

Alfvén Wave Propagation in Bismuth: Quantum Oscillations of the Fermi Surface

Abstract: Alfvén wave propagation in single crystal bismuth has been studied as a function of magnetic field to 105 kilogauss for frequencies between 13 and 18 kMc/sec. Small deviations from a linear dependence of wave velocity on magnetic field are found and are interpreted as quantum oscillations in the mass density of carriers. Theory and experiment are compared for the magnetic field along a bisectrix axis and along a two-fold axis. Good agreement is found for the first case, but significant discrepancies exist for the magnetic field along a two-fold axis.

Introduction

Alfvén waves are magnetohydrodynamic waves which propagate in a magnetic field in a plasma of high conductivity consisting of equal numbers of mobile positive and negative carriers. Their existence and properties were first predicted by Alfvén.¹ Experiments to verify this prediction have been attempted in gaseous plasmas^{2,3} and in liquid metals.^{3,4,5} In both cases there are many experimental difficulties which prevent an accurate comparison between theory and experiment. A compensated semi-metal such as bismuth or antimony provides a very well defined medium in which the propagation of Alfvén waves can be studied and theory and experiment compared, as has been proposed by Buchsbaum and Galt.⁶ The first direct observation of Alfvén wave propagation in a solid was by Williams.⁷ Other work on bismuth has been reported by Kirsch and Miller,^{8,9} Khaikin et al.,¹⁰ and Bartelink.¹¹ Alfvén wave propagation in antimony has been observed by Williams.¹² It is believed that the present experiments provide the clearest experimental observation of Alfvén waves, and the best fit between theory and experiment of any of the attempts to observe Alfvén wave behavior.

In the present paper we emphasize deviations from the simple theory for Alfvén waves, and attribute these deviations to periodic variations in the total number of carriers in bismuth with magnetic field. Other papers will describe in more detail the study of Alfvén waves in bismuth and

antimony,¹³ both from the viewpoint of verifying that true Alfvén wave behavior is being observed and also in order to obtain information about the band structures of these materials.

Theory

A simple theory which illustrates the properties of Alfvén waves in a completely classical situation is presented here. This treatment appears in many places in the literature.^{14,15,16} The equation of motion for n carriers of charge e and effective mass m^* in a magnetic field $\mathbf{H}$ is

$$nem^* d\mathbf{v}/dt = ne^2(\mathbf{E} + \mathbf{v} \times \mathbf{H}/c) - nem^* \mathbf{v}/\tau. \quad (1)$$

Here $nev = \mathbf{J}$, the current density, τ is the scattering time, and the other symbols have their usual meaning. If, for simplicity in the final result, we take $\mathbf{H}$ to be in the z direction and circularly polarized microwave radiation propagating along the z axis, this equation may be solved for the current $\mathbf{J}$. Since $\mathbf{J} = \hat{\sigma} \cdot \mathbf{E}$, we can obtain the tensor conductivity $\hat{\sigma}$, and thus the effective dielectric constant

$$\epsilon_{\text{eff}} = \epsilon_l - 4\pi i \hat{\sigma} / \omega. \quad (2)$$

When one obtains the conductivity from equation (1) under these conditions, there follows

$$\epsilon_{\text{eff}} = \epsilon_l - \frac{4\pi i n e^2 \tau / m^* \omega}{1 + i(\omega - \omega_c) \tau}. \quad (3)$$

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If $(\omega - \omega_c)\tau \gg 1$, one obtains

$$\epsilon_{eff} = \epsilon_l + \omega_p^2 / \omega(\omega_c - \omega), \quad (4)$$

where ω_p is the classical plasma angular frequency, given by

$$\omega_p^2 = 4\pi n e^2 / m^*, \quad (5)$$

for the single group of carriers under consideration at the moment.

For low magnetic fields and microwave frequencies $\omega_p \gg \omega$ and $\omega_p \gg \omega_c$, and the lattice dielectric constant is negligible; that is, the displacement current can be neglected. When $\omega_c = \omega$ the effective dielectric constant changes sign. At this point the material passes from metallic behavior, with a negative dielectric constant and thus an imaginary refractive index, to dielectric behavior, with a positive dielectric constant and a real refractive index. Of course, the point at which this transition occurs is the cyclotron resonance frequency as studied by Galt et al.¹⁴

If one increases the magnetic field, so that $\omega_c > \omega$, one finds that electromagnetic radiation can propagate in the medium. For a compensated material such as bismuth with equal numbers of electrons and holes this is an Alfvén wave, as pointed out by Buchsbaum and Galt.⁶ The effective dielectric constant for a medium with equal numbers of electrons and holes can be calculated using the results of Eq. (4). If one takes a term like the second term in Eq. (4) for each group of carriers, the result is

$$\epsilon_{eff} = \epsilon_l + \frac{\omega_{pe}^2}{\omega(\omega_{ce} - \omega)} + \frac{\omega_{ph}^2}{\omega(\omega_{ch} - \omega)}, \quad (6)$$

where ω_{ce} and ω_{ch} are the cyclotron frequencies of electrons and holes and ω_{pe} and ω_{ph} are the electron and hole plasma frequencies. Thus $\omega_{pe}^2 = 4\pi n_e e^2 / m_e^*$, where n_e is the number of electrons and m_e^* their effective mass. Also, $\omega_{ce} = -eH/m_e^*c$ and $\omega_{ch} = +eH/m_h^*c$. The present treatment assumes isotropic masses.

For $\omega_c > \omega$, one can expand the denominators in Eq. (6) to obtain

$$\epsilon_{eff} = \epsilon_l + \frac{\omega_{pe}^2}{\omega\omega_{ce}} \left\{ 1 + \frac{\omega}{\omega_{ce}} + \dots \right\} + \frac{\omega_{ph}^2}{\omega\omega_{ch}} \left\{ 1 + \frac{\omega}{\omega_{ch}} + \dots \right\} \quad (7)$$

which yields, when the values of ω_{pe}^2 , ω_{ph}^2 , ω_{ce} , and ω_{ch} are substituted,

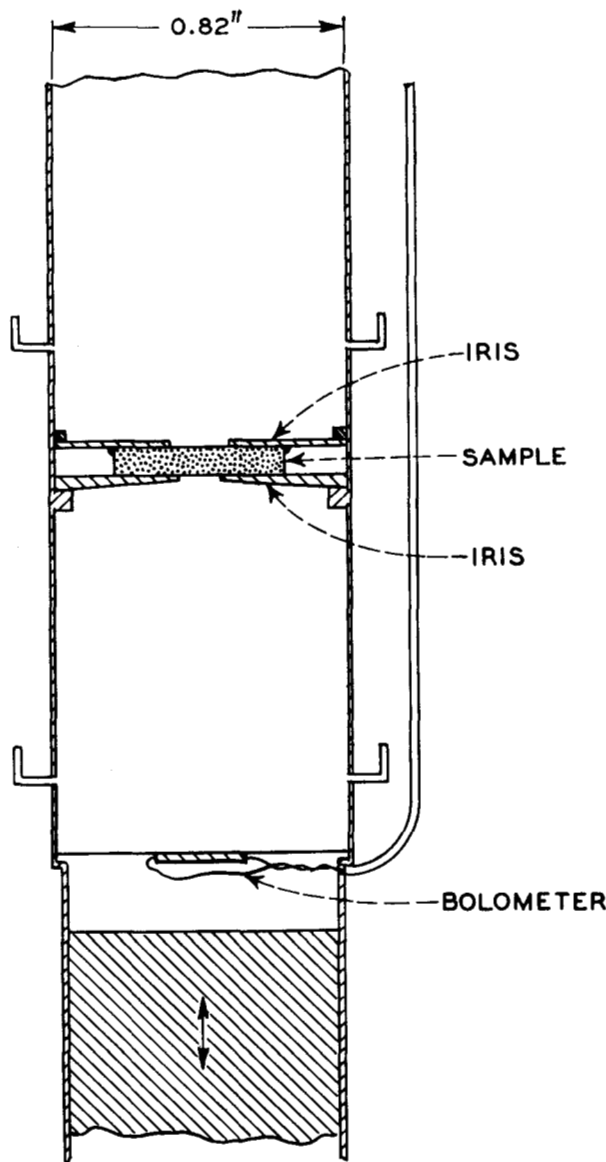
$$\epsilon_{eff} = \epsilon_l + \frac{4\pi c e}{H\omega} (n_h - n_e) + \frac{4\pi c^2}{H^2} (n_e m_e^* + n_h m_h^*). \quad (8)$$

For a compensated material, the second term, which describes Helicon wave behavior,¹⁷ vanishes. The third term is c^2/v_a^2 , where v_a is the Alfvén wave velocity, given by

$$v_a^2 = H^2 / 4\pi(n_e m_e^* + n_h m_h^*). \quad (9)$$

Alfvén has proposed a simple mechanical model for the behavior of these waves.^{1,18} Obviously for an anisotropic material such as bismuth the mass density expression, $(n_e m_e^* + n_h m_h^*)$, will be more complex, but the general physical model holds and, in particular, the linear dependence of effective dielectric constant on $1/H^2$ and its independence of frequency remain. These are the two features which the propagation of the observed wave must show if it is to be considered an Alfvén wave.

Figure 1 Schematic diagram of the sample holder arrangement. The entire region was evacuated and operated at an exchange gas pressure of He of approximately 20 μ Hg.



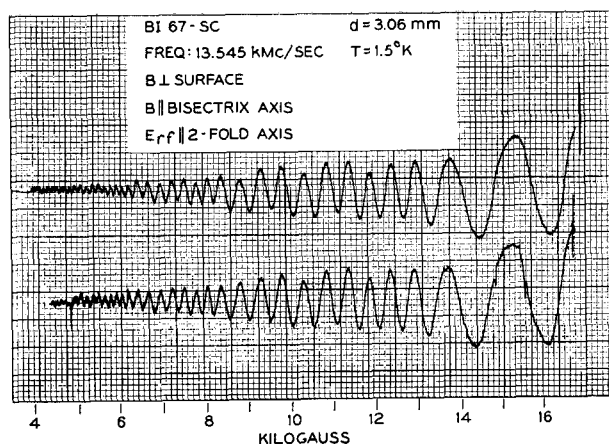


Figure 2 Interference fringes at 13.54 kMc/sec in a sample of bismuth with the magnetic field normal to the surface and parallel to a bisectrix axis. These are "leakage" fringes—interference between the micro-waves leaking around the sample and those which have passed once through the sample (see text).

Experimental

Figure 1 shows the experimental arrangement for the Alfvén wave experiments on bismuth. In all of the experiments described in this work the magnetic field is normal to the sample surface, but experiments have also been performed with the magnetic field parallel to the sample surface.⁷

Microwave radiation is incident on the sample surface from above. 12–18 kMc/sec was chosen for the microwave frequency in order to have $\omega\tau > 1$ but still allow $\omega_c > \omega$ to be reached at reasonable magnetic fields. The radiation transmitted through the sample is detected using a bolometer cut from a carbon resistor similar to that used by Boyle and Rodgers¹⁹ in the infrared. The microwaves were amplitude modulated at 27 cps, and the bolometer signal fed to a 27 cps, Baird Atomic coherent detector. A more detailed discussion of the experimental arrangement will be published elsewhere.¹³

We find interference fringes, or intensity maxima, in the signal transmitted, which can be called geometric resonances or Fabry-Perot type of interference fringes. The optical thickness of the sample varies with magnetic field and thus a series of fringes are observed. These fringes are formed by radiation reflected back and forth between the plane parallel faces of the sample. Also a certain amount of microwave power leaks around the sample, and leads to an interference pattern with the signal which has made one pass through the sample. These fringes have

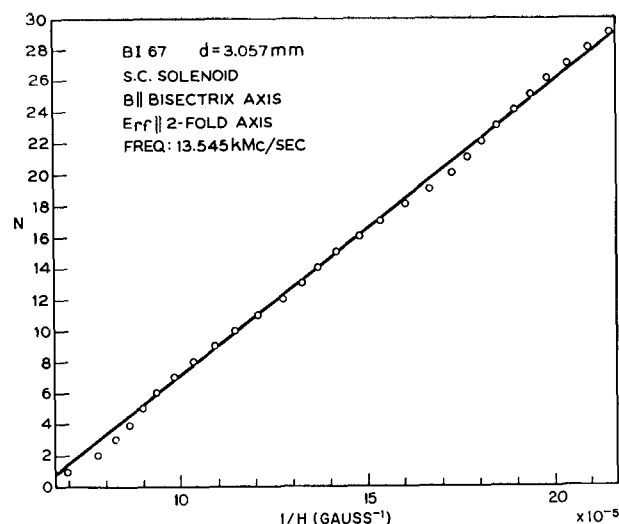


Figure 3 Plot of fringe index as a function of $1/H$ for the data of Fig. 2. The solid line is a least squares line through the data points.

twice the period of the Fabry-Perot fringes and cause little confusion. They will be called "leakage" fringes in this paper.

The optical thickness of the sample is calculated directly from the refractive index η , which is equal to $\epsilon_{\text{eff}}^{1/2}$. If $\epsilon_l \ll c^2/v_a^2$, then $\eta = c/v_a$ where v_a , the Alfvén velocity for an anisotropic medium, is given by $v_a = H/(4\pi m_0)^{1/2} [nf(m^*)]^{1/2}$. Here m_0 is the free electron mass and $f(m^*)$ is a function of the effective masses in units of the free electron mass which can be calculated from the magnetoconductivity tensor.¹⁸ $nf(m^*)$ is the mass density for the anisotropic case which replaces the simple expression in Eq. (8).

The interference relationship is $N\lambda_0 = 2d\eta$, where λ_0 is the free space wavelength and N the fringe index, or number of half wavelengths in a sample thickness. The $\cos \theta$ term usually found in such a formula is set equal to 1 because in a medium of very high refractive index the propagation is normal to the sample surface. Then we find

$$N = (2d/\lambda_0)(4\pi m_0)^{1/2}(c/H)[nf(m^*)]^{1/2}, \quad (10)$$

or

$$N = (A/H)[nf(n^*)]^{1/2}, \quad (11)$$

where A contains all of the constants. Therefore one expects that if fringe index N is plotted as a function of

$1/H$ a straight line should be obtained whose slope is $A [nf(m^*)]^2$. For some of these experiments⁷ excellent straight line behavior is found. We also find that the effective refractive index has no frequency dependence, and thus true Alfvén wave behavior is demonstrated.

Figure 2 shows typical experimental data taken with the magnetic field normal to the sample surface and parallel to a bisectrix axis. Arbitrarily the peak at highest field is assigned a fringe index of one, and fringe index is plotted as a function of $1/H$. This is shown in Fig. 3. Here is found the first indication of the deviations that are of interest in this paper. Small deviations from straight line behavior can be seen. This orientation of crystal axis with respect to the magnetic field results in the largest magnitude of this effect. The precision of the data is actually better than that indicated in Fig. 3. To demonstrate this, a least-squares straight line was drawn through the points in Fig. 3 and the vertical deviations of the data points from straight-line behavior was plotted as a function of $1/H$. This is shown in Fig. 4. A smooth, although complex, curve can be drawn through these points.

To account for this behavior it has been assumed that the mass density, $nf(m^*)$, is varying with magnetic field in an oscillatory way, related to the de Haas - van Alphen effect. However, these experiments measure a carrier mass density, which will be sensitive to changes in the position of the Fermi surface, rather than changes in the density of states at the Fermi surface. The theory which Smith

et al.²⁰ presented earlier at this conference yields directly the value of the Fermi energy as a function of magnetic field for bismuth, and therefore can be used to predict mass densities for comparison with these experiments.

In order to compare the experimental data with the theory, Eq. (11) is rearranged to yield:

$$nf(m^*) = (NH/A)^2. \quad (12)$$

One should note that this assumes a knowledge of the absolute fringe index. For the first set of data to be shown, this has been chosen to fit the shape of the highest field data. An error in this index will not affect the conclusions to be drawn in a major way. Its predominant effect is to shift the data curves vertically without changing their shape, except at the highest fields.

Fig. 5 shows the calculated Fermi energy for bismuth with the magnetic field parallel to the bisectrix direction, as a function of $1/H$, as calculated using the program of Smith et al.²⁰ There are four major peaks in the Fermi energy before it drops drastically at high magnetic fields. From this can be obtained a mass density as a function of magnetic field. The calculation includes the effects of non-parabolic bands. The calculated mass densities are compared in Fig. 6 with those obtained from the data using Eq. (12). Two theoretical curves are given. One shows the calculated mass densities for Smith's set of parameters, set 52, and the second for set 53, a small modification of set 52. The vertical fit is not significant,

Figure 4 Plot of the deviations of the data points in Fig. 3 from the least squares line as a function of $1/H$. Deviations are measured in fractions of a fringe.

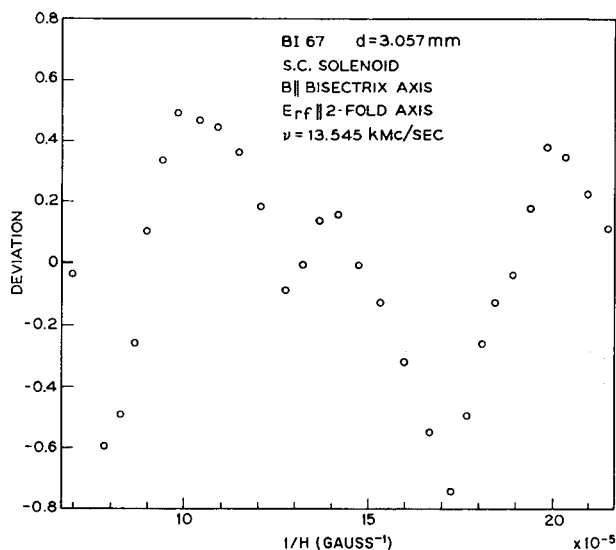


Figure 5 Plot of calculated Fermi energy as a function of $1/H$ for the magnetic field parallel to a bisectrix axis.

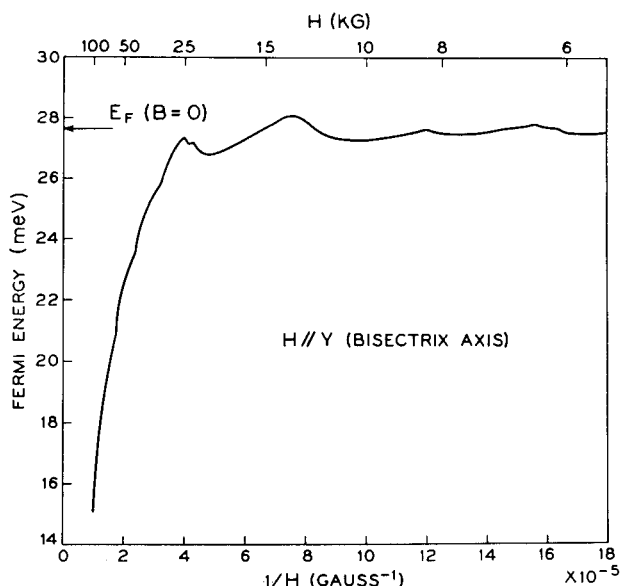
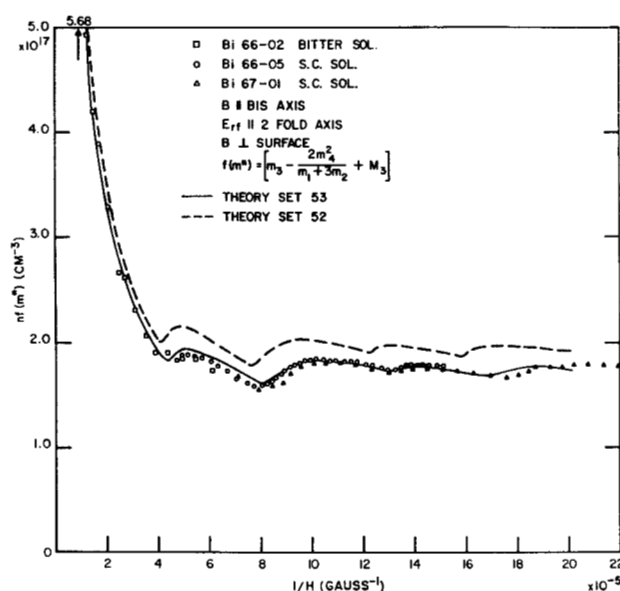


Table 1 Summary of parameters in sets 52 and 53 used to calculate the theoretical curves in Fig. 6. For the electron bands the effective mass at the top of the band is related to that at the bottom by the relation $m_{\text{top}} = [1 + (2E_F / E_G)] m_{\text{bot}}$. For details of the calculation, see Reference 20.

Effective masses (m_0 as unit)		
Electrons:	bottom of band	top of band
	$m_1 = 0.00113$	0.00521
	$m_2 = 0.26$	1.20
	$m_3 = 0.00443$	0.0204
	$m_4 = -0.0195$	-0.090
Holes:	$M_1 = 0.064$	
	$M_3 = 0.69$	

	Set 52	Set 53
$E_G = 15.3 \times 10^{-3}$ eV	14.4×10^{-3} eV	
$E_F = 27.6 \times 10^{-3}$ eV	26.0×10^{-3} eV	
$E_0 = 38.5 \times 10^{-3}$ eV	36.0×10^{-3} eV	
$n_h = 2.75 \times 10^{17}$ cm $^{-3}$	$n_h = 2.55 \times 10^{17}$ cm $^{-3}$	

Figure 6 Plot of experimental and theoretical mass density for the magnetic field parallel to a bisectrix axis. The high field data was taken in a Bitter solenoid, the low field data in a superconducting solenoid. The solid and dotted curves are calculated using the theory of Smith and parameter sets 52 and 53. The data points are calculated using Eq. (12). The vertical fit is not significant because of the uncertainty in absolute fringe index as discussed in the text.



because of the uncertainty in fringe index, but the fit in the position of the valleys in the mass density curves is. Table 1 summarizes the parameters used in these sets. On the basis of the data for this orientation, it seems that the fluctuations can be accounted for as a mass density fluctuation associated with variation in the position of the Fermi surface as a function of magnetic field.

With the magnetic field parallel to a two-fold axis of symmetry it is found that Alfvén waves with the $\mathbf{E}$ field of the microwaves parallel to either the three-fold or the bisectrix axis are suitable for handling in this manner. With the magnetic field along the bisectrix direction, only one of the two possible waves is convenient. The other has a very long period, and is harder to analyze. Figure 7 shows data to 105 kilogauss with the magnetic field parallel to a two-fold axis. These are pure Fabry-Perot type fringes. The data with the microwave field parallel to the three-fold axis and the $\mathbf{E}$ field parallel to the bisectrix axis is also good, and is not shown. Figure 8 is a plot of fringe index as a function of $1/H$ for the data of Fig. 7, and also for data at another frequency. The peaks at $1/H = 2.0 \times 10^{-5}$ (50 kilogauss) are at the same field for both frequencies. This permits the statement that the fringe indices for these two points must be in the ratio of the two frequencies, which is 14:11 (19:15 and 9:7 are other possible choices). Mass densities for these three possible choices of absolute fringe index will be plotted in the data figures.

Figure 9 shows the Fermi energy as a function of $1/H$ for the magnetic field parallel to a two-fold axis. Two major peaks occur before it falls off at high fields. Figure 10 gives the number of carriers in the individual bands as obtained in this calculation. The important point to note here is that the number of electrons in the band N_3^e goes down at high field. This is a general feature of the model and does not depend on the specific parameters chosen.

Figure 11 is a plot similar to Figure 6 of mass density as a function of $1/H$, for the magnetic field parallel to a two-fold axis and the microwave electric field parallel to a three-fold axis. The data are plotted for three choices of fringe index at the highest field ($N_1 = 3, 7, 12$) as discussed above. The theoretical mass density is the solid line. At high magnetic fields, the finite lattice dielectric constant should be included in the theoretical curve, and for $\epsilon_l = 100$ the actual curve would follow the dashed line. The data curve for $N_1 = 3$ can be ruled out on these grounds. The other two data curves have similar shapes, and rise at high fields while the theoretical curve drops until about 50 kilogauss. The shape of the theoretical curve is determined by the fact that the number of carriers in band N_3^e is dropping, and this band contributes the term $\frac{1}{3} m_2$, which is the dominant term in the mass density for this orientation.

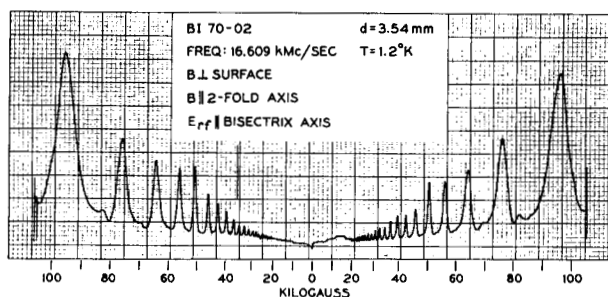


Figure 7 Fabry-Perot type interference fringes in bismuth at 16.61 kMc/sec with the magnetic field parallel to a two-fold axis.

Figure 8 Plot of fringe index as a function of $1/H$ for the data of Fig. 7 and also data at 13.04 kMc/sec.

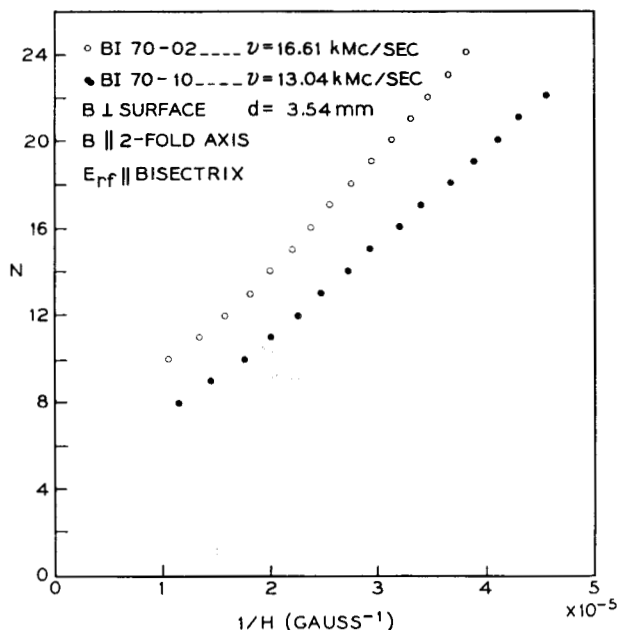


Figure 12 is a plot of mass density as a function of $1/H$ for the magnetic field along a two-fold axis and the microwave electric field parallel to a bisectrix axis. Again three choices of absolute fringe index are shown. Ruling out the choice $N_1 = 5$ as before, one is left with $N_1 = 10$ or 15. These have similar shapes, which do not agree with the valleys of the theoretical curve. In fact, if one stretches a point, there are indications of broad humps where the valleys should occur.

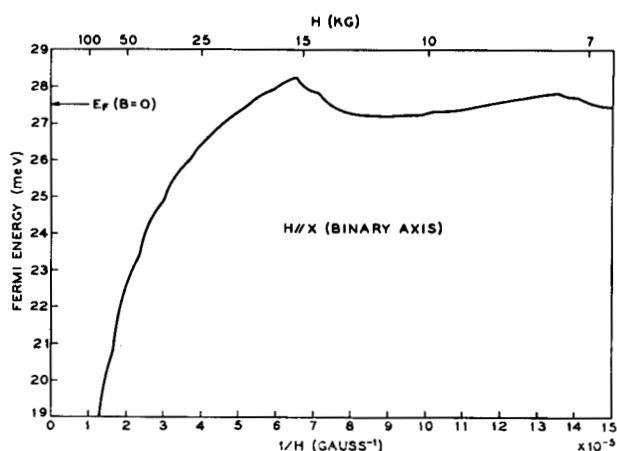
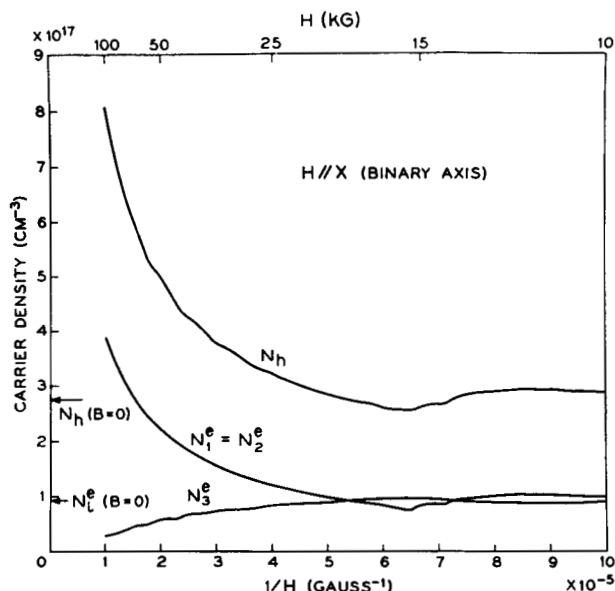


Figure 9 Plot of calculated Fermi energy as a function of $1/H$ for the magnetic field parallel to a two-fold axis.

Figure 10 Plot of the number of carriers in each of the bands in bismuth vs $1/H$ for the magnetic field parallel to a two-fold axis.



Summary and Conclusions

It is found that with the magnetic field along a bisectrix axis the valleys in the mass density curve can be accounted for using a two band non-parabolic model for the variation in Fermi energy with magnetic field. It is felt that the basic fact is established, that the deviations from straight line behavior in the Alfvén wave plots are due

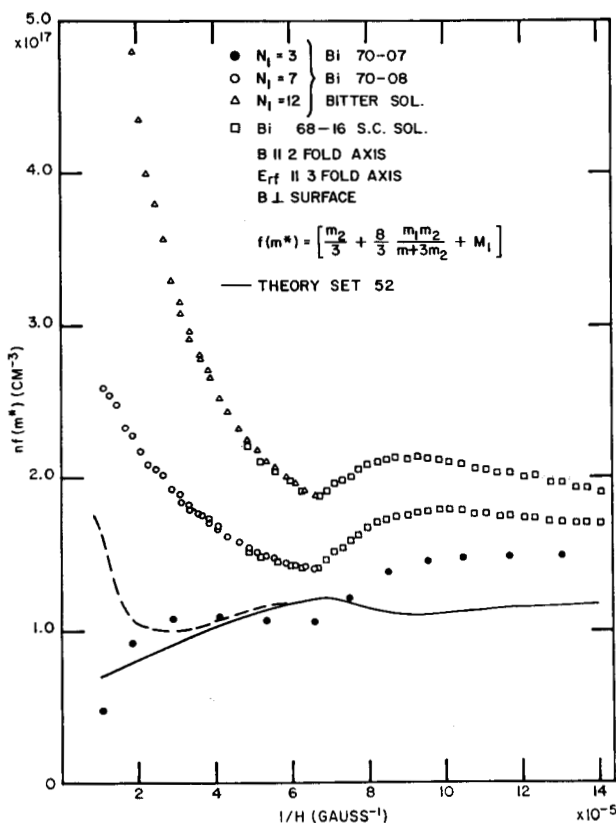


Figure 11 Plot of experimental and theoretical mass density for bismuth with the magnetic field parallel to a two-fold axis and the microwave electric field parallel to a three-fold axis. The solid curve is calculated using Smith's theory. The dotted curve indicates the modification of this curve at high magnetic field for an assumed lattice dielectric constant $\epsilon_1 = 100$. The three sets of data points are plotted for the choices shown of fringe index at highest field.

to quantum fluctuations in the Fermi surface leading to fluctuations in the mass density, both because n changes and also because the electron masses change since the bands are not parabolic.

However, when the magnetic field is along a two-fold axis, theory and experiment do not agree, and the disagreement is not of a character to be resolved by small changes in the parameters used. Since these parameters have been used to fit very detailed de Haas - van Alphen data,²⁰ no gross variations are allowed.

Several possible ways of accounting for this difficulty are under consideration. Because of the good fit for the bisectrix data, it is believed the method of analysis is valid. The postulation of a third band, the famous heavy holes, which might lie just at or above the Fermi surface

at zero field is being studied as one way out of the difficulty. One can also consider various complicated effects, such as field dependence of the overlap or gap energies or the band shape. Any such ideas, however, must not destroy the agreement between the present model of the band structure and other data on bismuth.

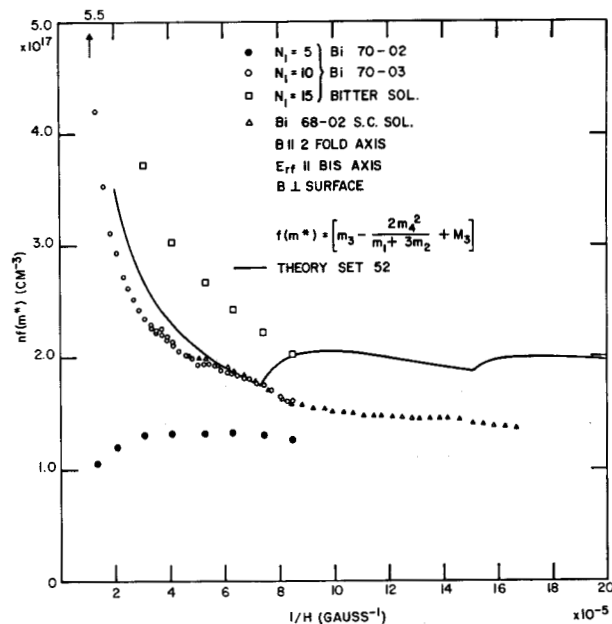
Acknowledgments

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Figure 12 Plot of experimental and theoretical mass density for bismuth with the magnetic field parallel to a two-fold axis and the microwave electric field parallel to a bisectrix axis. The solid curve is calculated from theory using parameter set 52. The three sets of points are plotted for the choices shown of fringe index at the highest field.



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Discussion

Unidentified questioner: What fraction of the incident microwave energy was converted to Alfvén waves, and what was the wavelength of the latter?

G. A. Williams: The fraction converted depends upon the magnetic field. Except at the highest fields, reflection coefficients are large and the conversion is quite small. At low fields, it is a question of reflection from a material whose effective dielectric constant is about 10,000. At the highest magnetic field there were 5 to 10 half wavelengths in a sample 3 mm thick. The wavelength varies linearly with magnetic field.

M. H. Cohen: Was the departure of the energy bands from parabolicity taken into account not only in the theory of the Fermi energy and concentration but also in the theory of the

dielectric constant of the charge carriers?

G. E. Smith: In calculating the mass density of the carriers, nonparabolicity and the resulting change of the effective mass were taken into account.

R. N. Dexter: To what extent did you force the theoretical fit to your data by assuming that no helicon term contributed?

Williams: The low-field data says that these samples are compensated to within a percent or so. That is to say, there is no frequency dependence to the Alfvén velocity. At high fields it would be more difficult to prove that point from the data. Because of curvature in the experimental curves at high fields, it is more difficult to compare data at various frequencies.