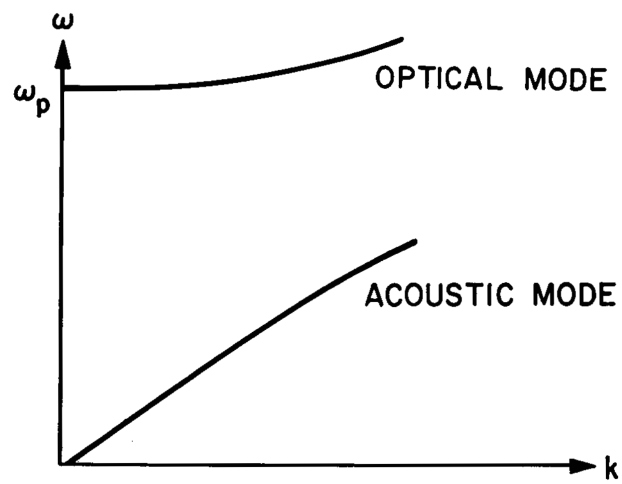
Acoustic Plasma Waves in Semimetals

Abstract: The acoustic plasma wave suffers severe Landau damping for equal-temperature carriers obeying Boltzmann statistics, but can be relatively weakly Landau damped in semimetals if in the propagation direction the Fermi velocities and masses of the two carriers are very unequal. Only the carriers with the smaller Fermi velocity are important in producing collision damping since the other carriers store no appreciable momentum. Some results for many-valley semimetals like bismuth are given, together with a discussion of the problem of exciting and detecting this essentially neutral and longitudinal wave. Experiments undertaken to detect the acoustic plasma wave by transmission through thin wafers of bismuth at 10 Gc/sec have been unsuccessful thus far, but have revealed the existence of a higher velocity wave of weak amplitude that has not yet been identified. A discussion is also given of some magnetic quantum effects that should be associated with the acoustic wave.

Introduction

As was first pointed out by Tonks and Langmuir,[1] a plasma containing two types of mobile charge carriers has two branches to its longitudinal oscillation spectrum, as shown in Fig. 1. In the higher frequency branch the two carriers oscillate out of phase (assuming they are of opposite charge) with the long wavelength limit at the so-called plasma frequency, while in the lower branch the carriers oscillate in phase, exhibiting at long wavelengths a dispersion relation like that of a sound wave. In analogy to the classification of lattice vibrations, Pines[2] has called the upper branch "optical" and the lower branch "acoustic." In the gas plasma field the lower branch is usually referred to as the ion wave or the ion acoustic wave. The acoustic branch was first discussed for solid state plasmas by Pines,[2] and more recently by Pines and Schrieffer[3] and by Harrison[4] in connection with the possibility of obtaining growing waves in a drifted plasma. Despite the considerable interest in these waves, they have not yet been observed in solids and were only recently seen in gas plasmas.[5,6] The reason lies both in the large damping usually associated with this mode and in the experimental difficulty of exciting and detecting an essentially neutral longitudinal wave. As will be discussed in the next section, the damping can theoretically be reduced to a tolerable level by working at liquid helium temperature with a high-purity semimetal, while it should be possible to overcome the excitation-detection problem by measuring the transmission through thin samples using

Figure 1. **Dispersion relation for longitudinal waves in a two-component plasma.**



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a very sensitive microwave detection method to be described. Experiments performed at 10 Gc/sec using wafers of bismuth about 100μ thick have been unsuccessful thus far in detecting the acoustic plasma wave, but have revealed the existence of a higher velocity wave of weak amplitude, whose origin will be speculated upon in Section 6.

2. Properties of acoustic mode

The condition for longitudinal plasma oscillations is that the total (longitudinal) dielectric constant vanish:

$$\epsilon_{\mathrm{tot}} = \epsilon_{\mathrm{lattice}} + \epsilon_{\mathrm{elect}} + \epsilon_{\mathrm{holes}} = 0, \tag{1}$$

when $\epsilon_{\mathrm{elect}}$ and $\epsilon_{\mathrm{holes}}$ refer to the intraband contributions only, the interband parts being included in the lattice term. For collision-free carriers in a spherical, parabolic band at 0°K, the long-wavelength dielectric constant, determined either classically or quantum mechanically,[7,8] is

$$\epsilon(\omega, k) = (3\omega_p^2/k^2 v_F^2)[1 - (\omega/kv_F)\tanh^{-1}(kv_F/\omega)] \tag{2}$$

$$\simeq \begin{cases} -(\omega_p^2/\omega^2) & \text{for } \omega/k \gg v_F \quad \text{(3a)} \\ (k_{FT}^2/k^2)(1 - i\pi\omega/2kv_F) & \text{for } \omega/k \ll v_F, \quad \text{(3b)} \end{cases}$$

where v_F is the Fermi velocity, $\omega_p = (4\pi ne^2/m)^{\frac{1}{2}}$, and $k_{FT} = \sqrt{3}\,\omega_p/v_F$ is the Fermi-Thomas screening wave vector. These equations are also valid for ellipsoidal bands with $\mathbf{k}$ along a principal axis if the values of v_F and m corresponding to that direction are used. When the propagation is not along a principal axis, one must in general deal with a dielectric tensor, as will be discussed later.

In the optical branch the phase velocity v_p of the wave is greater than v_F for both carriers, and the positive dielectric constant of the lattice is counterbalanced by the negative contributions of the two carriers. In the acoustic mode v_p is intermediate between the Fermi velocities of the holes and electrons. The carriers with the smaller v_F give a negative contribution to ϵ and store energy in the form of kinetic energy resulting from their average velocity. The carriers with the larger v_F tend to adiabatically screen the slower carriers; their dielectric constant is positive and corresponds to a storage of potential energy through a variation in density. At long wavelengths the dielectric constant of the lattice can be neglected for this mode.

It is important to note that ϵ for the carriers with $v_F > v_p$ has an imaginary part and hence that the acoustic wave is damped even when collisions are neglected. This effect, known as Landau damping,[9] arises from the strong coupling of the wave to those carriers travelling at velocity v_p and which therefore see a nearly static field for a long time. The same phenomenon is met in many other situations, such as in acoustic attenuation, and may also be described quantum mechanically in terms of single particle excitations. In a semiconductor where Maxwell-Boltzmann statistics are obeyed it is difficult to achieve small Landau damping. Designating the two carrier types[10] by + and − and assuming that the + carriers are the heavier, the requirements for small damping are[3]

$$m_-/m_+ \ll n_-/n_+ \ll T_-/T_+ \,. \tag{4}$$

Generally it is quite difficult in a solid to have $T_- \gg T_+$ because of collisions with the lattice, while m_- is not much less than m_+ in existing semiconductors. Also if the lattice is cooled to reduce collision damping n_- will probably differ from n_+ by several orders of magnitude unless additional nonthermal carriers are supplied. However, in a semimetal where Fermi statistics are obeyed the slow carriers with $v_F < v_p$ will produce negligible Landau damping near 0°K; while if $v_F \gg v_p$ for the other carriers their damping will be small since relatively few will be travelling in synchronism with the wave. Assuming that the + carrier is the slower and that $v_{F-} \gg v_p$, but not necessarily that $v_{F+} \ll v_p$, we find from (1) and (2) that the ratio $\eta = v_p/v_{F+}$ is given by

$$\frac{\eta}{2}\ln\frac{\eta+1}{\eta-1} = \frac{1}{F_0} + 1, \tag{5}$$

where $F_0 = (k_{FT+}/k_{FT-})^2$. The notation has been chosen to show the analogy with Landau's theory[11] of zero sound in liquid He[3], the two cases being mathematically equivalent since in the acoustic plasma wave the adiabatic screening of the holes by the electrons leads to an effective short range repulsion between the holes.[12] If $F_0 \gg 1$ then $\eta \gg 1$ and

$$v_p \approx v_{F+}(F_0/3)^{\frac{1}{2}} = \omega_{p+}/k_{FT-} = (m_- n_+/3m_+ n_-)^{\frac{1}{2}} v_{F-}\,. \tag{6}$$

If $F_0 \lesssim 1$ then v_p is only slightly greater than v_{F+} and is given by

$$v_p \approx v_{F+}[1 + 2\exp(-2 - 2/F_0)]. \tag{7}$$

In either case the necessary and sufficient conditions for small Landau damping near 0°K are

$$v_{F+} \ll v_{F-} \quad \text{and} \quad m_-/m_+ \ll n_-/n_+ \,. \tag{8}$$

Bismuth meets these requirements fairly well if $\mathbf{k}$ is in the trigonal direction, the value of v_p for this orientation being about 8×10^6 cm/sec.

We next consider the damping due to collisions. In terms of the usual phenomenological collision time τ, the dielectric constant including collisions is

$$\epsilon(\omega, k) = \frac{3\omega_p^2\tau}{i\omega(1 + i\omega\tau)}\frac{1}{a^3}(a - \tan^{-1} a) \times \left[1 - \frac{i}{\omega\tau a}(a - \tan^{-1} a)\right]^{-1}, \tag{9}$$

where $a = kv_F\tau/(1 + i\omega\tau)$.

In obtaining (9) it has been assumed that the distribution relaxes to a local equilibrium distribution with zero average velocity but with the perturbed density. One often sees (9) without the factor in the square brackets; this results from incorrectly assuming a relaxation to the thermal equilibrium density, which would not conserve particle number. When $\omega\tau \gtrsim 1$ we find

$$\epsilon(\omega, k) \approx \begin{cases} -(\omega_p^2/\omega^2)(1 - i/\omega\tau)^{-1} & \\ \qquad \text{for} \quad \omega/k \gg v_F & (10a) \\ (k_{FT}^2/k^2)(1 - i\pi\omega/2kv_F) & \\ \qquad \text{for} \quad \omega/k \ll v_F\,. & (10b) \end{cases}$$

Hence, hole collisions (still assuming holes are the slower carrier) can produce appreciable damping; but electron collision damping is negligible in comparison with electron Landau damping. This is to be expected physically since the electrons do not carry appreciable momentum; they store energy through their density perturbations.[13] From (10a) it is easily found that when $\omega\tau_+ \gg 1$ the wave will decay as a result of hole collisions by $1/e$ in a distance $\delta = v_p\tau_+$, independent of the frequency. For bismuth with $\mathbf{k}$ in the trigonal direction $\delta \sim 10^{-2}$ cm for $\tau_+ \sim 10^{-9}$ sec. Landau damping from the electrons would lead to a decay length of about 5×10^{-3} cm in this same configuration at 10 Gc/sec.

All of the preceding discussion has been for spherical energy surfaces or for ellipsoidal surfaces with propagation along a principal axis. If we have an energy ellipsoid with axes labelled 1, 2, 3 with $\mathbf{k}$ in the 1-3 plane at an angle θ to axis 3, a straightforward modification of the usual spherical band results[7] gives for the components of the dielectric tensor in the ellipsoidal coordinate system

$$\begin{aligned} \epsilon_{11} &= (\bar{m}/m_1)(\cos^2\theta\ m_3^{-1}\epsilon'_t + \sin^2\theta\ m_1^{-1}\epsilon'_l) \\ \epsilon_{13} &= \epsilon_{31} = (\bar{m}/m_1 m_3)\cos\theta\sin\theta\,(\epsilon'_l - \epsilon'_t) \\ \epsilon_{33} &= (\bar{m}/m_3)(\cos^2\theta\ m_3^{-1}\epsilon'_l + \sin^2\theta\ m_1^{-1}\epsilon'_t) \end{aligned} \tag{11}$$

where

$$\begin{aligned} \epsilon'_l &= \frac{4\pi ne^2\tau}{i\omega(1+i\omega\tau)}\frac{3}{a^3}(a - \tan^{-1}a) \\ &\quad \times \left[1 - \frac{i}{\omega\tau a}(a - \tan^{-1}a)\right]^{-1} \\ \epsilon'_t &= \frac{4\pi ne^2\tau}{i\omega(1+i\omega\tau)}\frac{3}{2a^3}[(a^2+1)\tan^{-1}a - a] \\ \bar{m} &= (\cos^2\theta\ m_3^{-1} + \sin^2\theta\ m_1^{-1})^{-1} \\ a &= (v_{F3}^2\cos^2\theta + v_{F1}^2\sin^2\theta)^{\frac{1}{2}}\,k\tau/(1+i\omega\tau). \end{aligned} \tag{12}$$

For bismuth with $\mathbf{k}$ in the trigonal direction the total dielectric tensor is still diagonal and for $a \gg 1$ the electron contribution to the longitudinal component is still given by (10b), but with $k_{FT}^2 = 4\pi N(E_F)e^2$, where $N(E_F)$ is the total denisty of states at the Fermi surface of the electron ellipsoids. In determining the Landau damping $v_F = (v_{F3}^2\cos^2\theta + v_{F1}^2\sin^2\theta)^{\frac{1}{2}}$ should be used, with θ the tilt angle of the ellipsoids.

In general, for $\mathbf{k}$ not along a crystallographic axis of high symmetry, purely longitudinal waves will no longer exist. Although this introduces considerable algebraic complexity into the theory, the presence of transverse field components is useful since they greatly aid in excitation. It should also be mentioned that in bismuth, because of the existence of three electron ellipsoids, there will generally be a total of four plasma waves plus the two electromagnetic waves.

3. Excitation

Reflection or surface impedance measurements cannot be used to detect the existence of the acoustic plasma mode because the wave is excited so much more weakly than the transverse skin depth wave. However, transmission measurements with the sample mounted as a diaphragm in a waveguide should be feasible since the longitudinal wave decays quite slowly in comparison with the transverse wave (although on an absolute basis rather rapidly, as indicated above). Consider first a semimetal with spherical energy bands. An analysis assuming specular reflection at the two surfaces has been carried out for this case using techniques similar to those employed in treatments of the anomalous skin effect;[14] but in view of the recently published work of Platzman and Buchsbaum[15] on transmission of transverse waves through plasma slabs the details will not be presented. It is of course obvious that if we were dealing with a purely transverse wave incident on a slab of infinite transverse extent there would be no coupling to the longitudinal wave. However, in the actual problem coupling will occur as a result of both the waveguide boundaries and the finite transverse dimensions of the sample. In fact if the sample fills the entire cross section of the guide it is clear that a strictly longitudinal normal mode cannot exist, because of the boundary conditions at the waveguide walls, and hence the coupling does not vanish. On the other hand, if the sample is mounted on a frame and illuminated through an aperture, as if the case experimentally, then even a purely transverse and uniform incident wave wi!l couple because of the diffracted longitudinal waves that propagate at an angle to the normal.

We can determine the coupling in the second case by the following simple argument, which may be generalized to cover transverse waves as well. Expressing the power carried by the longitudinal acoustic wave in terms of the dielectric constant,[16] we have

$$P = -\frac{\omega}{16\pi}\frac{\partial\epsilon}{\partial k}\,|E|^2 = \frac{\omega}{8\pi}\frac{k_{FT-}^2}{k^3}\,|E|^2. \tag{13}$$

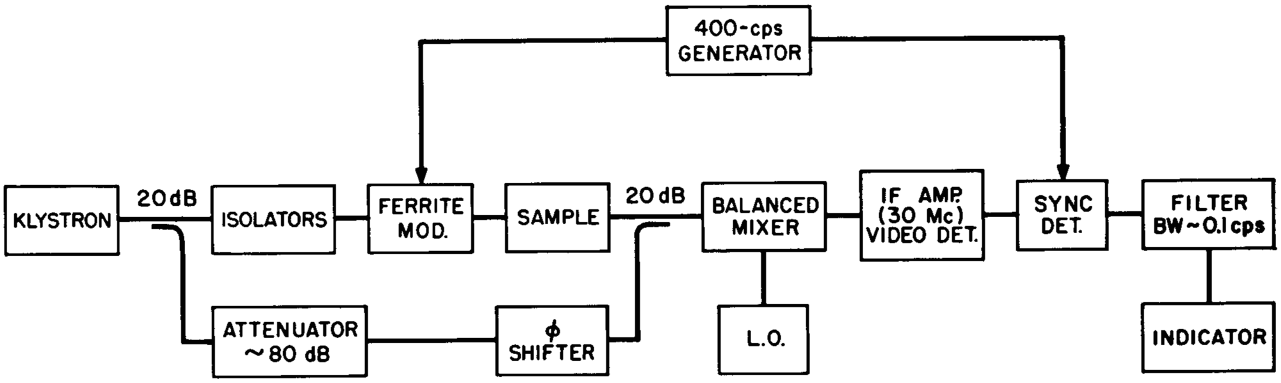


Figure 2. **Block diagram of microwave circuit for detecting transmitted signals of the order of 10^{-20} w.**

If **k** is in the *x-z* plane, with the normal to the surface in the *z*-direction, then the component of the Poynting vector at the surface which supplies this power is

$$\frac{c}{8\pi} E_x[H_y(k_x)]^* = \frac{c}{8\pi} E[H_y(k_x)]^* k_x/k, \tag{14}$$

where $H_y(k_x)$ is the Fourier component of H_y. Equating the two expressions gives

$$P(k_x) = \frac{c^2}{8\pi} \frac{k k_x^2}{\omega k_{FT-}^2} |H_y(k_x)|^2. \tag{15}$$

If the sample is of half-width L_x and uniformly illuminated, then $|H_y(k_x)|^2$ is proportional to $(\sin k_xL_x/k_xL_x)^2$.

It is easy to see that the above situation would lead to a very large diffraction of the transmitted wave and considerable destructive interference at the far surface of the sample. However, the actual case in bismuth is one of ellipsoidal energy surfaces. As mentioned in the preceding Section, one can easily produce a small admixture of transverse field components by simply orienting the sample slightly off of the trigonal axis. This can give a larger coupling than either mechanism considered above and one which is free from excessive diffraction effects. It is also possible to produce a mixed transverse-longitudinal wave by applying a small magnetic field.

4. Detection

A block diagram of the microwave circuit is shown in Fig. 2. The detection system employs a conventional microwave receiver and synchronous detector with one unusual feature: some of the unmodulated microwave signal is added to the modulated signal going to the mixer. This arrangement permits a signal of about 10^{-20}W to be detected with a one-second integration time. Equally important for this experiment, it can also be used to measure the phase shift (within multiples of 2π) through the sample. One can view the circuit either as converting the problem of detecting a weak signal into the problem of detecting a weak modulation or as a poor man's correlator (or matched filter). The first point of view is fairly obvious and the ability to detect 10^{-20}W or less is well known in paramagnetic resonance work.[17] However, it is rather instructive to see the circuit from the other aspect and appreciate why a small modulation is easy to detect.

The optimum procedure for extracting a signal of known waveform from white additive Gaussian noise consists simply of cross correlating the received signal plus noise with the known signal waveform.[18] This is also known as a matched filter and gives a signal-to-noise of $P_s\Delta t/kT_N$, where P_s is the average input signal power to the receiver, Δt is the integration time, and T_N is the noise temperature of the receiver. For a good *X*-band receiver with, say, a 7 db noise figure, $T_N \approx 1200°$K, and hence for $\Delta t = 1$ sec. $P_s \sim 10^{-20}$W for unity signal-to-noise. It will now be shown that the circuit of Fig. 2 is equivalent to an optimum linear filter.

Let $m(t)$ be the modulation waveform, ω_0 the IF frequency, and assume for simplicity that the detector is a square-law device (the nonlinearity is the only essential feature). Then the operation of detection, phase sensitive detection at $m(t)$, and low-pass filtering (integration) gives an output at time t proportional to

$$\int_{t-\Delta t}^{t} m(t')[m(t') \cos \omega_0 t' + A \cos \omega_0 t' + n(t')]^2 \, dt', \tag{16}$$

where $n(t)$ is the noise and $A \cos \omega_0 t$ is the large unmodulated, but properly phase shifted, signal added in at the receiver input. The term proportional to A^2 gives a negligible output after integration, so the dominant contribution is

$$2A \int_{t-\Delta t}^{t} m(t') \cos \omega_0 t' [m(t') \cos \omega_0 t' + n(t')] \, dt', \tag{17}$$

which is just the optimum filtering. Note that without the addition of $A \cos \omega_0 t$ cross terms of the noise with itself and with the signal would come through the low-pass filter and degrade the signal-to-noise in proportion to the square root of the ratio of the IF bandwidth to the low-pass filter bandwidth.

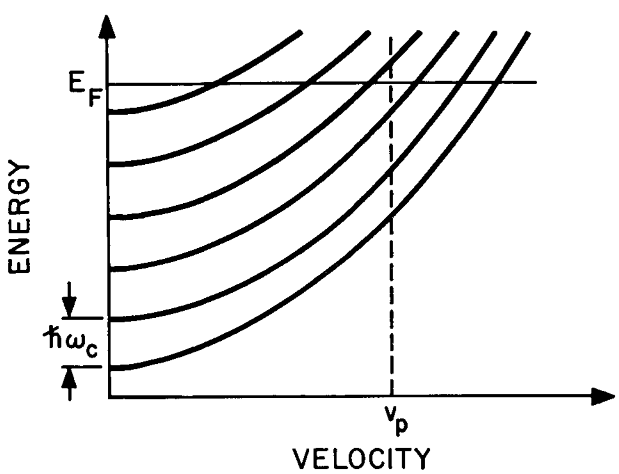
This same detection scheme can be used for waveforms other than pure sinusoids; the signal-to-noise will still be the optimum if non-additive sources of noise, such as gain fluctuations, are negligible. We have in fact achieved the theoretical limit with the above circuit.

5. Sample preparation and mounting

The requirements on the sample are rather severe. It should be about $1 \times 1 \times 0.01$ cm in size, work- and strain-free in order to minimize collision damping, with faces smooth and parallel to about 2 microns. Conventional grinding and polishing techniques cannot be used since bismuth is very soft. Success has been achieved by etching with glacial acetic acid and concentrated nitric acid on Dextilose paper in a manner similar to that used by Sullivan[19] for GaAs. The resulting surface is shiny, but slightly rough ($\sim$0.5 micron), and rather flat. Another technique which has been used is that of growing single-crystal slabs to size between microscope slides on a hot stage.[20] The surfaces are flat, but show some dimples that seem to come from adsorbed gas.

In order to avoid straining, the sample is mounted on a thicker oriented bismuth frame, which is soldered with a Bi-Cd solder onto a metal plate inserted into the waveguide. The metal structure is of type 310 stainless steel, which has the same total thermal expansion as bismuth[21] between 4.2° and 300°K.

Figure 3. **Diagram showing Landau level formation in a magnetic field, with the energy plotted against the velocity parallel to the magnetic field.** *Dotted line shows a value of the phase velocity v_p for which there will be no Landau damping.*



6. Discussion of results

No indication has been found thus far of a transmitted acoustic plasma wave, despite the use of both magnetic fields and crystallographic orientation to enhance the coupling. The difficulty may lie in a degradation of τ by the sample preparation or mounting procedure. A check at larger longitudinal magnetic fields showed that the effective τ for Alfvén wave propagation was only 5×10^{-11} sec, whereas the starting bismuth crystal had a residual resistivity ratio ($R_{300°}/R_{4.2°}$) of about 200, corresponding to a much larger τ. This problem is now being investigated.

A transmitted wave has been seen, however. It is very weak (190-200 db transmission loss), and over the frequency range 8.8 to 9.8 Gc/sec it has a phase velocity near 10^8 cm/sec, about an order of magnitude greater than that expected for the acoustic mode. More work will be required to make a precise measurement, but these preliminary results are suggestive of a wave travelling approximately at the Fermi velocity of the electrons. If this is so, two possibilities come to mind. One is the long, non-exponential tail on the transverse skin depth excitation, which was derived theoretically by Reuter and Sondheimer.[14] The other is a zero-sound wave, which Gorkov and Dzyaloshinskii[22] have recently discussed for metals. The latter showed that spin-dependent oscillations should exist for vanishingly small Fermi-liquid corrections, while spinless waves should exist in this limit if certain symmetry conditions are met, which apparently can be satisfied for propagation along the trigonal direction in bismuth. In this connection, it is worth mentioning that, although Gorkov and Dzyaloshinskii suggest infrared as the most feasible range in which to see zero-sound waves, the greater sensitivity and easier sample tolerances in the microwave region probably favor that part of the spectrum. An interesting question that is raised by this work is the effect of the Fermi-liquid correction on the non-exponential tail of the Reuter-Sondheimer solution.

7. Magnetic quantum effects

Classically or semiclassically there is no effect of a longitudinal magnetic field on longitudinal waves. However, quantum mechanically the field produces a series of one-dimensional Landau sub-bands, as shown in Fig. 3 for a simple parabolic band. If $\hbar\omega_c \gg kT$ and $\omega_c\tau \gg 1$, so that the Landau levels are well formed, one should expect giant oscillations as a function of magnetic field to occur in the Landau damping of the acoustic plasma wave, just as has been predicted[23] and recently seen[24] in ultrasonic attenuation in metals. The reason is simply that Landau damping will be present only if there are electron states at the Fermi surface with a velocity in the propagation direction equal to the phase velocity of the wave. For the case shown by the dotted line in Fig. 3 no attenuation can occur.

In addition to this effect it may be possible for a large longitudinal magnetic field to create one or more lightly damped acoustic modes in a one-carrier plasma, such as a metal. In a sense each Landau sub-band acts as a separate carrier; those with $v_F > v_p$ give a positive contribution to ϵ, while those with $v_F < v_p$ give a negative contribution, with the magnitude of ϵ becoming very large for $v_F \approx v_p$.[23] Hence, just as in a multicomponent plasma, there will be $(n - 1)$ acoustic plasma waves when there are n Landau sub-bands intersecting the Fermi surface, the phase velocities of the waves being interspersed between the sub-band Fermi velocities. Of course, for a nonzero temperature and a finite τ most of these waves will be strongly damped, but the ones with intermediate phase velocities are the most weakly damped and may be observable. As the applied field approaches zero, these modes become infinitely dense and go over into the van Kampen modes.[25] In fact this can be used as a physical argument for the existence of the van Kampen modes as an alternative to the usual electron beam approach.[26]

References and footnotes

1. L. Tonks and I. Langmuir, *Phys. Rev.* **33**, 195 (1929).
2. D. Pines, *Can. J. Phys.* **34**, 1379 (1956).
3. D. Pines and J. R. Schrieffer, *Phys. Rev.* **124**, 1387 (1961).
4. M. J. Harrison, *J. Phys. Chem. Solids* **23**, 1079 (1962).
5. I. Alexeff and R. Neidigh, *Phys. Rev. Letters* **7**, 223 (1961); *Phys. Rev.* **129**, 516 (1963).
6. A. Y. Wong, N. D'Angelo and R. W. Motley, *Phys. Rev. Letters* **9**, 415 (1962).
7. J. Lindhard, *Dan. Mat. Fys. Medd.* **28**, No. 8 (1954).
8. H. Ehrenreich and M. Cohen, *Phys. Rev.* **115**, 786 (1959).
9. L. D. Landau, *J. Phys. USSR* **10**, 25 (1946).
10. It should be noted that an acoustic plasma wave can result from two carriers of the same charge, even from carriers in the two valleys of the same band.
11. L. D. Landau, *Soviet Phys.—JETP* **5**, 101 (1957).
12. This analogy in the form of (5) to zero sound in He^3 is not to be confused with the possibility of real zero sound, as discussed in Sec. 6.
13. The same result may also be deduced from the value of the plasmon-electron and plasmon-hole coupling coefficients given by Pines,[2] since quantum mechanically the collision damping may be described in terms of a two-step process involving a single-particle excitation and simultaneous scattering.
14. G. E. H. Reuter and E. H. Sondheimer, *Proc. Roy. Soc.* (*London*) **A195**, 336 (1949).
15. P. M. Platzman and S. J. Buchsbaum, *Phys. Rev.* **132**, 2 (1963).
16. W. P. Allis, S. J. Buchsbaum and A. Bers, *Waves in Anisotropic Plasmas*, MIT Press, Cambridge, Mass., 1963, Ch. 8.
17. G. Feher, *Bell Syst. Tech. J.* **36**, 449 (1957).
18. See, for example, W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise* McGraw-Hill Book Co., New York, 1958, p. 244.
19. M. V. Sullivan, *J. Electrochem. Soc.* **110**, 585 (1963).
20. A. N. Friedman and S. H. Koenig, *IBM J. Res. Develop.* **4**, 158 (1960).
21. E. F. Cave and L. V. Holroyd, *J. Appl. Phys.* **31**, 1357 (1960).
22. L. P. Gorkov and I. E. Dzyaloshinskii, *Soviet Phys.—JETP* **17**, 1111 (1963).
23. V. L. Gurevich, V. G. Skobov and Yu. A. Firsov, *Soviet Phys.—JETP* **13**, 552 (1961); J. J. Quinn and S. Rodriguez, *Phys. Rev.* **128**, 2487 (1962).
24. Y. Shapira and B. Lax, *Phys. Rev. Letters* **12**, 166 (1964).
25. N. G. van Kampen, *Physica* **21**, 949 (1955).
26. J. M. Dawson, *Phys. Rev.* **118**, 381 (1960).

Discussion

R. Bowers: How are you proposing to couple transverse waves to the longitudinal acoustic plasma wave?

A. L. McWhorter: I am sorry that I did not have time to discuss this point, but it is covered in the written form of the paper. First of all, except when the propagation is exactly along a symmetry direction, an acoustic plasma wave in bismuth will actually have transverse components to which an external transverse wave can directly couple. In addition, in a finite sample the boundary conditions will produce transverse components, or one can apply a small magnetic field to mix the transverse and longitudinal waves.

D. Pines: Can one detect the existence of the acoustic plasma wave by using ultrasonic attenuation?

McWhorter: I do not think so, but I have not made any calculations. The coupling between the acoustic plasma wave and the sound wave should be small because of the large difference in their velocities.